UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2005/2006

November 2005

EEE 228E – SIGNAL AND SYSTEM
[Isyarat Dan Sistem]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of NINE pages of printed material and TEN pages APPENDIX before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat bercetak dan SEPULUH muka surat LAMPIRAN sebelum anda memulakan peperiksaan].

This paper contains SIX questions.

[Kertas soalan ini mengandungi ENAM soalan].

Instructions: Answer **FIVE** questions.

*[**Arahan:** Jawab **LIMA** soalan].*

Answer to any question must start on a new page.

[Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru].

All questions must be answered in English. However, ONE question can be answered in bahasa Malaysia.

[Jawab semua soalan dalam bahasa Inggeris. Walau bagaimanapun, SATU soalan dibenarkan dijawab dalam bahasa Malaysia].

...2/-

1. (a) The input-output relation of a semiconductor diode is represented by
Hubung kait masukan-keluaran suatu diod semikonduktor diwakili oleh

$$i(t) = a_0 + a_1 v(t) + a_2 v^2(t) + a_3 v^3(t) + ...$$

where $v(t)$ is the applied voltage, $i(t)$ is the current flowing through the diode and $a_0, a_1, a_2, ...$ are constant. Does this diode have memory?

yang mana $v(t)$ ialah voltan yang dikenakan, $i(t)$ ialah arus yang mengalir melalui diod, dan $a_0, a_1, a_2, ...$ ialah pemalar. Adakah diod ini mempunyai ingatan?

(4%)

(b) *Sketch the indicated product $g(t)$ of these functions in Figure 1.*
Lakarkan hasil darab $g(t)$ yang dinyatakan untuk fungsi dalam Rajah 1.

(8%)

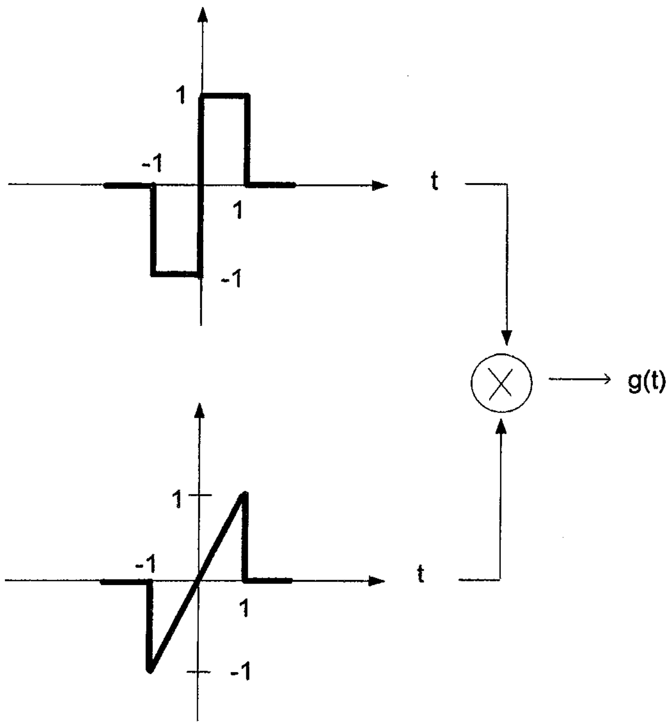

Figure 1
Rajah 1

...3/-

(c) For the modulator described by

Untuk pemodulat yang ditakrifkan oleh

$$y(t) = [A + x(t)]\cos \omega_c t$$

with the input $x(t)$ and output $y(t)$, determine whether the system is linear and time invariant. Justify your answer.

dengan masukan $x(t)$ dan keluaran $y(t)$, tentukan sama ada sistem ini linear dan lelurus masa. Jelaskan jawapan anda.

(8%)

2. (a) Determine if the following discrete-time signals are periodic. If periodic, find the fundamental discrete-time period N_0.

Tentukan sama ada isyarat masa-diskret berikut berkala. Jika berkala, cari kala asas masa-diskret, N_0.

