

**EVIDENCE OF TIME-DIVERSIFICATION IN MALAYSIA: AN EMPIRICAL
STUDY ON THE RELATIONSHIP BETWEEN RETURN AND INVESTMENT
TIME HORIZON**

by

TAN ENG LAM

**Research report in partial fulfillment of the requirements
for the degree of Masters of Business Administration**

May 2006

ACKNOWLEDGEMENTS

I would like to take this opportunity to extend my appreciation to Associate Professor Datin Dr. Ruhani Ali, of the School of Management, University Science Malaysia, for providing me with her invaluable guidance and supervision to pursue on this research study. Along the development of this report, she has read the scripts and identified several problem areas. Her experience and knowledge assisted me in accomplishing this research study.

Last but not least, my heartfelt appreciations to my lovely wife, Goh Sing Sing, for her full support, understanding and encouragement throughout the course of my study.

TABLE OF CONTENTS

	<u>Page</u>
Title page	i
Acknowledgements	ii
Table of Contents	iii
List of Tables	v
List of Figures	vi
Abstrak	vii
Abstract	viii
Chapter 1: INTRODUCTION	1
1.1 Introduction	1
1.2 Problem Statement	2
1.3 Research Objectives and Questions	4
1.4 Significance of the Study	4
1.5 Organization of Report	6
Chapter 2: LITERATURE REVIEW	7
2.1 Conceptual Foundation	7
2.2 Time Diversification	9
2.2.1 <i>Time Diversification Underlying by Expected Utility Theory</i>	9
2.2.2 <i>Time Diversification Underlying by Option Pricing Theory</i>	11
2.2.3 <i>Time Diversification with Mean Reversion Evidence</i>	13
2.3 Theoretical Framework and Hypothesis Development	15
Chapter 3: REASERCH METHODOLOGY	16

3.1 Research Methodology and Measurement of Variables	16
3.2 Part 1 Analysis for Hypothesis Testing and Mean Reversion	16
3.3 Part 2 Analysis on Time Diversification	23
3.4 Sample Data	26
Chapter 4: RESULTS	27
4.1 Introduction	27
4.2 Descriptive Statistics	27
4.3 Part 1 Analysis Results for Hypothesis Testing and Mean Reversion	28
4.4 Part 2 Analysis Result on Time Diversification	37
Chapter 5: CONCLUSION AND RECOMMENDATIONS	39
5.1 Recapitulation and Discussion	39
5.2 Conclusion	40
5.3 Implication of the Study	41
5.4 Limitation of the Study	42
5.5 Suggestions for Further Studies	42
REFERENCES	44
APPENDICES	48

LIST OF TABLES

	<u>Page</u>
Table 4.1 Descriptive Statistics	28
Table 4.2 Log (Volatility) vs Log (Investment Time Horizon) Linear Regression Results	35
Table 4.3 Log (Sharp Ratio) vs Log (Investment Time Horizon) Linear Regression Results	36

LIST OF FIGURES

	<u>Page</u>	
Figure 4.1	KLCI – Daily (4 Jan 1999 to 30 Dec 2005)	29
Figure 4.2	KLCI Log Returns – Daily (4 Jan 1999 to 30 Dec 2005)	29
Figure 4.3	Term Structure of Volatilities	30
Figure 4.4	Term Structure of Returns	31
Figure 4.5	Risk-Return Trade-Off	32
Figure 4.6	Term Structure of Sharpe Ratios	33
Figure 4.7	Optimized Risky Investment Allocation (α^*) vs Investment Time Horizon under constant Relative Risk Aversion Assumption	38

ABSTRAK

Penyelidikan ini bertujuan untuk mengenalpasti sama ada pemelbagaian masa wujud di Pasaran Saham Malaysia. Tumpuan penyelidikan ini adalah pemelbagaian masa dengan menggunakan Indeks Komposit Kuala Lumpur dari tahun 1999 sehingga tahun 2005. Penyelidikan ini merangkumi 1712 data Indeks Komposit Kuala Lumpur. Bahagian pertama kajian bertujuan untuk mengenalpasti sama ada pasaran saham mengikut perjalanan rambang dan kemeruapan dan Nisbah Sharpe dapat ditentukan dengan menggunakan aturan '*Square Root of Time*'. Sepanjang kajian ini, bukti untuk 'putaran purata' akan juga diperhatikan. Sejumlah 1.45 juta data log pulangan telah dihasilkan, untuk pelbagai jangka masa pelaburan, yang merangkumi 1 sehingga 1518 hari. Bahagian kedua kajian menggunakan '*Mean-Varian Analysis*' dan bertujuan untuk menentukan bahagian nisbah untuk pelaburan ekuiti yang dapat memaksimumkan utiliti dengan tanggapan rintangan risiko yang malar. Bahagian kajian ini bertujuan meninjau kesahihan pemelbagaian masa dalam pasaran ekuiti Malaysia. Hasil kajian menunjukkan pasaran ekuiti Malaysia tidak mengikuti perjalanan rambang dan dengan sedemikian, aturan '*Square Root of Time*' tidak dapat dipakai. Didapati bukti yang menampirkan kemungkinan wujud 'putaran purata' dengan kitaran urusanniaga dalam lingkungan 4.6 ke 5.3 tahun. Selain daripada itu, terdapat bukti yang menyokong kesahihan pemelbagaian masa di dalam pasaran ekuiti Malaysia. Pelabur boleh meraih keuntungan yang lebih tinggi dengan risiko rendah untuk jangka panjang pelaburan di dalam lingkungan 1300 ke 1518 hari. Tiada data yang mencukupi untuk meneruskan kajian bagi pelaburan jangka masa melebihi 1518 hari.

ABSTRACT

This study is an empirical study on the validity of time diversification in the Malaysian equity market. This study focuses exclusively on time diversification using Kuala Lumpur Composite Index (KLCI) data, from 1999 to 2005. A total of 1712 KLCI data were included in this study. The first part of this study attempts to determine whether the market follows random walk and hence can utilize the Square Root of Time Rule to predict the volatility and Sharpe Ratio. Along the first part analysis, evidence of mean reversion will be observed. A total of 1.45 million log return data were generated, corresponding to various investment time horizons, ranging from 1-day to 1518-days. The second part of this study utilized Mean-Variance Analysis and attempts to determine the equity investment allocation ratio that will maximize one's utility under constant risk aversion. This part of the study will indicate the validity of time diversification in the Malaysian equity market. The results showed that Malaysian equity market does not follow random walk and hence the Square Root of Time Rule does not apply. There are evidences that indicate the possible presence of mean reversion in the market with business cycle estimated between 4.6 to 5.3 years. Furthermore, there are evidences to support the validity of time diversification in Malaysian equity market. Investors with long time horizon can capitalize on higher equity gain at reduced risk with 1300-days to 1518-days investment time horizon. There are not enough data to support analysis with investment time horizon beyond 1518-days.

