UNIVERSITI SAINS MALAYSIA

First Semester Examination
2011/2012 Academic Session

January 2012

ESA 380/3 – Orbital Mechanics
[Mekanik Orbit]

Duration : 3 hours
[Masa : 3 jam]

Please check that this paper contains **NINE (9)** printed pages, **THREE (3)** pages appendix and **FIVE (5)** questions before you begin the examination.
*Sila pastikan bahawa kertas soalan ini mengandungi **SEMBILAN (9)** mukasurat bercetak, **TIGA (3)** mukasurat lampiran dan **LIMA (5)** soalan sebelum anda memulakan peperiksaan.*

Instructions : Answer **ALL q**uestions.
Arahan : *Jawab **SEMUA** soalan.*

1. **Appendix Table A.1**/*Lampiran Jadual A.1.* [**2 pages**/*mukasurat*]
2. **Appendix A.2**/*Lampiran Jadual A.2.* [**1 page**/*mukasurat*]

Answer all questions in **English** OR **Bahasa Malaysia.**
*Menjawab semua soalan dalam **Bahasa Inggeris ATAU Bahasa Malaysia.***

Answer to each question must begin from a new page.
Jawapan untuk setiap soalan mestilah dimulakan pada mukasurat yang baru.

In the event of any discrepancies, the English version shall be used.
Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

Answer **ALL** questions.
*Jawab **SEMUA** soalan.*

1. [a] Consider the two-body problem illustrated in **Figure 1**. If a force **T** (such as rocket thrust) acts on m_2 in addition to the mutual force of gravitation $\mathbf{F}_{21}$, define that

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3} + \frac{\mathbf{T}}{m_2}$$

*Pertimbangkan permasalahan dua jasad yang ditunjukkan dalam **Rajah 1**. Jika daya **T** (contohnya sistem pendorong roket) bertindak ke atas m_2 selain dari daya saling tarikan gravity $\mathbf{F}_{21}$, tentukan bahawa*

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3} + \frac{\mathbf{T}}{m_2}$$

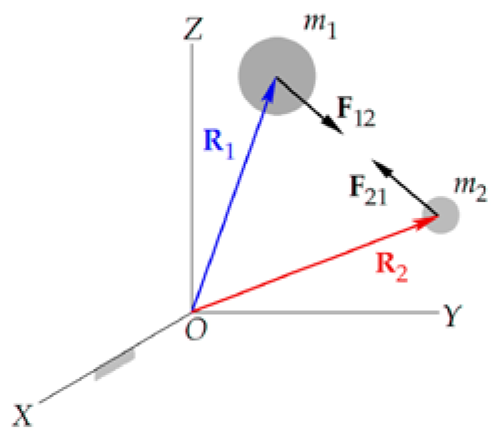

Figure 1/*Rajah 1*

(20 marks/*markah*)

[b] Relative to an earth-centered non-rotating frame, the position and velocity vectors of a spacecraft are

$$\mathbf{r}_0 = 3450\hat{\mathbf{i}} - 1700\hat{\mathbf{j}} + 7750\hat{\mathbf{k}}\ km$$
$$\mathbf{v}_0 = 5.4\hat{\mathbf{i}} - 5.4\hat{\mathbf{j}} + 1.0\hat{\mathbf{k}}\ km$$

Relatif kepada bingkai kaku pusat bumi, vector kedudukan dan halaju kapal angkasa adalah

$$\mathbf{r}_0 = 3450\hat{\mathbf{i}} - 1700\hat{\mathbf{j}} + 7750\hat{\mathbf{k}}\ km$$
$$\mathbf{v}_0 = 5.4\hat{\mathbf{i}} - 5.4\hat{\mathbf{j}} + 1.0\hat{\mathbf{k}}\ km$$

[i] Calculate the distance and speed of the spacecraft after the true anomaly changes by $82°$, see appendix.

Kirakan jarak dan kelajuan selepas sudut 'true anomaly' berubah sebanyak $82°$, *lihat lampiran.*

[ii] Show that the specific angular momentum h and total energy ε are conserved.