(i) $g[k] = \cos\left(\frac{2\pi k}{5}\right) + \cos\left(\frac{2\pi k}{7}\right)$

(ii) $g[k] = e^{j(2\pi k/3)} + e^{j(2\pi k/4)}$

(6%)

(b) A discrete-time function $g[k]$ as shown in Figure 2 is defined by

Suatu fungsi masa-diskret $g[k]$ seperti yang ditunjukkan dalam Rajah 2 didefinasikan oleh

$$g[k]=\begin{cases}-2, & k<-4\\ k, & -4\le k<1\\ \dfrac{4}{k}, & 1\le k\end{cases}$$

Sketch

Lakarkan

(i) $g[2-k]$

(ii) $g[2k]$ (8%)

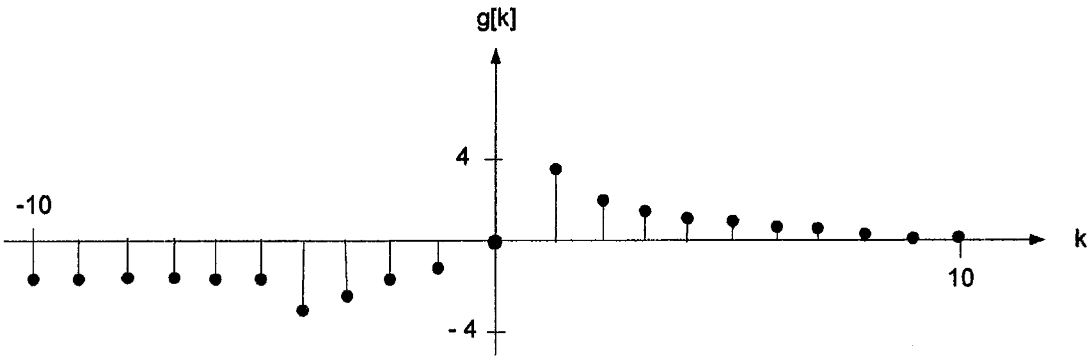

Figure 2
Rajah 2

(c) Time compression for discrete-time functions is similar to time compression for continuous-time functions in that the function occurs faster in time. But in the case of discrete-time functions there is another effect called decimation that has meaning only for discrete-time functions. Explain fully.

Pemampatan masa untuk fungsi masa-diskret agak sama dengan pemampatan masa fungsi masa-selanjar dengan fungsi tersebut berlaku lebih cepat dalam masa. Tetapi dalam kes fungsi masa-diskret, terdapat kesan lain dipanggil 'decimation' yang mempunyai makna hanya untuk fungsi masa-diskret. Terangkan dengan lengkap.

(6%)

...5/-

3. (a) Sketch the even and odd components of this discrete-time function.

Lakarkan komponen genap dan ganjil untuk fungsi masa-diskret ini.

$$g[k] = u[k] - u[k-4]$$

(6%)

(b) An operation that appears frequently in continuous-time system analysis is the convolution of a continuous-time signal with an impulse. Use the sampling property of the impulse to show that convolution of a function $f(t)$ with a unit impulse results in the function $f(t)$ itself.

Suatu operasi yang selalu dilihat dalam analisis sistem masa-selanjar ialah konvolusi suatu isyarat masa-selanjar dengan suatu dedenyut. Guna ciri pensampelan dedenyut untuk menunjukkan bahawa konvolusi suatu fungsi $f(t)$ dengan suatu dedenyut yunit menghasilkan fungsi $f(t)$ itu sendiri.

(6%)

(c) Find and sketch.
Cari dan lakarkan.

(i) $rect(t) \; * \; \delta(t)$

(ii) $rect(t) \; * \; \delta(t-1)$

(8%)

4. (a) Suppose the input $x(t)$ and impulse response $h(t)$ of an linear time-invariant system are, respectively, given by

Andaikan masukan $x(t)$ dan sambutan dedenyut $h(t)$ suatu sistem lelurus tak ubah-masa masing-masing diberikan oleh

$$x(t) = u(t-1) - u(t-2)$$

...6/-

and
dan

$$h(t) = e^{-t}\ u(t)$$

Find and sketch the output of this system.
Cari dan lakarkan keluaran sistem ini.

(10%)

(b) Consider the modulator depicted in Figure 3.

Pertimbangkan pemodulat yang ditunjukkan dalam Rajah 3.