REFERENCES

- Aminuddin, A.B. (1994). Risk-return Relationship in KLSE Stocks. *University of Science Malaysia*.
- Athanassakos, G. (1997). Investing for Retirement and the Risk of Long Holding Periods (Time Diversification). *Canadian Investment Review*, 10(1), 43-50.
- Barnes, P. (1986). Thin Trading and Stock Market Efficiency: The Case of the Kuala Lumpur Stock Exchange. *Journal of Business Finance & Accounting*, 13(4), 609-617.
- Black, K. (2001). *Business Statistics: Contemporary Decision Making*. South-Western College.
- Bodie, Z. (1995). On the Risk of Stocks in the Long Run. *Financial Analysts Journal*, 51(3), 18-22.
- Bodie, Z., Kane, A., and Marcus, A.J. (2005). *Investments*. McGraw-Hill.
- Butler, K.C., and Damian, D.L. (1991). Risk, Diversification, and the Investment Horizon, *Journal of Portfolio Management*, 17(3), 41-47.
- Chaudhuri, K., and Wu, Y. (2003). Mean Reversion in Stock Prices: Evidence from Emerging Markets. *Managerial Finance*, 29(10), 22-37.
- Cochran, S.J., and DeFina, R.H. (1994). International Evidence on Mean Reversion in Stock Prices. *Quarterly Journal of Business and Economics*, 33(2), 79-85.
- Connelly, T.J. (1996). The Time Diversification Controversy. *Journal of Financial Planning*, 19(1), 20-23.
- Dempsey, M., Hudson, R., Littler, K., and Keasey, K. (1996). On the Risk of Stocks in the Long Run: A Resolution to the Debate? *Financial Analysts Journal*, 52(5), 57-62.
- Dempsey, M.J. (2002). The Nature of Market Growth, Risk, and Return. *Financial Analysts Journal*, 58(3), 45-59.
- Elton, E.J., Gruber, M.J., Brown, S.J., and Goetzmann, W.N. (2003). *Modern Portfolio Theory and Investment Analysis*. John Wiley & Sons.
- Evensky, H. (1997). Time Diversification. *Financial Planning*, 27(2), 53-60.
- Fama, E. and French, K. (1988). Permanent and Temporary Components of Stock Prices. *Journal of Political Economy*, 96, 246-273.
- Ferguson, R., and Leistikow, D. (1996). On the Risk of Stock in the Long Run: A Comment. *Financial Analysts Journal*, 67-68.

- Fisher, K.L., and Statmen, M (1999). A Behavioral Framework for Time Diversification. *Financial Analysts Journal*, 55(3), 88-97.
- Gallati, R.R. (2003). 15.433 Investments: Class2: Securities, Random Walk on Wall Street. Retrieved February 20, 2006, from Massachusetts Institute of Technology, MIT's OpenCourseWare website:
<http://ocw.mit.edu/OcwWeb/Sloan-School-of-Management/15-433InvestmentsSpring2003/LectureNotes/index.htm>
- Hakim, S. and Naeime, S. (2000). Mean-reversion across MENA stock markets: implications for portfolio allocations. Retrieved February 20, 2006, from Economic Research Forum for the Arab Countries's website:
<http://www.erf.org.eg/html/bfinance4.pdf>
- Hansson, B., and Persson, M. (2000). Time Diversification and Estimation Risk. *Financial Analysts Journal*, 55-62.
- Harold Bierman, JR. (1997). Portfolio Allocation and the Investment Horizon. *Journal of Portfolio Management*, 23(4), 51-55.
- Hodges, C.W., Taylor, W.R.L. and Yoder, J.A. (1997). Stocks, Bonds, the Sharpe Ratio, and the Investment Horizon. *Financial Analyst Journal*, 53(6), 74-80.
- Hodges, C.W., Taylor, W.R.L., and Yoder, J.A. (2002). The Pitfalls of Using Short-interval Betas for Long-run Investment Decisions. *Financial Service Review* 11, 85-95.
- Holton, G.A. (1992). Time: The Second Dimension of Risk. *Financial Analysts Journal*, 48(6), 38-45.
- Howe, T.S., and Mistic, D.L. (2003). Taxes, Time Diversification, and Asset Choice at Retirement. *Journal of Economics and Finance*, 27(3), 404-421.
- IImanen, A., Byrne, R., Gunsekera, H., and Minikin, R. (2004). Which Risks Have Been Best Rewarded? Duration, Equity Market, and Short-Dated Credit Risk. *The Journal of Portfolio Management*, 53-57.
- Jaggia, S., and Thosar, S. (2000). Risk Aversion and Investment Horizon: A New Perspective on the Time Diversification Debate. *The Journal of Psychology and Financial Markets*, 1(3&4), 211-215.
- Johnson, S., Kochhar, K., Mitton, T., and Tamirisa, N. (2006). Malaysian Capital Controls: Macroeconomics and Institutions. *International Monetary Fund (IMF) 2006 Working Paper*. Retrieved February 26, 2006, from:
<http://www.imf.org/external/pubs/ft/wp/2006/wp0651.pdf>
- Jorion, P. (2003). The Long-Term Risks of Global Stock Markets. *Financial Management*, 32(4), 5-26.

- Kochman, L., and Goodwin, Randy. (2001). Updating the Case against Time Diversification: A Note. *The Mid-Atlantic Journal of Business*, 37(2/3), 139-141.
- Kritzman, M. (1993). What Practitioners Need to Know About Return and Risk. *Financial Analysts Journal*, 14-17.
- Kritzman, M. (1994). What Practitioners Need to Know About Time Diversification. *Financial Analysts Journal*, 50(1), 14-18.
- Kritzman, M., and Rich, D. (1998). Beware of Dogma. *Journal of Portfolio Management*, 24(4), 66-77.
- Lanjong, N. (1983). Efficiency of the Malaysian Stock Market and Risk-Return Relationship. Unpublished Ph.D. Thesis, Leuven.
- Lee, W.Y. (1990). Diversification and Time: Do Investment Horizons Matter? *The Journal of Portfolio Management*, 21-26.
- Lee, Y.S. (1998). Optimum Portfolio: Return, Risk and Performance. *University of Science Malaysia*.
- Levy, H. (1984). Measuring Risk and Performance over Alternative Investment Horizons. *Financial Analysts Journal*, 61-68.
- Levy, H., and Spector, Y. (1996). Cross-Asset versus Time Diversification. *Journal of Portfolio Management*, 22(3), 24-34.
- Levy, H., and Cohen, A. (1998). On the Risk of Stocks in the Long Run: Revisited, *Journal of Portfolio Management*, 24(3), 60-68.
- Madhusoodanan, T.P. (1997). Time Diversification: The Indian Evidence. *Journal of Financial Management & Analysis*, 10(2), 11-22.
- McQueen, G. (1992). Long-Horizon Mean-Reverting Stock Prices Revisited. *Journal of Financial & Quantitative Analysis*, 27(1), 1-18.
- Merrill, C., and Thorley, S. (1996). Time Diversification: Perspectives from Option Pricing Theory. *Financial Analysts Journal*, 52(3), 13-19.
- Milevsky, M.A. (1999). Time Diversification, Safety-First and Risk. *Review of Quantitative Finance and Accounting*, 12(3), 271-281.
- Milevsky, M.A.(2002). Space-time Diversification: Which Dimension is better? *Journal of Risk*, 5(2), 27-45.
- Mukherji, S. (2002). Stocks, Bonds, Bills, Wealth, and Time Diversification. *Journal of Investing*, 11(2), 39-52.

- Olsen, R.A., and Khaki, M. (1998). Risk, Rationality, and Time Diversification. *Financial Analysts Journal*, 54(5), 58-63.
- Poterba, J.M., and Summers, L.H. (1998). Mean Reversion in Stock Prices: Evidence and Implications. *Journal of Financial Economics*, 22(1), 27-33.
- Ratner, M., Arbelaez, H., and Leal, R. (1997). A Time Diversification Approach to the Emerging Stock Markets of Latin America: Argentina, Brazil, Chile, and Mexico. *The International Executive*, 39(5), 707-725.
- Reichenstein, W., and Rich, S.P. (1994). Predicting Long-Horizon Stock Returns: Evidence and Implications. *Financial Analysts Journal*, 50(1), 73-76.
- Return document. (n.d.). Retrieved February 8, 2006, from:
<http://www.riskglossary.com/link/return.htm>
- Samuelson, P.A (2001). Lifetime Portfolio Selection by Dynamic Stochastic Programming. *The Review of Economics and Statistics*; 239-246.
- Samuelson, P.A (1994). The Long-Term Case for Equities: and How It Can Be Oversold. *Journal of Portfolio Management*, 21(1), 15-24.
- Samuelson, P.A (1990). Asset Allocation Could Be Dangerous to Your Health: Pitfall in Across-Time Diversification. *Journal of Portfolio Management*, 16(3), 5-8.
- Samuelson, P.A. (1989). The Judgment of Economic Science on Rational Portfolio Management: Indexing, Timing, and Long-horizon effects. *Journal of Portfolio Management*, 16(1), 4-12.
- Sing, T.F., Liow, K.H. and Chan. W.J. (2002). Mean Reversion of Singapore Property Stock Prices towards their Fundamental Values. *Journal of Property Investment & Finance*, 20(4), 374-385.
- Strong, N., and Taylor, N. (2001). Time Diversification: Empirical Tests. *Journal of Business Finance & Accounting*, 28(3&4), 263-302.
- Taylor, R., and Brown, D.J. (1996). On the Risk of Stocks in the Long Run: A Note. *Financial Analysts Journal*, 69-71.
- Thorley, S.R. (1995). The Time-Diversification Controversy. *Financial Analysts Journal*, 68-76.
- Van Eaton, R.D., and Conover, J.A. (1998). Misconceptions about Optimal Equity Allocation and Investment Horizon. *Financial Analysts Journal*, 54(2), 52-59.
- Volatility document. (n.d.). Retrieved February 8, 2006, from:
<http://www.riskglossary.com/link/volatility.htm>

CHAPTER 1	1
1.1 INTRODUCTION	1
1.2 PROBLEM STATEMENT	2
1.3 RESEARCH OBJECTIVES AND QUESTIONS	4
1.4 SIGNIFICANCE OF THE STUDY	4
1.5 ORGANIZATION OF REPORT	6
CHAPTER 2	7
2.1 CONCEPTUAL FOUNDATION	7
2.2 TIME DIVERSIFICATION	9
2.2.1 <i>Time Diversification Underlying by Expected Utility Theory</i>	9
2.2.2 <i>Time Diversification Underlying by Option Pricing Theory</i>	11
2.2.3 <i>Time Diversification with Mean Reversion Evidence</i>	13
2.3 THEORETICAL FRAMEWORK AND HYPOTHESIS DEVELOPMENT	15
CHAPTER 3	16
3.1 RESEARCH METHODOLOGY AND MEASUREMENT OF VARIABLES	16
3.2 PART 1 ANALYSIS FOR HYPOTHESIS TESTING AND MEAN REVERSION	16
3.3 PART 2 ANALYSIS ON TIME DIVERSIFICATION	23
3.4 SAMPLE DATA	26
CHAPTER 4	27
4.1 INTRODUCTION	27
4.2 DESCRIPTIVE STATISTICS	27
4.3 PART 1 ANALYSIS RESULTS FOR HYPOTHESIS TESTING AND MEAN REVERSION	28
4.4 PART 2 ANALYSIS RESULTS FOR TIME DIVERSIFICATION	37
CHAPTER 5	39
5.1 RECAPITULATION AND DISCUSSION	39
5.2 CONCLUSION	40
5.3 IMPLICATION OF THE STUDY	41
5.4 LIMITATIONS OF THE STUDY	42
5.5 SUGGESTIONS FOR FURTHER STUDIES	42

CHAPTER 1

INTRODUCTION

1.1 Introduction

The primary principle which modern portfolio theory is based on is the random walk hypothesis, which simply states that the movement of asset prices follows an unpredictable path. This path is referred to as a trend that is based on long-term nominal growth of corporate earnings per share, but fluctuations around the trend are random. However, there are in general three forms of the hypothesis, namely weak, semi-strong and strong form. Within these different market forms, the modern portfolio theory recommends that diversifying the security investments across different classes of security or asset is a method to reduce the risk bared. When financial resources are scarce for investment, an investor has to decide the strategy to allocate the resources on various types of investment vehicle, such as bond, treasury-bills, equity and so on. However, the decided diversification strategy is bounded to a specific investment horizon. The decision-making becomes harder when the effect of investment time horizon on the investment gain is not known. Considering one-year versus ten or twenty year investment horizon, how would this impact the decision made when ultimately, what an investor is looking for is to maximize the investment gain.