Tunjukkan bahawa 'specific angular momentum' h *dan jumlah tenaga* ε *adalah terpelihara.*

(50 marks/*markah*)

[c] If the eccentricity of the elliptical orbit depicted in **Figure 2** is 0.5, calculate, in terms of the period T, the time required to fly from P to B. Given the mean anomaly and eccentric anomaly of this type of orbit are as follow

$$M_e = E - e\sin E$$

$$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right)$$

*Jika keeksentrikan orbit elips yang dipaparkan dalam **Rajah 2** bersamaan dengan* 0.5*, kira, dalam sebutan tempoh* T*, masa yang diperlukan untuk terbang dari titik* P *ke* B*. Diberi 'mean anomaly' dan 'eccentric anomaly' orbit jenis ini adalah seperti berikut*

$$M_e = E - e\sin E$$

$$E = 2\tan^{-1}\left(\sqrt{\frac{1-e}{1+e}}\tan\frac{\theta}{2}\right)$$

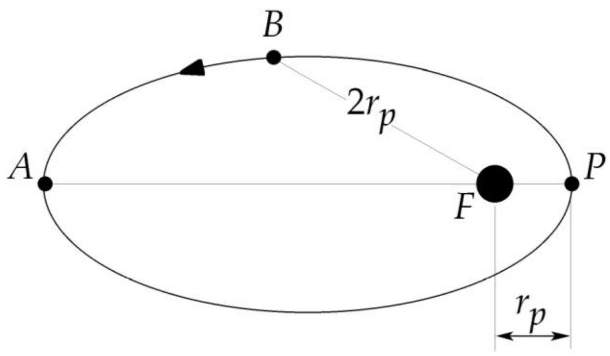

Figure 2/*Rajah 2*

(30 marks/*markah*)

2. [a] [i] Show the orientation of the orbit in three dimensions.

Tunjukkan orientasi orbit dalam tiga dimensi.

[ii] State the classical orbital elements

Nyatakan elemen-elemen orbit klasik.

(15 marks/*markah*)

[b] Given the vector position of the satellite with respect to the geocentric equatorial frame is $\mathbf{r} = -6634.2\hat{\mathbf{I}} - 1261.8\hat{\mathbf{J}} - 5230.9\hat{\mathbf{K}}\ (km)$, the eccentricity vector is $\mathbf{e} = -0.40907\hat{\mathbf{I}} - 0.48751\hat{\mathbf{J}} - 0.63640\hat{\mathbf{K}}\ (km)$, and the satellite is flying towards perigee, calculate the inclination of the orbit.

Diberi vektor posisi sebuah satelit sehubungan dengan bingkai equatorial adalah $\mathbf{r} = -6634.2\hat{\mathbf{I}} - 1261.8\hat{\mathbf{J}} - 5230.9\hat{\mathbf{K}}\ (km)$, *vector keeksentrikan adalah* $\mathbf{e} = -0.40907\hat{\mathbf{I}} - 0.48751\hat{\mathbf{J}} - 0.63640\hat{\mathbf{K}}\ (km)$, *dan satelit terbang menuju ke perigee, kirakan sudut condong orbit.*

(20 marks/*markah*)

[c] For a spacecraft, the following orbital parameters are given: $e = 1.5$; perigee altitude $= 300\ km$; $i = 35°$; $\Omega = 130°$; $\omega = 115°$. Calculate **r** and **v** at perigee relative to

Sebuah kapal angkasa, diberi parameter-parameter orbit seperti berikut: $e = 1.5$, perigee altitude $= 300\ km$; $i = 35°$; $\Omega = 130°$; $\omega = 115°$. *Kirakan* **r** *dan* **v** *pada perigee sehubungan dengan*

[i] The perifocal reference frame

Bingkai rujukan perifokal

[ii] The geocentric equatorial frame

Bingkai geopusat equatorial

(65 marks/*markah*)

3. [a] **Figure 3** shows how the transfer from the smaller circular orbit to the larger one by Hohmann transfer (dashed red line) and Bi-elliptical Hohmann transfer (solid red line). Describe the condition when the Bi-elliptical Hohmann transfer becomes more efficient than Hohmann transfer.

Rajah 3 *menunjukkan bagaimana perpindahan daripada orbit bulat kecil kepada orbit bulan besar dengan menggunakan teknik pemindahan Hohmann dan teknik pemindahan Bi-elliptical Hohmann. Terangkan keadaan bila teknik pemindahan Bi-elliptical Hohman menjadi lebih cekap berbanding teknik pemindahan Hohmann.*

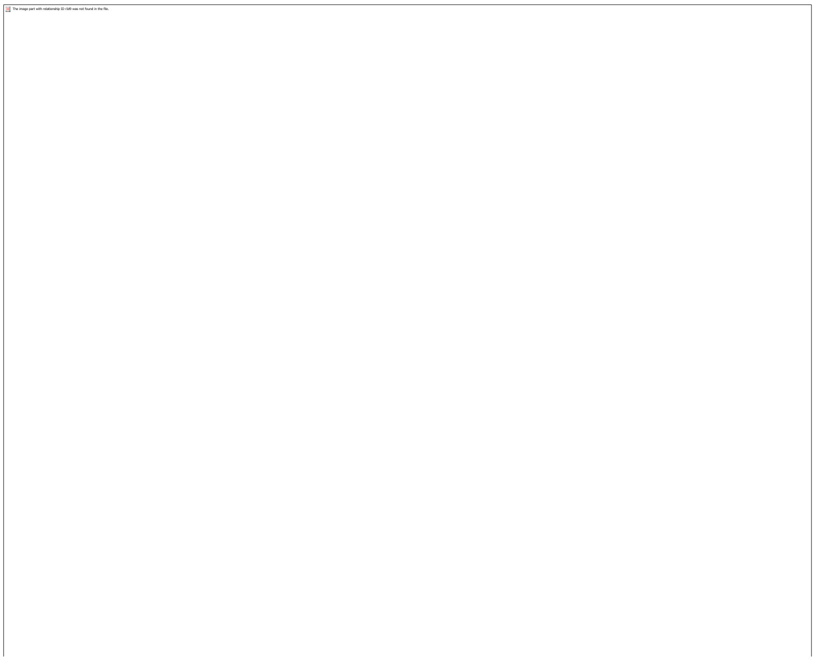

Figure 3/*Rajah 3*

(20 marks/*markah*)

[b] In **Figure 4**, a spacecraft in circular orbit 1 of radius r leaves for infinity on parabolic trajectory 2 and returns from infinity on a parabolic trajectory 3 to a circular orbit 4 of radius $12r$. Find the total delta-v required for this non-Hohmann orbit change maneuver.

Dalam ***Rajah 4****, sebuah kapal angkasa berada pada orbit bulat 1 dengan jejari r meninggalkan orbit ke infiniti melalui trajektori parabola 2 dan kembali dari infiniti melalui trajektori parabola 3 ke orbit bulat 4 dengan jejari* $12r$*. Cari jumlah* delta-v *yang diperlukan oleh perubahan orbit bukan Hohmann ini.*

(40 marks/*markah*)

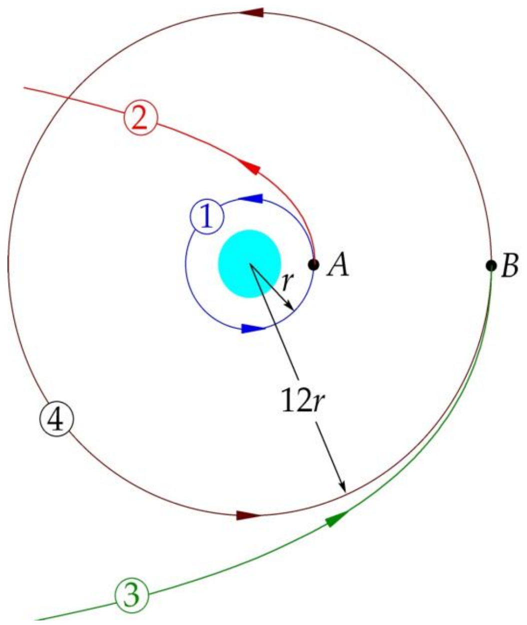

Figure 4/*Rajah 4*

[c] With a single impulsive maneuver, an earth satellite changes from a $400\ km$ circular orbit inclined at $60°$ to an elliptical orbit of eccentricity $e = 0.5$ with an inclination of $40°$. Calculate the minimum required delta-v.

Dengan melaksanakan satu 'impulsive maneuver', sebuah satelit yang mengelilingi bumi berubah dari $400\ km$ *orbit bulat yang mempunyai sudut condong* $60°$ *kepada orbit elip dengan keeksentrikan* $e = 0.5$ *dan sudut condong* $40°$. *Kira jumlah* delta-v *yang diperlukan.*

(40 marks/*markah*)

4. The space station, spacecraft A and B are all in the same circular orbit of $350\ km$ altitude. Spacecraft A is $600\ km$ behind the space station and Spacecraft B is a $600\ km$ ahead of the space station. At the same instant, both spacecrafts apply a delta-v so as to arrive at the space station in one revolution of their phasing orbits. See **Figure 5**.

*Stesen angkasa dan kapal angkasa A dan B kesemuanya berada dalam orbit bulat yang sama dengan altitude $350\ km$. Kapal angkasa A berada $600\ km$ di belakang kapal angkasa manakala kapal angkasa B berada $600\ km$ di hadapan stesen angkasa. Pada masa yang sama, kedua-dua kapal angkasa mengenakan 'delta-v' supaya kedua-duanya sampai sampai pada stesen angkasa dalam satu pusingan lengkap orbit fasa masing-masing. Lihat **Rajah 5**.*

Figure 5/*Rajah 5*

[a] Calculate the times required for each spacecraft to reach the space station.

Kira masa yang diperlukan bagi kedua-dua kapal angkasa untuk sampai kepasa stesen angkasa.

(40 marks/*markah*)

[b] Calculate the total delta-v requirement for each spacecraft.

Kira jumlah keperluan 'delta-v' untuk setiap kapal angkasa.

(60 marks/*markah*)

5. Estimate the total delta-v requirement for a Hohmann transfer from Earth to Mercury, assuming a $150\ km$ circular parking orbit at Earth and a $150\ km$ circular capture orbit at Mercury. Furthermore, assume that the planets have coplanar circular orbits with radii equal to the semi major axes listed in **Table A.1**.(Appendix)

*Anggarkan jumlah keperluan 'delta-v' untuk perpindahan Hohmann dari Bumi ke Merkuri dengan menganggap orbit bulat sementara pada Bumi adalah $150\ km$ dan orbit bulat tangkapan pada Merkuri adalah $150\ km$. Seterusnya, anggap bahawa kedua-dua planet mempunyai orbit bulat pada satah yang sama dengan radii bersamaan dengan paksi semi-major tersenarai dalam **Jadual A.1**.(Lampiran)*

(100 marks/*markah*)

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APPENDIX A.2/*LAMPIRAN A.2*

Langrage coefficients in terms of the change in true anomaly,

$$f = 1 - \frac{\mu r}{h^2}(1 - \cos\Delta\theta)$$

$$g = \frac{r r_0}{h}\sin\Delta\theta$$

$$\dot{f} = \frac{\mu}{h}\frac{1 - \cos\Delta\theta}{\sin\Delta\theta}\left[\frac{\mu}{h^2}(1 - \cos\Delta\theta) - \frac{1}{r_0} - \frac{1}{r}\right]$$

$$\dot{g} = 1 - \frac{\mu r_0}{h^2}(1 - \cos\Delta\theta)$$

where,

$$r = \frac{h^2}{\mu}\frac{1}{1 + \left(\frac{h^2}{\mu r_0} - 1\right)\cos\Delta\theta - \frac{h v_{r0}}{\mu}\sin\Delta\theta}$$

$$v_{r0} = \frac{\mathbf{r}_0 \cdot \mathbf{v}_0}{r_0}$$

$$h = r_0 v_{\perp 0} = r_0\sqrt{v_0^2 - v_{r0}^2}$$