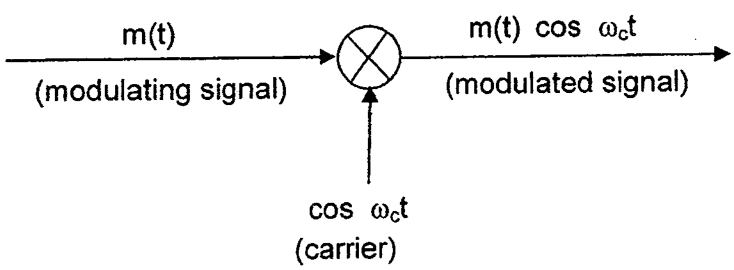

Figure 3
Rajah 3

For a modulating signal $m(t) = \cos \omega_m t$,
Untuk isyarat modulat $m(t) = \cos \omega_m t$,

(i) Give and sketch the spectrum of the modulating signal, M(w) if
Beri dan lakarkan spektrum isyarat modulat, M(w) jika

$$m(t) \overset{\Im}{\Leftrightarrow} M(w)$$

(4%)

...7/-

(ii) In the frequency-domain, find the double sideband signal and sketch its spectrum. Identity the upper and lower sidebands. Is this called the double sideband-suppressed carrier? Why?

Dalam domain frekuensi, cari isyarat jalur sisi kembar dan lakarkan spektrumnya, nyatakan jalur sisi atas dan bawah. Adakah ini dipanggil jalur sisi kembar pembawa-tertindas? Mengapa?

(6%)

5. (a) Find the continuous-time Fourier transform of the convolution of 10 sin(t) with $2\delta(t+4)$ with two different methods. (Using table and properties of Fourier transform given in the appendix).

Cari jelmaan Fourier masa-selanjar untuk konvolusi 10 sin(t) dengan $2\delta(t+4)$ *dengan dua kaedah berlainan. (Guna ciri-ciri dan jadual jelmaan Fourier yang diberikan dalam lampiran).*

(i) Method 1. Do the convolution first and find the continuous-time Fourier transform of the result.

Cara 1. Lakukan konvolusi dahulu dan cari jelmaan Fourier masa-selanjar keputusan konvolusi tersebut.

(6%)

(ii) Method 2. Do the continuous-time Fourier transform first to avoid the convolution.

Cara 2. Lakukan jelmaan Fourier masa-selanjar dahulu untuk mengelakkan konvolusi.

(6%)

...8/-

(b) Compute the inverse discrete Fourier transform, IDFT if the rectangular form of a 4-point DFT is given by

Kira jelmaan Fourier disket songsang, IDFT jika DFT 4-titik dalam bentuk segiempat diberikan oleh

$$X_r = \begin{cases} 6 & ,r=0 \\ -1-j & ,r=1 \\ 0 & ,r=2 \\ -1+j & ,r=3 \end{cases}$$

The IDFT of X_r is

IDFT untuk X_r ialah

$$x[k] = \frac{1}{N}\sum_{r=0}^{N-1} X_r \, e^{jr\Omega k}, \Omega = \frac{2\pi}{N} \quad k = 0,1,2,\ldots,N-1$$

(8%)

6. (a) Consider the discrete-time domain block diagram of a discrete-time system in Figure 4.

Pertimbangkan gambarajah blok domain masa-diskret untuk suatu sistem masa-diskret dalam Rajah 4.

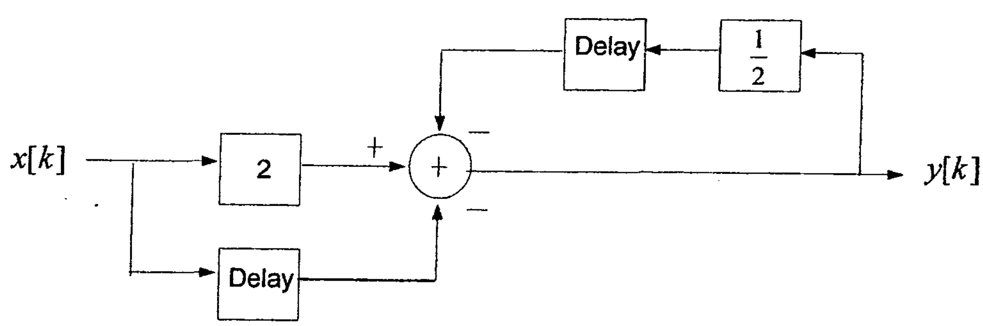

Figure 4
Rajah 4

...9/-

(i) Write the difference equation describing the response $y[k]$ to the excitation $x[k]$.