Investment practitioners in the west commonly recommended that an investor with a long investment horizon, for instance someone saving for retirement or pension, tilt their portfolio toward stocks and away from fixed income securities (Hansson & Persson, 2000). Most practitioners take this view as given that the longer an investor holds risky assets, such as stocks, the more investor will benefit from what is often

called as time diversification. Time diversification is usually defined as above-average returns tend to offset below-average returns over long investment time horizon (Kritzman, 1994; Madhusoodanan, 1997). The point underlying time diversification is if equity returns are independent from one investment period to another, then the losses from low return periods will be offset by high return periods.

Indeed, Thorley (1995) has pointed out most practitioner-oriented research assumes the validity of time diversification and concerns itself with measuring its economic significance. However, research of a more academic nature (Bodie, 1995; Samuelson, 1994), has repeatedly challenged the validity of time diversification. The academicians have generally used economic models and theory based on risk aversion and expected utility to reject the time diversification notion as a logical fallacy.

Thus, an important practical question that financial theory should address is how the investment horizon affects investment allocation decision-making. Given the huge quantity of research and definitive answers in other areas of investments, such a basic question remains unresolved is surprising (Thorley, 1995). There have been many researches done on this topic within the western investment context. However, relatively there are few researches done on the same issue within the Malaysian investment market.

1.2 Problem Statement

In the Malaysia context, there have been many studies done related to risk-return relationships of various Kuala Lumpur Stock Exchange (KLSE) main board stocks (Aminuddin, 1994; Lee, 1998). These studies examine return, risk and performance relationship of selected stocks to determine possible optimum portfolio. These studies suggested some insights on asset allocation and that one should

diversify across different asset classes. However, any asset diversification strategy pertains to a given investment time horizon (Madhusoodanan, 1997). Hence, it is important to study whether is it enough to just diversify across different classes of equity asset or security, or if investment time horizon also plays a key role that affects the risk-return trade-off. In addition, the significant role of investment time horizon in diversification strategy varies with different countries. Jorion (2003) has shown the different characteristics of different countries in his 2003 paper. Many researches have been done to determine the applicability of time diversification in various countries such as USA, Latin Americas and India (Butler & Domian, 1991; Lee, 1990; Madhusoodanan, 1997; Ratner, Arbelaez & Leal, 1997). These researches were done with recognition that the asset allocation decision is the important financial decision facing individual investors and also observing more and more corporate pension, saving plans and even individual insurance planning are making shift from defined benefits to defined contributions. Individuals are increasingly having to execute the critical decision of how to allocate their retirement savings, insurance with investment-linked between risky equity and risk-free investments such as bond funds or fixed-income savings.

The above background information and findings suggest a continued need to study and understand the situation in Malaysia whether or not time diversification is applicable and how will the time affect the equity investment return. Numerous studies have been carried out on time diversification concept in the other countries, as shown in literature review. However there is lack in similar work done in the Malaysian context. This understanding will be very important for the Malaysian investors. For this reason, the research using Bursa Malaysia Main Board Kuala Lumpur Composite Index (KLCI) is proposed.

1.3 Research Objectives and Questions

The focus of this research is to examine whether investment time horizon is applicable in diversifying asset investments in Malaysia. In order to better understand the issue, the following objectives are formulated: -

1. To study the characteristics of Malaysia's equity returns over different investment horizon and if there is mean reversion over time. Specifically:

The study will look at the characteristics of:

- a. Term structure of volatility,
- b. Term structure of return,
- c. Term structure of Sharpe Ratio,
- d. Risk-return trade-off pattern.

The study will also look at the characteristics of:

- a. Equity investment allocation changes over time horizon under constant relative risk aversion assumption.

Bearing on the above objectives, this study attempts to answer the following questions:

1. Does Malaysia's equity market exhibit random walk?
2. Does Malaysia's equity market show mean reversion evidence over time?
3. Does Malaysia's equity market show evidence of time diversification applicability?

1.4 Significance of the Study

From investors' point of view, investors would like to understand whether they can utilize different investment time horizon to diversify their asset investments

in Malaysia effectively. It is hoped that the result of the study will bring significant meaning to investors in the following ways:

1. In investment community, using time horizon to reduce risk has been a common believe and practice, hence, this study is significant to address issues relating to this belief.
2. The result of the study may provide insight, in particular, to brokerage houses and investment services that when providing data, such as systematic risk, should take time horizon into consideration and hence should offer an array of data based on alternative horizons.
3. To provide reference to investment-linked life insurance policy holders when determining preference for their policy's investment risk profile. Young life insurance policy holders have normally had very long period of policy premium to pay. With investment-linked featuring low, medium and high risk investment profile; with high risk profile having more allocation in equity investment; this study will provide reference to these young policy holders to determine their choice of risk profile.
4. To provide reference to younger generation in early retirement planning. Young people has long investment time horizon. This study can serve as reference that will be beneficial to them as to whether they should plan for more allocation in equity investment for long period horizon.

In summary, the findings can provide useful reference that effectively affects the portfolio allocation of individual investor, private investors, corporate investors and mutual funds investors in the Malaysian stock markets.

1.5 Organization of Report

The background and purpose of the study are provided in Chapter 1. The remaining chapters are organized as follows. Chapter 2 covers the previous related researches, theoretical framework and hypotheses developed. Chapter 3 reviews the research methodology, data collection criteria and statistical analysis methods. Chapter 4 tabulates and analyzes results obtained and verifies stated hypotheses. Chapter 5 discusses the result, states the limitation of study, proposes potential future research area and concludes the findings of this study.

CHAPTER 2
LITERATURE REVIEW

2.1 Conceptual Foundation

The long debated time diversification subject have been documented in many literatures such as Bodie (1995), Butler and Domian (1991), Dempsey, Hudson, Littler and Keasey (1996), Kritzman (1993, 1994), Kritzman and Rick (1998), Lee (1990), Levy and Cohen (1998), Merrill and Thorley (1996), Samuelson (1989, 1990, 1994), Taylor and Brown (1996), and Thorley (1995). This debate is primarily between the academicians and investment practitioners. For risky asset such as equity, investors are concern with the diversification strategies they should adopt. For investors who has long time horizon such as planning their retirement, optimizing asset allocation to achieve maximum return at minimize risk over long period of horizon is their ultimate aim. In particular, investors would like to know whether it is sufficient to consider diversification only in terms of across different classes of assets, or the investment time horizon also affects the risk and return of an investment portfolio (Madhusoodanan, 1997).

Holton (1992) has pointed out that if the stock price is assumed to follow random walk time series and the returns are independent and are identically distributed from one period to the next, the annualized stock return can be derived by multiplying the number of trading days in the year to daily returns, while the stock returns volatility will increase with time and the proportionality constant is the square root of time. Following the same logic, the annualized volatility can be estimated from daily volatility by multiplying it with square root of the number of trading days.

$$\sigma_{annualized} = n \times \sigma_{daily} \quad \text{-----} \quad (2.1)$$

$$\sigma_{annualized} = \sqrt{n} \times \sigma_{daily} \quad \text{-----} \quad (2.2)$$

Where,

Π = stock return;

σ = stock return volatility;

and n = number of trading days in year.

This result actually originated from Albert Einstein's Brownian motion of movement of particles study (Gallati, 2003; Madhusoodanan, 1997). The key point is if the actual volatility increase is not according to equation (2.2), it can be regarded as deviation from the random walk time series assumptions. This property is commonly known as **Square Root of Time Rule** (Volatility document (n.d.). Retrieved February 8, 2006, from <http://www.riskglossary.com/link/volatility.htm>). When this happen, it has several ramifications to the investment business. The most important implication is the applicability of time diversification concept. According to Madhusoodanan (1997), Sharpe ratio is regarded as an ideal measurement to examine the pattern in risk-return trade-off. Sharpe ratio measures the return per unit of risk and it links equation (2.1) and (2.2) directly. Hence, it is important to study the characteristic measure of volatility and Sharpe ratio. The significance of the study is it will discover if Square Root of Time Rule is followed and hence implied if the random walk theory applied. Following to this, it leads to conclusion about time diversification applicability.

In the mean time, over the last decade, an increasing interest in the discussion on mean reversion has been witnessed. Initiated from the work of Poterba and Summers (1988) and Fama and French (1988) who documented mean reversion in stock market returns during time horizon greater than one year. According to Madhusoodanan (1997), the mean reversion is shown in a time series of a stock

returns, in which if the series exhibit high return in a period and revert back to low return in the following period or vice versa. Following to that, many researches have been done to analyze the implications of their findings on the efficient market hypothesis, such as Jorion (2003), Kritzman (1994), Madhusoodanan (1997), Mukherji (2002), Sing, Liow and Chan (2002), and Thorley (1995). One thing for sure, the mean reversion property has significant implications for optimal asset allocations and hence, link to important aspect of time diversification.

2.2 Time Diversification

According to Evensky (1997), the original formation of time diversification is attributed to Peter Bernstein, whose two basic premises were:

“The longer the investment horizon, the larger the percentage of the portfolio that should be invested in stock and other high-return assets. In the long run, an investor can be reasonably sure that a higher volatility portfolio will earn more than a lower volatility portfolio (p. 54).”

Kritzman (1994) denotes that time diversification is the phenomenon of when above-average returns likely to reduce the effect or cancel out the effect of below-average returns over long time horizon. From practical standpoint, it implies that investor who invests over long investment time horizon has less likelihood of losing money compare to investor who invests for short investment time horizon.

2.2.1 Time Diversification Underlying by Expected Utility Theory

One of the all time famous academicians, whom does not believe and support time diversification, Paul A. Samuelson, has written many landmark articles about this subject (Samuelson, 1989; 1990; 1994). Under three conditions, Samuelson has

shown that investor who intends to maximize the expected utility, should not move more allocation to risky investment assets on the basis of their time horizon. These conditions are, firstly, if investors have constant relative risk aversion, which means that they maintain the same percentage exposure to risky assets regardless of changes in wealth. Secondly, if investment returns follow a random walk or in other words, they are independent and identically distributed. The third condition is if future wealth depends only on investment results and not on human capital or consumption habits.

However, there have been several other researchers, particularly investment practitioners, whom have addressed and disagree with the theoretical arguments against time diversification. Among them, Kritzman (1994), Kritzman and Rich (1998), Levy and Spector (1996), and many others.

Kritzman (1994) denotes what Samuelson has derived to against time diversification is a mathematical truth, if the assumptions hold. He pointed out in real life situation, an investor may not believe risky assets follow a random walk pattern. If assets returns demonstrated mean reverting process, then *“the terminal wealth dispersion will increase at slower rate than implied by lognormal distribution”* (p. 17). Hence, a rational investor, who is more risk averse than log wealth, will increase the exposure to risk when investment horizon expanded. In his following paper, Kritzman and Rich (1998) highlighted that many of the critics to Samuelson are not encouraged by the mathematical truth, but on the grounds that annualized returns volatility decrease with time and the probability of loss also reduce with time. On the other hands, Kritzman and Rich also recognized that there are also many researches dwelled too much unnecessary details in the meaning of risk and measurement of risk. Instead, they used the pedagogical tool of binomial trees to demonstrate the impact of horizon on expected utility and objectively showed one’s preference for risky asset.

They concluded if returns are independently and identically distributed, then the annualized return standard deviation will diminish as the time horizon expanded. Two other mathematical truths they concluded are:

“The probability of loss for positive expected return assets diminishes with time, and the dispersion of terminal wealth increases with time (p.71).”

However, Kritzman and Rich iterated that individual’s perceived risk is really depending on individual perception. Another important finding from them is under a mean reverting process, one who is more averse to risk than the degree of risk aversion implicit in a log wealth utility function, will be led to favor risky assets over a long horizon, regardless if one is indifferent between a riskless and a risky asset over a short horizon. This finding strongly support the advocates of time diversification as the historical evidence of stocks mean reversion is clearly shown in Fama and French (1998); Poterba and Summers (1988).

2.2.2 Time Diversification Underlying by Option Pricing Theory

Bodie (1995) presented a new angle to look into this issue. With novel approach, Bodie indicated that if investing in common stocks is less risky if the investment is held over a long period of time, than the cost of insuring against earning less than risk-free rate of interest should reduce as the investment time horizon expanded. Bodie used option pricing theory to demonstrate his point. He showed that the level of risk in stocks increases rather than decreases with the length of time horizon. He claimed the result is held both under the assumption of random walk and mean-reverting process for stock returns. However, many scholars have expressed their disagreement to Bodie, both from the academic as well as the investment practitioners.

Taylor and Brown (1996) argued that no research is presented to indicate this worst-case pattern, pattern used as an example in Bodie (1995), has happened. Furthermore, they disagree with the simplification done when using Black-Scholes model. They claimed that the assumption of constant one-period standard deviation will ensure the result Bodie desired. They showed that with different holding period and the equivalent standard deviation, the cost per dollar insured actually declined.

Merill and Thorley (1996) are in favor of the application of option pricing methodology in time diversification debate because the derived prices are independent of any specific model of investor utility or risk aversion. They noted option pricing theory provides quantifiable cost associated with the elimination of specific market risk. However, they disagree with Bodie's conclusion to rule out time diversification. They pointed out Bodie managed to show the insured cost increased over longer period but failed to point out that it is increased in less-than-proportional, considering the equity returns, on average, increase at much higher rates of about nine percent. At this rate, the value of equity investment to be insured in 10th year will be more than twice of the value in the first year on the average. As a result, the authors reiterate that the fair cost of equity insurance on a per annum and per value insured basis, is much lower for longer period commitment.

Dempsey, Hudson, Littler and Keasey (1996) joined the time diversification using option pricing discussion bandwagon. Their research result is against Bodie as well. Dempsey et al. (1996) argued that Bodie's put option prices can not be regarded as a representation of market risk measurement. The reason for this is the price for put option is not just an indicative of market risk, but also for an extra market feature, the market's reward for risk that an insurance writer on a stock can expect to gain.

Levy and Cohen (1998) intend to close the gap between the findings of Bodie (1995) and Merrill and Thorley (1996). Levy and Cohen proved that methodology of using options to measure risk is apparently dependent on the part of distribution taken into consideration. They pointed out Bodie essentially considers only the left-hand side or downside of the distribution while Merrill and Thorley considered only the right-hand side. Levy and Cohen's analysis were done by taking into consideration the whole distribution of returns with integration of the time-value of money concept. The authors stated it is possible that the put option value indeed increases with the investment horizon, but the mean return also increases, and if the whole return distribution is considered, all risk avoiders may prefer the distribution of return corresponding to the longer investment horizon despite the fact that the put value increases when horizon lengthen.

2.2.3 Time Diversification with Mean Reversion Evidence

Undoubtedly, both the investment practitioners and researchers of a more academic nature, agreed that if stocks returns show indication of mean reversion, than validity of time diversification is applicable. Works, like Samuelson and Kritzman, are more like a simulated research on returns that are assumed perfectly random and lognormal. On the other hand, works done related to mean-reversion are mainly driven by historical evidence.

Since the work of Fama and French (1998) and Poterba and Summers (1988), who documented mean reverting characteristic in stock market returns for time horizon more than a year, there have been many researches witnessed to investigate the implications of their findings on the efficient market hypothesis and the relevance to time diversification. Among them are Chaudhuri and Wu (2003); Jorion (2003);

Kritzman (1998); Madhusoodanan (1997); Samuelson (1994); Sing, Liow and Chan (2002); Strong and Taylor (2001); and Thorley (1995). Thorley in his 1995 paper has devised the advantages and shortcomings of different methodology in analyzing time diversification, other than option pricing theory, as Bodie only introduced that method in late 1995. Thorley has made detail review on practitioner risk measures, mean-variance optimization, expected utility theory, and methodology utilizing historical data. In his comment using historical data, Thorley noted that even an investor with constant relative risk aversion would allocate more portion of his investment to equity market upon knowing the historical equity return trend. Hakim and Neaime (2000) acknowledged the mean reversion property has significant effects in optimizing asset allocations. The mean reversion is defined as bad returns are likely to be followed by periods of good returns in a stock market. They did a study on stock markets of Middle East and North Africa areas. They showed that the volatility of stocks is dampened by a high speed of reversion. They recommended this result should be fully utilized by investors to employ tactical asset allocation strategies and especially when investment horizon is one of the elements in consideration.

Madhusoodanan (1997) states that majority of previous analysis on diversification strategies are based on the risk-return trade-off of different asset classes. He indicates that past arguments against time diversification holds only in perfect efficient market conditions. Evidences from Indian equity market however shows that its market is not perfectly efficient and sign for mean reversion also existed. Along this line of thought, he set to test the validity of time diversification in Indian market. With 18 years of daily Bombay stock index data, he studied the term structure of volatility, risk-return trade-off, mean-variance analysis, and Sharpe ratios. The results reveal that Indian market does not follow random walk, hence head

towards market inefficiency indication. The research results also show sign of mean reversion. From these results, he concluded evidence of time diversification is presence in India. Thus investment risks can be reduced with time diversification.

2.3 Theoretical Framework and Hypothesis Development

Studies on Malaysian market efficiency have shown conflicting results, evidenced from risk-return relationship analysis by Lanjong (1983), and Barnes (1986). These analyses ignored the time horizon factor. Their results have shown weak-form efficiency in Malaysian market. These evidences of market inefficiency indicate the risk-return is deviating from the theoretical predictions. Hence, in other words, indication to deviation from random walk hypothesis. These suggest that there may be evidence of mean reversion in Malaysian stock market. As a result, it also suggests that there may be some scope to benefit from time diversification in the Malaysian stock market.

To search for evidence of time diversification applicability, first must make sure if Malaysian equity market exhibit random walk under the consideration of various time horizons. To develop the hypothesis to test this, it is first assumed that the Malaysian equity market follows random walk. Hence, the volatility and Sharpe ratio will follow the **Square Root Time Rule** as explained by the equation (2.1) and (2.2). As a result, the hypotheses developed are:

H1: The Malaysian stock market's volatilities, with respected to various time horizon, follow Square Root Time Rule.

H2: The Malaysian stock market Sharpe ratios, with respected to various time horizon, follow Square Root Time Rule.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Research Methodology and Measurement of Variables

The study undertaken is divided into two parts. The first part is for the Square Root of Time Rule hypothesis testing and the determination of mean reversion evidence. The second part is for the determination of time diversification. For both part, an empirical study approach, based on historical data, is selected.

3.2 Part 1 Analysis for Hypothesis Testing and Mean Reversion

Part 1 analysis is carried out to identify whether there is any significant evidence to indicate sign of mean reversion in the Bursa Malaysia stock market performance. In summary, for this part, there are total of seven steps involved in the analysis. These steps are developed to test H1 hypothesis. The Bursa Malaysia KLCI is assumed to follow random walk. By defining the KLCI returns in continuously compounded, using natural logarithm, the returns pertaining to designated time horizon is assumed independently and identically distributed and the returns are also assumed to follow normal distribution. With these assumptions, the returns and volatilities can be projected as per the discussion in section 2.1. The seven steps are developed for testing whether or not the returns and volatilities can be projected as such. The detail discussions of the seven steps are as shown below.

Step 1: The sample data's daily performance and return are plotted against time to depict an overview of the empirical data used. Following Madhusoodanan (1997), the continuously compounded return from the KLCI index is calculated as per equation (3.1).

$$R_t = \ln (I_t / I_{t-1}) \quad \text{-----} \quad (3.1)$$

Where,

I_t = the index value on the day t , and

I_{t-1} = the index value for the day before I_t .

To further illustrate the choice of this formula, consider a single day is a unit of time. Then the daily KLCI are depicted as a time series stochastic process. Let I_t be the KLCI index at end of day-(t), and I_{t-1} be the KLCI index at the end of day-($t-1$). The return from the change in the indexes may be calculated using a simple return or log return (Return document. (n.d.). Retrieved February 8, 2006, from <http://www.riskglossary.com/link/return.htm>). The log denotes a natural logarithm. Simple return is commonly known as arithmetic return whereas log return is commonly known as geometric return. The geometric approach is regarded as an excellent measure of past performance. Furthermore, the geometric approach always produces return less than the arithmetic approach. This will constitute a downward-biased estimator of the index expected return in any future year (Bodie, Kane & Marcus, 2005, p. 865).

Hence, in general, log return over N days, can be calculated as such:

$$\text{Log Return over } N \text{ days horizon} = \sum_{t=1}^N R_t \quad \text{-----} \quad (3.2)$$

Step 2: The term structure of volatility is examined. The term structure is defined as pattern of the analyzed variable depicted over time (Bodie, Kane & Marcus, 2005, p. 487). The volatility of a KLCI index, I_{t-1} , is defined as the standard deviation of the index log return for I_t (Holton, 1992). This is further interpreted as:

$$\text{Volatility} = \text{std} [\ln (I_t / I_{t-1})] \quad \text{-----} \quad (3.3)$$

This definition is in-line with Madhusoodanan (1997). Hence, this analysis is carried out by first calculating the standard deviations of the KLCI log return. The exact formula used to calculate standard deviation is as such:

$$\sigma_N = \left\{ \left(\frac{1}{n-1} \right) \sum_{t=1}^n (R_t - \mu)^2 \right\}^{1/2} \text{-----} \quad (3.4)$$

where,

N = the investment time horizon measured in number of day,

n = the number of samples corresponding to N -days of time investment horizon,

R_t = the log return sample corresponding to N , determined from equation (3.2),

μ = the average log return of the total samples, and

σ_N = the standard deviation corresponding to N -days time investment horizon.

Equation (3.4) is an equally weighted standard deviation formula. This calculation is repeated from $N=1$ to N equals to a period depending on the samples availability. This process will generate the series of actual volatilities corresponding to the designated investment time horizon. According to Holton (1992), it is advisable to use rolling time series because it generates better measurements. Furthermore, if staggered time series is used, it will generate significantly less samples for this analysis. Moreover, from a practical stand-point, the investors can invest at any window and for any time horizon. Hence, staggering time series will not be able to reflect the actual investment situation but rolling time series is more reflective to the actual situation. Finally, rolling time series can remove the market seasonality factors such as Chinese New Year's effect, and etcetera.

The plot of the actual volatilities over different investment horizon (days) is known as term structure of volatility. The foundation of this analysis is based on the notion that theoretically, the annualized volatilities should be constant regardless the frequency of return calculation. By examining the term structure of volatility, it is able to access if any deviation from the assumed random walk efficiency theory. To carry out this test, the theoretically expected standard deviation values are also calculated and plotted on the same graph. Then the comparison between the actual values and expected values is carried out and the differences are visualized and commented. At step 6, the test for significance difference will be executed.

The expected volatility is determined from the sequence shown at below. To utilize equation (2.2), first, the daily volatility needs to be estimated. In order to get a good estimation, maximum samples should be utilized to calculate the volatility. In this case, normally all the samples are used. Hence,

$$\sigma_{est-daily} = \left\{ \left(\frac{1}{\sum N - 1} \right) \sum_{t=1}^{\sum N} (R_t - \mu)^2 \right\}^{1/2}, \text{ where } \sum N \text{ is the total number of}$$

daily sample, in another words, the number of single day of investment horizon. $\sigma_{est-daily}$ is the corresponding volatility.

Then from equation (2.2), $\sigma_{annualized} = \sqrt{(TR)} \times \sigma_{est-daily}$, where TR = number of trading days per year. Using $\sigma_{annualized}$, the expected volatility for any n days investment horizon, σ_{n-days} , can be determined as such:

$$\sigma_{annualized} = \sqrt{\frac{TR}{n}} \sigma_{n-days}. \Rightarrow \text{But } \sigma_{annualized} = \sqrt{(TR)} \times \sigma_{est-daily}. \text{ Substitute}$$

this equation will result in:

$$\therefore \sqrt{(TR)} \times \sigma_{est-daily} = \sqrt{\frac{(TR)}{n}} \times \sigma_{n-days}$$

Hence, the expected volatility for any n days investment horizon,

$$\sigma_{n - days} = \sqrt{n} \times \sigma_{est - daily} \quad \text{-----} \quad (3.5)$$

Step3: In step 3, the term structure of returns is analyzed. The expected and actual returns of one-day investment horizon to n-days investment horizon are calculated and depicted over n. This is to observe the return pattern and also to compare with the expected values and pattern. The actual return is calculated as per equation (3.6)

$$\mu_N = \left\{ \left(\frac{1}{n} \right) \sum_{t=1}^n (R_t) \right\} \quad \text{-----} \quad (3.6)$$

where,

$N =$ the investment time horizon measured in number of day,

$n =$ the number of samples corresponding to N -days of time investment horizon,

$R_t =$ the log return sample corresponding to N , determined from equation (3.2),

$\mu_N =$ the actual log return of the total samples, corresponding to N -days time investment horizon.

The expected return is calculated as shown in below. To utilize equation (2.1), first, the average daily log return needs to be estimated. In order to get a good estimation, maximum samples should be utilized to calculate the average daily log return. In this case, normally all the samples are used. Hence,

$$\mu_{est-daily} = \left\{ \left(\frac{1}{\sum N} \right) \sum_{t=1}^{\sum N} (R_t) \right\}, \text{ where } \sum N \text{ is the total number of daily sample.}$$

$\mu_{est-daily}$ is the average daily log return of the samples. Then from equation (2.1),

$R_{\text{annualized}} = \sqrt{(TR)} \times \mu_{\text{est-daily}}$, where TR = number of trading days per year. Using $R_{\text{annualized}}$, the expected return for any n days investment horizon, $R_{n\text{-days}}$, can be determined as such:

$$R_{\text{annualized}} = \sqrt{\frac{TR}{n}} R_{n\text{-days}} \Rightarrow \text{But } R_{\text{annualized}} = \sqrt{(TR)} \times \mu_{\text{est-daily}} \cdot \text{Substitute}$$

this equation will result in:

$$\therefore \sqrt{(TR)} \times \mu_{\text{est-daily}} = \sqrt{\frac{(TR)}{n}} \times R_{n\text{-days}}$$

$$\text{Hence, } R_{n\text{-days}} = \sqrt{n} \times \mu_{\text{est-daily}} \text{ ----- (3.7)}$$

Step 4: In this step, the representation of risk-return trade-off is done through plotting of return against the volatility. This will provide an interaction view between return and volatility. The interaction pattern can be used to determine if there is any bounded pattern. Ideally, if the equation (2.1) and (2.2) are true, then the log returns to volatilities plot would follow the shape of equation “ $y = x^2$ ”, which is a parabolic shape. However, if there is mean reversion occurred, the actual plot would deviate from parabolic shape, instead, exhibit more like a bounded pattern, which is when return increases, volatility decreases, instead of increases.

Step 5: It is important to examine the pattern in risk-return trade-off. For this purpose, Sharpe ratio is regarded as an ideal measurement according to Madhusoodanan (1997). Sharpe ratio is derived from dividing the average return by the standard deviation of the returns. Sharpe ratio measures the return per unit of risk. Assuming the risk-free rate is ignored, a modified Sharpe Ratio formula is used. The modified formula for Sharpe ratio as in follow:

$$\text{Sharpe Ratio} = \text{Return} / \text{volatility} \text{ ----- (3.8)}$$

(Bodie, Kane & Marcus, 2005, p.868.). Sharpe Ratio is preferred to Treynor measure and Jensen's alpha due to standard deviation is used as risk measure. The Sharpe-ratio is computed and depicted over various investment horizons. The actual Sharpe ratios are compared to the expected Sharpe Ratio. The same formula is used to calculate the expected Sharpe Ratio using the expected return and expected volatility.

Step 6: This is the step whereby test will be conducted to verify H1. In order to determine the statistical significance of the findings from the term structure volatility test, regression on the logarithm of the volatilities with logarithm of the length of investment horizon will be carried out. The length of the investment horizon will be determined from the term structure analysis carried out prior to this statistical significance study. The regression model is derived as follows:

From equation (2.2), take the base 10 logarithm for both sides of the equation.

Hence, $\log(\sigma_{n\text{-days}}) = \log(\sqrt{n} \times \sigma_{\text{daily}})$. This is further derived to become, $\log(\sigma_{n\text{-days}}) = 0.5\log(n) + \log(\sigma_{\text{daily}})$, where n is the corresponding time horizon and σ_{daily} is derived daily volatility of the total samples and is a constant figure.

Hence, the plot of $\log(\sigma_{n\text{-days}})$ against $\log(n)$ is expected to follow a straight line. Hence, in general:

$$\text{Log(volatility)} = A + B * \text{Log(investment time horizon)} \quad \text{-----} \quad (3.9)$$

If KLCI's volatilities follow the Square Root of Time Rule, the coefficient, B, will closely follow the value of 0.5. The t-distribution test will be carried out to test the actual coefficient against the ideal value of 0.5.

Step 7: Following Step 6, the statistical significance of Sharpe-ratio will be carried out using regression study as well. The logarithm of Sharpe-ratio and length of investment horizon are used in this regression study to examine the statistical significance of this measure. The regression model is derived as in follow :-

Substitute equation (2.1) and (2.2) into equation (3.8) results into “Sharpe Ratio” = $(n\pi_{daily})/(\sqrt{n} \times \sigma_{daily}) = \sqrt{n} \times (\pi_{daily} / \sigma_{daily})$. Take the base 10 logarithm for both sides of the equation. Hence, $\log(\text{Sharpe Ratio}) = 0.5\log(n) + \log(\pi_{daily} / \sigma_{daily})$, where n is the corresponding time horizon and $(\pi_{daily} / \sigma_{daily})$ is derived from the total samples and is a constant figure. Hence, the plot of $\log(\text{Sharpe Ratio})$ against $\log(n)$ is expected to follow a straight line. Hence, in general:-

$$\text{Log(Sharpe Ratio)} = A+B*\text{Log(investment time horizon)} \text{-----} \quad (3.10)$$

If KLCI’s Sharpe ratios follow the Square Root of Time Rule, the coefficient, B, will closely follow the value of 0.5. The t-distribution test will be carried out to test the actual coefficient against the ideal value of 0.5.

3.3 Part 2 Analysis on Time Diversification

Part 2 analysis is carried out to identify whether there is any significance evidences to indicate the applicability of Time Diversification in the KLCI stock market performance. Mean-variance optimization is adopted for this purpose due to its popularity as the most widely used model for portfolio optimization in the capital markets (Madhusoodanan, 1997). According to Madhoosoodanan (1997), this method is a good check on time diversification. This is because it is commonly assumed that return mean and variance increase proportionally with investment time horizon. For the case where variance increases faster than the mean, the α^* is expected to decrease as the time horizon increases. α^* is the proportion of funds invested in the risky asset which will maximize the utility function. The utility function, U, which increases with the mean of returns and decreases with the variance, is calculated as per below:

$$U = E(R_c) - 0.5A\text{Var}(R_c) \text{-----} \quad (3.11)$$

(Bodie, Kane & Marcus, 2005, p.168) where,

- The return value will be expressed in decimal, rather than percentage,
- A is investor's specific risk-aversion parameter,
- R_c is the combined and weighted average of an investor's choice allocation between risk-free investment, with return rate of R_F ; and risky investment with unknown return of R_R . The R_c is calculated as such:

$$R_c = (1-\alpha)R_F + \alpha R_R \text{ ----- (3.12)}$$

where α is the proportion of funds invested in the risky asset.

- $E(R_c)$ is the expected portfolio return calculated for corresponding investment time horizon.
- $Var(R_c)$ is the portfolio variance for the corresponding time horizon.

Thorley (1995) recognizes that under the assumptions of normal distribution and random walk, as the time horizon increases, the α^* would eventually converge to zero value. Owing to this, Madhusoodanan (1997) adopted this method as a good check for time diversification applicability. The condition for time diversification applicability is very apparent when the α^* value will increase instead, as the time horizon increases.

To derive the formula for α^* , substitute equation (3.12) into equation (3.11).

The utility function becomes:

$$U = E[(1-\alpha)R_F + \alpha R_R] - 0.5A \text{Var}[(1-\alpha)R_F + \alpha R_R]$$

$$U = (1-\alpha)E(R_F) + \alpha E(R_R) - 0.5A(1-\alpha)^2 \text{Var}(R_F) - 0.5A \alpha^2 \text{Var}(R_R)$$