Tulis persamaan bezaan yang menghuraikan sambutan $y[k]$ kepada ujaan $x[k]$.

(5%)

(ii) Find the transfer function in the z-domain, $H[z]$.

Cari fungsi pindah dalam domain-z, $H[z]$.

(5%)

(b) Using the partial-fraction expansion method, find the inverse z-transform of $X[z]$.

Menggunakan kaedah kembangan pecahan-separa, cari jelmaan-z songsang untuk $X[z]$.

$$X[z]=\frac{Z^2\left(z-\frac{1}{2}\right)}{\left(z-\frac{2}{3}\right)\left(z-\frac{1}{3}\right)\left(z-\frac{1}{4}\right)}$$

Evaluate $x[k]$ for k = 0, 1, 2

Kira $x[k]$ untuk k = 0, 1, 2

(10%)

ooo0ooo

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\frac{1}{\lvert a \rvert} F\left(\frac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi} F_1(\omega) * F_2(\omega)$
Time differentiation	$\frac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\int_{-\infty}^{t} f(x)\,dx$	$\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2 + b^2}, \qquad \theta = \tan^{-1}\left(\tfrac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j \sin x$$

$$\cos x = \tfrac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \tfrac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos\left(x \pm \tfrac{\pi}{2}\right) = \mp \sin x$$

$$\sin\left(x \pm \tfrac{\pi}{2}\right) = \pm \cos x$$

$$2 \sin x \cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \tfrac{1}{4}(3 \cos x + \cos 3x)$$

$$\sin^3 x = \tfrac{1}{4}(3 \sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \tfrac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \tfrac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$a \cos x + b \sin x = C \cos(x + \theta)$$

in which $C = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$

B.7-7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax-1)$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)$$

B.7-8 Differentiation Table

$\frac{d}{dx}f(u) = \frac{d}{du}f(u)\frac{du}{dx}$	$\frac{d}{dx}a^{bx} = b(\ln a)a^{bx}$
$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$	$\frac{d}{dx}\sin ax = a\cos ax$
$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}\cos ax = -a\sin ax$
$\frac{dx^n}{dx} = nx^{n-1}$	$\frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$
$\frac{d}{dx}\ln(ax) = \frac{1}{x}$	$\frac{d}{dx}(\sin^{-1} ax) = \frac{a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}\log(ax) = \frac{\log e}{x}$	$\frac{d}{dx}(\cos^{-1} ax) = \frac{-a}{\sqrt{1-a^2x^2}}$
$\frac{d}{dx}e^{bx} = be^{bx}$	$\frac{d}{dx}(\tan^{-1} ax) = \frac{a}{1+a^2x^2}$

B.7-9 Some Useful Constants

$\pi \approx 3.1415926535$

$e \approx 2.7182818284$

$\frac{1}{e} \approx 0.3678794411$

$\log_{10} 2 = 0.30103$

$\log_{10} 3 = 0.47712$

B.7-10 Solution of Quadratic and Cubic Equations

Any quadratic equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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TABLE 2.1: Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t-T)$	$f(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\dfrac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\dfrac{1}{2}t^2e^{\lambda t}u(t)$
7	$t^n u(t)$	$e^{\lambda t}u(t)$	$\dfrac{n!\,e^{\lambda t}}{\lambda^{n+1}}u(t) - \sum_{j=0}^{n}\dfrac{n!\,t^{n-j}}{\lambda^{j+1}(n-j)!}u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\dfrac{m!n!}{(m+n+1)!}t^{m+n+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^m e^{\lambda t}u(t)$	$t^n e^{\lambda t}u(t)$	$\dfrac{m!\,n!}{(n+m+1)!}t^{m+n+1}e^{\lambda t}u(t)$
11	$t^m e^{\lambda_1 t}u(t)$	$t^n e^{\lambda_2 t}u(t)$	$\sum_{j=0}^{m}\dfrac{(-1)^j m!(n+j)!\,t^{m-j}e^{\lambda_1 t}}{j!(m-j)!(\lambda_1-\lambda_2)^{n+j+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\sum_{k=0}^{n}\dfrac{(-1)^k n!(m+k)!\,t^{n-k}e^{\lambda_2 t}}{k!(n-k)!(\lambda_2-\lambda_1)^{m+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\dfrac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

TABLE 9.1: Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$	
1	$\delta[k-j]$	$f[k]$	$f[k-j]$	
2	$\gamma^k u[k]$	$u[k]$	$\left[\frac{1-\gamma^{k+1}}{1-\gamma}\right]u[k]$	
3	$u[k]$	$u[k]$	$(k+1)u[k]$	
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\frac{\gamma_1^{k+1}-\gamma_2^{k+1}}{\gamma_1-\gamma_2}\right]u[k] \quad \gamma_1 \neq \gamma_2$	
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\frac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^k u[-(k+1)]$	$\lvert\gamma_2\rvert > \lvert\gamma_1\rvert$
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\frac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1-\gamma_2}{\gamma_2}k\gamma_1^k\right]u[k]$	$\gamma_1 \neq \gamma_2$
7	$ku[k]$	$ku[k]$	$\frac{1}{6}k(k-1)(k+1)u[k]$	
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$	
9	$\gamma^k u[k]$	$ku[k]$	$\left[\frac{\gamma(\gamma^k-1)+k(1-\gamma)}{(1-\gamma)^2}\right]u[k]$	
10	$\lvert\gamma_1\rvert^k \cos(\beta k+\theta)u[k]$	$\gamma_2^k u[k]$	$\frac{1}{R}\left[\lvert\gamma_1\rvert^{k+1}\cos[\beta(k+1)+\theta-\phi]-\gamma_2^{k+1}\cos(\theta-\phi)\right]u[k]$	γ_2 real
			$R = \left[\lvert\gamma_1\rvert^2+\gamma_2^2-2\lvert\gamma_1\rvert\gamma_2\cos\beta\right]^{1/2}$	
			$\phi = \tan^{-1}\left[\frac{(\lvert\gamma_1\rvert\sin\beta)}{(\lvert\gamma_1\rvert\cos\beta-\gamma_2)}\right]$	

Table 11.1: (Unilateral) z-Transform Pairs

	$f[k]$	$F[z]$
1	$\delta[k-j]$	z^{-j}
2	$u[k]$	$\frac{z}{z-1}$
3	$ku[k]$	$\frac{z}{(z-1)^2}$
4	$k^2u[k]$	$\frac{z(z+1)}{(z-1)^3}$
5	$k^3u[k]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^{k-1}u[k-1]$	$\frac{1}{z-\gamma}$
7	$\gamma^k u[k]$	$\frac{z}{z-\gamma}$
8	$k\gamma^k u[k]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$k^2\gamma^k u[k]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma\rvert^k \cos\beta k\, u[k]$	$\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^k \sin\beta k\, u[k]$	$\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$	$\frac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k] \qquad \gamma=\lvert\gamma\rvert e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$ $r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$	$\frac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

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Table 11.2

$\mathcal{Z}$- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k-m]u[k-m]$	$\frac{1}{z^m}F[z]$
	$f[k-m]u[k]$	$\frac{1}{z^m}F[z] + \frac{1}{z^m}\sum_{k=1}^{m} f[-k]z^k$
	$f[k-1]u[k]$	$\frac{1}{z}F[z] + f[-1]$
	$f[k-2]u[k]$	$\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$
	$f[k-3]u[k]$	$\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$
Left-shift	$f[k+m]u[k]$	$z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k+1]u[k]$	$zF[z] - zf[0]$
	$f[k+2]u[k]$	$z^2F[z] - z^2f[0] - zf[1]$
	$f[k+3]u[k]$	$z^3F[z] - z^3f[0] - z^2f[1] - zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z\frac{d}{dz}F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$
Initial value	$f[0]$	$\lim_{z\to\infty} F[z]$
Final value	$\lim_{N\to\infty} f[N]$	$\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle.