UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

January 2012

EKC 314 – Transport Phenomena
[Fenomena Pengangkutan]

Duration : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains NINE printed pages and SEVEN printed pages of Appendix before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak dan TUJUH muka surat Lampiran sebelum anda memulakan peperiksaan ini.]

Instruction: Answer **FOUR (4)** questions. Section A is **COMPULSORY**. Answer any **TWO (2)** questions from Section B. All questions carry the same marks.

*[**Arahan:** Jawab **EMPAT (4)** soalan. Bahagian A **WAJIB**. Jawab mana-mana **DUA (2)** soalan dari Bahagian B. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

<u>Section A</u> : Answer <u>ALL</u> questions.
Bahagian A: Jawab <u>SEMUA</u> soalan.

1. [a] τ_{yx} is defined as the momentum flux of a flow of a liquid in *x*-direction. Compute the steady-state, τ_{yx} in lb_f/ft^2 when the lower plate velocity *V* in Figure Q.1.[a]. is 1 ft/s in the positive *x*-direction, the plate separation *Y* is 0.001 ft and the fluid viscosity, μ is 0.7 cp.

τ_{yx} *didefinisikan sebagai fluks momentum bagi suatu aliran cecair pada arah–*x. *Kirakan keadaan mantap,* τ_{yx} *dalam* $lb_{kaki}/kaki^2$ *apabila halaju plat bawah* V *dalam Gambarajah S.1.[a]. adalah 1 kaki/s pada arah–*x *positif. Pemisahan plat* Y *adalah 0.001 kaki dan kelikatan bendalir,* μ *adalah 0.7cp.*

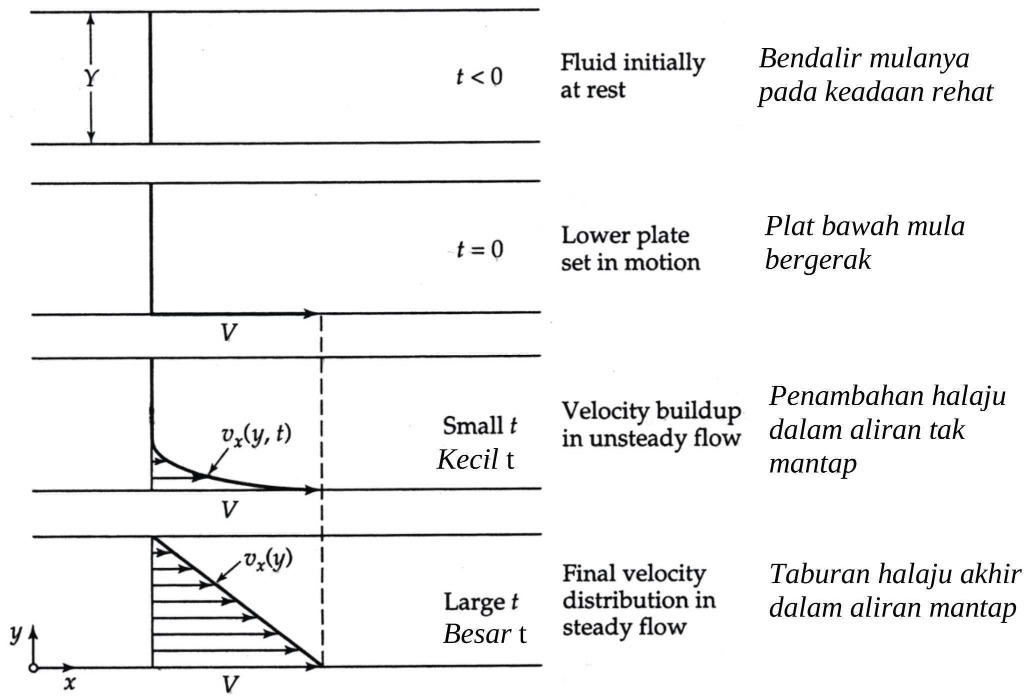

Figure Q.1.[a]: The formation of laminar velocity profile for a fluid contained between two plates.

Gambarajah S.1.[a].: Pembentukan profil halaju laminar bagi bendalir yang terletak di antara dua lapisan plat.

[6 marks/markah]

[b] Derive the following continuity relationship for single phase fluid flow:
Terbitkan hubungan keterusan bagi aliran bendalir satu fasa:

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) \right)$$

where ρ is the fluid density, t is time and v_x, v_y, and v_z, are the velocities in the x, y and z-directions, respectively. To what form does the above equation reduce into for an incompressible (constant density) fluid?

di mana, ρ *adalah ketumpatan bendalir,* t *adalah masa dan* v_x, v_y, *dan* v_z, *adalah masing-masing halaju pada arah* x, y *dan* z. *Kepada bentuk apakah persamaan di atas dapat dipermudahkan bagi bendalir ketidakboleh-mampatan (ketumpatan malar)?*

[13 marks/markah]

[c] In the mass transport theory, the mass flux j_A is generally given as;
Di dalam teori pemindahan jisim, fluks jisim, j_A *biasanya diberikan oleh;*

$$j = -\rho D \nabla \omega$$

Show that for a binary mixture of two components A and B, only **<u>ONE</u>** diffusivity is needed to describe the diffusional behaviour of a binary mixture.

Tunjukkan bagi campuran perduaan komponen A dan B, hanya ***<u>SATU</u>*** *kemeresapan diperlukan bagi menggambarkan tingkahlaku kemeresapan campuran perduaan.*

[6 marks/markah]

2. [a] Estimate the value of diffusivity D_{AB}, for the system of methane-ethane at 293 K and 1 atm using the following methods:

Anggarkan nilai kemeresapan, D_{AB}, *bagi sistem metana-etana pada suhu 293 K dan 1 atm menggunakan kaedah-kaedah berikut:*

[i] the correlation developed between the kinetic theory and the corresponding-state argument given by;

kolerasi yang diterbitkan di antara teori kinetik dan keadaan-sepadan diberi oleh;

$$\frac{pD_{AB}}{(p_{cA}p_{cB})^{1/3}(T_{cA}T_{cB})^{5/12}(\frac{1}{M_A}+\frac{1}{M_B})^{1/2}} = a\left(\frac{T}{\sqrt{T_{cA}T_{cB}}}\right)^b$$

where p represents the pressure in atm, D_{AB} is the diffusivity between two components, T represents the temperature in Kelvin and M is the components' relative molecular mass. The dimensionless constants a and b are values obtained from experimental observation. (The Lennard-Jones parameter and properties table may be used).

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di mana p *merujuk kepada tekanan dalam atm,* D_{AB} *ialah kemeresapan di antara dua komponen,* T *adalah suhu dalam Kelvin dan* M *adalah jisim molekul relatif komponen-komponen tersebut. Pemalar tanpa dimensi* a *dan* b *adalah nilai-nilai yang diperolehi daripada pemerhatian ujikaji. (Parameter Lennard-Jones dan jadual sifat boleh digunakan).*

[6 marks/markah]

[ii] the Chapman-Enskog relation given by;
hubungan Chapman-Enskog diberi sebagai;

$$\sigma_{AB} = \frac{1}{2}(\sigma_A + \sigma_B) \quad \text{and} \quad \varepsilon_{AB} = \sqrt{\varepsilon_A \varepsilon_B}$$

with the diffusivity given by;
dengan kemeresapan diberi oleh;

$$D_{AB} = 0.0018583\sqrt{T^3\left(\frac{1}{M_A} + \frac{1}{M_B}\right)}\frac{1}{p\sigma_{AB}^2 \Omega_{D,AB}}$$

[6 marks/markah]

[b] A gas is initially supplied at temperature T_1 and pressure P_1, later required to be heated to temperature T_2. Assuming the gas behaves as an ideal gas and undergoes adiabatic frictionless process, develop a relationship between the temperatures and pressures so that the required pressure P_2 could be determined. The specific heat capacity of the gas can be assumed constant and independent of temperature, while the momentum flux τ and the heat flux q are negligible.

Suatu gas pada mulanya dibekalkan pada suhu T_1 *dan tekanan* P_1. *Kemudian, ia perlu dipanaskan kepada suhu* T_2. *Dengan menganggap gas itu sebagai gas unggul dan menjalani proses adiabatik tanpa geseran, hasilkan satu hubungan antara suhu dan tekanan, supaya tekanan* P_2 *boleh ditentukan. Muatan haba tentu bagi gas tersebut boleh dianggap malar dan tidak bergantung pada suhu, manakala fluks momentum* τ *dan fluks haba* q *boleh diabaikan.*

The general form of equation of change for temperature is given by:
Bentuk umum bagi persamaan pertukaran bagi suhu diberi oleh:

$$\rho \hat{C}_P \frac{DT}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) - \left(\frac{\partial ln\rho}{\partial lnT}\right)_P \frac{DP}{Dt}$$

Ideal gas law: $PM_r = \rho RT$
Hukum gas unggul: $PM_r = \rho RT$

[8 marks/markah]

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[c] It was found that the specific heat capacity of methane gas does not vary significantly from temperature 300 K to 350 K. Determine the pressure required if methane gas (CH_4) is to be heated from initial condition of T_1 = 300 K and P_1 = 101.3 kPa to final temperature T_2 = 350 K by adiabatic frictionless compression.

Telah didapati bahawa muatan haba tentu bagi gas metana tidak berubah dengan ketara dari suhu 300 K hingga 350 K. Tentukan tekanan yang diperlukan jika gas metana (CH_4) dipanaskan dari keadaan awal pada T_1 = *300 K dan* P_1 = *101.3 kPa kepada suhu akhir* T_2 = *350 K menerusi mampatan adiabatik tanpa geseran.*

Data for methane:
Data untuk gas metana:

$\hat{C}_P$ = 2.3 J/(g K) for temperature range 300 K $\leq T \leq$ 350 K
$\hat{C}_P$ = 2.3 J/(g K) bagi julat suhu 300 K $\leq T \leq$ 350 K

M_r = 16 g/mol
M_r = 16 g/mol

Universal gas constant R = 8.314 J/(mol K)
Pemalar gas semesta R = 8.314 J/(mol K)

[5 marks/markah]

<u>Section B</u> : Answer any <u>TWO</u> questions.
<u>Bahagian B:</u> Jawab mana-mana <u>DUA</u> soalan.

3. Heat is flowing steadily through an annular wall of inside radius r_0 and outside radius r_1, as shown in the Figure Q.3.

Haba mengalir secara mantap melalui dinding berbentuk anulus berjejari dalam r_0 *dan berjejari luar* r_1, *seperti yang ditunjukkan dalam Rajah S.3.*

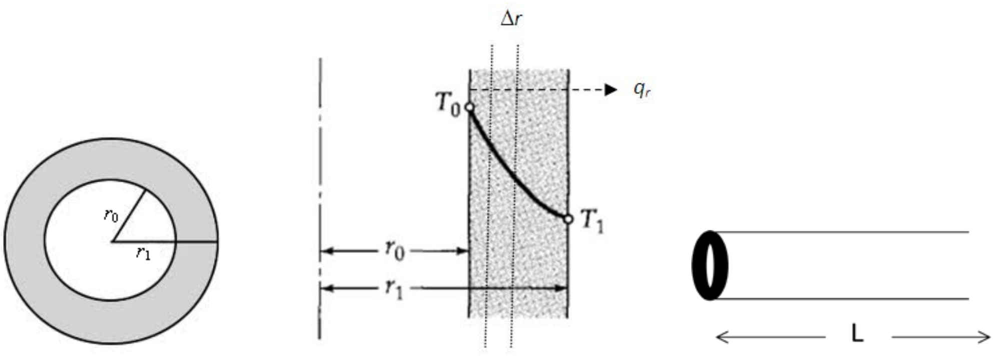

Figure Q.3. Annular wall of inside radius r_0, outside radius r_1 and length L

Rajah S.3.: Dinding berbentuk anulus berjejari dalam r_0 *dan berjejari luar* r_1 *dan panjang* L

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The thermal conductivity k of the wall varies linearly with temperature T from k_0 at T_0 to k_1 at T_1, and is given by:

Kekonduksian haba k bagi dinding berubah secara linear dengan suhu T dari k_0 pada T_0 kepada k_1 pada T_1, dan diberi oleh:

$$\frac{k - k_0}{k_1 - k_0} = \frac{T - T_0}{T_1 - T_0}$$

[a] Write the general statement of energy balance over a unit element of volume.
Tulis pernyataan umum imbangan tenaga bagi satu unit elemen isipadu.

[2 marks/markah]

[b] By applying energy balance on a cylindrical shell of thickness Δr and length L, show that:
Dengan menggunakan imbangan tenaga pada kerangka silinder berketebalan Δr dan panjang L, tunjukkan:

$$\frac{d}{dr}(rq_r) = 0$$

where q_r is the energy flux in radial direction and r representing radial position in the wall.
di mana q_r *ialah fluks tenaga dalam arah jejarian dan* r *mewakili kedudukan jejarian dalam dinding.*

[6 marks/markah]

[c] With the definition of dimensionless temperature $\theta = \frac{T-T_0}{T_1-T_0}$, write an algebraic expression showing the variation of temperature in the wall in terms of θ and r. Let C_1 be the constant of integration of the equation in [b].

Dengan menggunakan takrifan suhu tak berdimensi $\theta = \frac{T-T_0}{T_1-T_0}$, *tuliskan ungkapan algebra yang menunjukkan perubahan suhu di dalam dinding dalam sebutan θ dan r. Biarkan C_1 sebagai pemalar integrasi bagi persamaan dalam [b].*

[7 marks/markah]

[d] Use boundary conditions to solve for C_1 and develop expression for the heat flow Q through the wall.

Gunakan keadaan sempadan untuk menyelesaikan C_1 dan bangunkan ungkapan bagi aliran haba Q menerusi dinding.

[10 marks/markah]

4. Figure Q.4 shows a droplet of liquid A, of radius r_1, is suspended in a stream of gas B. We postulate that there is spherical stagnant gas film of radius r_2 surrounding the droplet. The concentration of A in the gas phase is x_{A1} at $r = r_1$ and x_{A2} at the outer edge of the film, $r = r_2$.

Gambarajah S.4. menunjukkan suatu titisan cecair A dengan jejari r_1, *yang terampai di dalam aliran gas B. Kita menaakulkan bahawa terdapat genangan filem gas sfera dengan jejari* r_2 *menyelaputi titisan tersebut. Kepekatan bahan A dalam fasa gas adalah* x_{A1} *pada* $r = r_1$ *dan* x_{A2} *pada sisi luaran filem,* $r = r_2$.

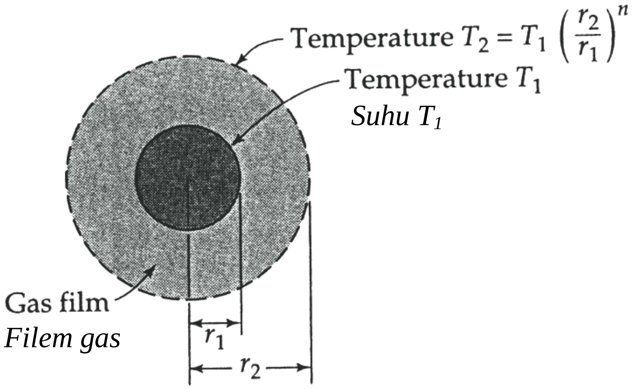

Figure Q.4.: Diffusion through a hypothetical spherical stagnant gas film surrounding a droplet of liquid A.

Gambarajah S.4.: Kemeresapan melalui hipotesis genangan filem gas menyelaputi titisan cecair A.

[a] By a shell balance, show that for steady-state diffusion r^2N_{Ar} is a constant within the gas film, and set the constant equal to $r^2_1N_{Ar1}$ the value at the droplet surface.

Dengan menggunakan imbangan kerangka, tunjukkan kemeresapan, r^2N_{Ar} *pada keadaan mantap adalah pemalar dalam filem gas dan tetapkan pemalar tersebut sama dengan* $r^2{}_1N_{Ar1}$ *iaitu nilai pada permukaan titisan.*

[8 marks/markah]

[b] Show that using the flux balance equation, and the result obtained in [a], gives the following equation;

Tunjukkan dengan menggunakan persamaan imbangan fluks, dan hasil yang diperolehi daripada [a] membawa kepada persamaan berikut;

$$r_1^2 N_{Ar1} = -\frac{cD_{AB}}{1-x_A} r^2 \frac{dx_A}{dr}$$

[8 marks/markah]

[c] Integrate this equation between the limits r_1 and r_2 to get;
Kamirkan persamaan di atas di antara limit-limit r_1 *dan* r_2 *bagi menghasilkan;*

$$N_{Ar1} = \frac{cD_{AB}}{r_2 - r_1}\left(\frac{r_2}{r_1}\right)\ln\frac{x_{B2}}{x_{B1}}$$

What is the limit of this expression when;
Apakah limit bagi persamaan yang terhasil apabila;

$$r_2 \to \infty$$

[9 marks/markah]

5. A part of a lubrication system consists of two circular disks between which a lubricant flows radially (refer to the Figure Q.5.). The flow takes place because of a modified pressure difference P_1 - P_2 between the inner and outer radii r_1 and r_2 respectively.

Suatu bahagian daripada sistem pelincir terdiri daripada dua cakera bulat di mana pelincir mengalir secara jejari (rujuk kepada Gambarajah S.5.). Aliran tersebut berlaku disebabkan oleh perubahan perbezaan tekanan $P_1 - P_2$ *di antara jejari-jejari dalaman dan luaran* r_1 *dan* r_2 *masing-masing.*

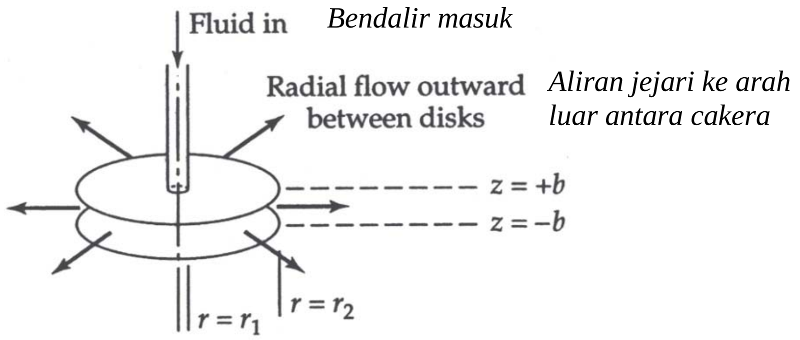

Figure Q.5.: Outward radial flow in the space between two parallel, circular disks.
Gambarajah S.4: Aliran jejari keluar di antara dua ruang selari cakera bulat.

[a] Write the equation of continuity and motion for this flow system, assuming steady-state, laminar, incompressible Newtonian flow. Consider only the region $r_1 \le r \le r_2$ and a flow that is radially directed.

Tuliskan persamaan keterusan dan pergerakan bagi sistem aliran ini, dengan menganggapkan keadaan mantap, laminar dan aliran ketidakbolehmampatan Newtonian. Pertimbangkan hanya kawasan $r_1 \le r \le r_2$ *dan aliran adalah berarah secara jejari.*

[7 marks/markah]

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[b] Show how the equation of continuity enables one to simplify the equation of motion to give;

Tunjukkan bagaimanakah persamaan keterusan membolehkan seseorang untuk meringkaskan persaman pergerakan bagi menghasilkan;

$$-\rho\frac{\phi^2}{r^3} = -\frac{dP}{dr} + \mu\frac{1}{r}\frac{d^2\phi}{dz^2}$$

in which $\phi = r v_r$ is a function of z only. Why is ϕ independent of r?
(ρ is the density of the lubricant).
di mana $\phi = r v_r$ *adalah fungsi kepada* z *sahaja. Kenapakah* ϕ *tidak bergantung kepada* r*? (*ρ *adalah ketumpatan bendalir pelincir)*

[8 marks/markah]

[c] It can be shown that no solution exists for the equation in S.5.[b] unless the nonlinear term containing ϕ is omitted. Omission of this term corresponds to the “creeping flow assumption”. Show that for creeping flow, equation in S.5.[b]. can be integrated with respect to r to give;

Boleh ditunjukkan bahawa tiada penyelesaian bagi persamaan pada S.5.[b]. melainkan fungsi tak-linear yang mengandungi ϕ *dihapuskan. Penghapusan fungsi ini berhubungkait dengan anggapan aliran rayap. Tunjukkan bahawa bagi aliran rayap, persamaan S.5.[b]. boleh dikamirkan kepada* r *untuk menghasilkan;*

$$0 = (P_1 - P_2) + \left(\mu \ln\frac{r_2}{r_1}\right)\frac{d^2\phi}{dz^2}$$

[3 marks/markah]

[d] Show that the mass flow rate is given by;
Tunjukkan bahawa kadar aliran jisim diberikan oleh;

$$\dot{m} = \frac{4\pi(P_1 - P_2)b^3\rho}{3\mu\ln\frac{r_2}{r_1}}$$

[7 marks/markah]

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Appendices

Appendix A: Conversion Factors

Given a quantity in these units:	Multiply by:	To get quantity in these units:
Pounds	453.59	Grams
Kilograms	2.2046	Pounds
Inches	2.5400	Centimeters
Meters	39.370	Inches
Gallons (U.S.)	3.7853	Liters
Gallons (U.S.)	231.00	Cubic inches
Gallons (U.S.)	0.13368	Cubic feet
Cubic feet	28.316	Liters
Kelvins	1.800000	Degrees Rankine
Degrees Rankine	0.555556	Kelvins

Table A.1

Table A.2

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<u>Appendix B</u>: Equation of Motion in Terms of τ

The general equation is in the vector form of:

$$\rho D\mathbf{v}/Dt = -\nabla p - [\nabla \cdot \tau] + \rho \boldsymbol{g}$$

Cartesian coordinates (x, y, z):[a]

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Table B.1

Cylindrical coordinates (r, θ, z):[b]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Table B.2

Spherical coordinates (r, θ, ϕ):[c]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$- \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta r}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi r} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2 \cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\phi\phi}\cot\theta}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi \cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\phi}) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\phi}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\phi} + \frac{(\tau_{\phi r} - \tau_{r\phi}) + \tau_{\phi\theta}\cot\theta}{r}\right] + \rho g_\phi$$

Table B.3

<u>Appendix C</u>: Equation of Motion for a Newtoniam Fluid with Constant ρ and μ

The general equation is in the vector form of:

$$\rho D\mathbf{v}/Dt = -\nabla p + \boldsymbol{\mu}\nabla^2\mathbf{v} + \rho\boldsymbol{g}$$

Cartesian coordinates (x, y, z):

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table C.1

Cylindrical coordinates (r, θ, z):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Table C.2

Spherical coordinates (r, θ, ϕ):

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial v_r}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial \phi^2}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$+ \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right] + \rho g_\phi$$

Table C.3

<u>Appendix D</u>: Some Ordinary Differential Equations and Their Solutions

Equation	Solution
$\dfrac{dy}{dx}=\dfrac{f(x)}{g(y)}$	$\int g\,dy=\int f\,dx+C_1$
$\dfrac{dy}{dx}+f(x)y=g(x)$	$y=e^{-\int f\,dx}(\int e^{\int f\,dx}g\,dx+C_1)$
$\dfrac{d^2y}{dx^2}+a^2y=0$	$y=C_1\cos ax+C_2\sin ax$
$\dfrac{d^2y}{dx^2}-a^2y=0$	$y=C_1\cosh ax+C_2\sinh ax$ or $y=C_3e^{+ax}+C_4e^{-ax}$
$\dfrac{1}{x^2}\dfrac{d}{dx}\left(x^2\dfrac{dy}{dx}\right)+a^2y=0$	$y=\dfrac{C_1}{x}\cos ax+\dfrac{C_2}{x}\sin ax$
$\dfrac{1}{x^2}\dfrac{d}{dx}\left(x^2\dfrac{dy}{dx}\right)-a^2y=0$	$y=\dfrac{C_1}{x}\cosh ax+\dfrac{C_2}{x}\sinh ax$ or $y=\dfrac{C_3}{x}e^{+ax}+\dfrac{C_4}{x}e^{-ax}$
$\dfrac{d^2y}{dx^2}+a\dfrac{dy}{dx}+by=0$	Solve the equation $n^2+an+b=0$, and get the roots $n=n_+$ and $n=n_-$. Then (a) if n_+ and n_- are real and unequal, $y=C_1\exp(n_+x)+C_2\exp(n_-x)$ (b) if n_+ and n_- are real and equal to n, $y=e^{nx}(C_1x+C_2)$ (c) if n_+ and n_- are complex: $n_\pm=p\pm iq$, $y=e^{px}(C_1\cos qx+C_2\sin qx)$
$\dfrac{d^2y}{dx^2}+2x\dfrac{dy}{dx}=0$	$y=C_1\int_0^x\exp(-\bar{x}^2)\,d\bar{x}+C_2$
$\dfrac{d^2y}{dx^2}+3x^2\dfrac{dy}{dx}=0$	$y=C_1\int_0^x\exp(-\bar{x}^3)\,d\bar{x}+C_2$
$\dfrac{d^2y}{dx^2}=f(x)$	$y=\int_0^x\int_0^{\bar{x}}f(\bar{\bar{x}})d\bar{\bar{x}}\,d\bar{x}+C_1x+C_2$
$\dfrac{1}{x}\dfrac{d}{dx}\left(x\dfrac{dy}{dx}\right)=f(x)$	$y=\int_0^x\dfrac{1}{\bar{x}}\int_0^{\bar{x}}\bar{\bar{x}}f(\bar{\bar{x}})d\bar{\bar{x}}d\bar{x}+C_1\ln x+C_2$
$\dfrac{1}{x^2}\dfrac{d}{dx}\left(x^2\dfrac{dy}{dx}\right)=f(x)$	$y=\int_0^x\dfrac{1}{\bar{x}^2}\int_0^{\bar{x}}\bar{\bar{x}}^2f(\bar{\bar{x}})d\bar{\bar{x}}d\bar{x}-\dfrac{C_1}{x}+C_2$
$\dfrac{d^2y}{dx^2}=h(y)$	$x=\int_0^y\dfrac{d\bar{y}}{\sqrt{2\int_0^{\bar{y}}h(\bar{\bar{y}})d\bar{\bar{y}}+C_1}}+C_2$
$x^3\dfrac{d^3y}{dx^3}+ax^2\dfrac{d^2y}{dx^2}+bx\dfrac{dy}{dx}+cy=0$	$y=C_1x^{n_1}+C_2x^{n_2}+C_3x^{n_3}$, where the n_k are the roots of the equation $n(n-1)(n-2)+an(n-1)+bn+c=0$, provided that all roots are distinct.

Table D

<u>Appendix D</u>: Some Ordinary Differential Equations and Their Solutions (cont'd)

Error Function:

The error function is defined as

$$\operatorname{erf} x = \frac{\int_0^x \exp(-\bar{x}^2)\, d\bar{x}}{\int_0^\infty \exp(-\bar{x}^2)\, d\bar{x}} = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\bar{x}^2)\, d\bar{x}$$

$$\frac{d}{dx} \operatorname{erf} u = \frac{2}{\sqrt{\pi}} \exp(-u^2) \frac{du}{dx}$$

Appendix E: Lennard-Jones Potential Parameters and Critical Properties

		Lennard-Jones parameters			Critical properties[g,h]				
Substance	Molecular Weight M	σ (Å)	ε/κ (K)	Ref.	T_c (K)	p_c (atm)	$\tilde{V}_c$ (cm^3/g-mole)	$\mu_c \times 10^6$ (g/cm · s)	$k_c \times 10^6$ (cal/cm · s · K)
Light elements:									
H_2	2.016	2.915	38.0	*a*	33.3	12.80	65.0	34.7	—
He	4.003	2.576	10.2	*a*	5.26	2.26	57.8	25.4	—
Noble gases:									
Ne	20.180	2.789	35.7	*a*	44.5	26.9	41.7	156.	79.2
Ar	39.948	3.432	122.4	*b*	150.7	48.0	75.2	264.	71.0
Kr	83.80	3.675	170.0	*b*	209.4	54.3	92.2	396.	49.4
Xe	131.29	4.009	234.7	*b*	289.8	58.0	118.8	490.	40.2
Simple polyatomic gases:									
Air	28.964[i]	3.617	97.0	*a*	132.4[i]	37.0[i]	86.7[i]	193.	90.8
N_2	28.013	3.667	99.8	*b*	126.2	33.5	90.1	180.	86.8
O_2	31.999	3.433	113.	*a*	154.4	49.7	74.4	250.	105.3
CO	28.010	3.590	110.	*a*	132.9	34.5	93.1	190.	86.5
CO_2	44.010	3.996	190.	*a*	304.2	72.8	94.1	343.	122.
NO	30.006	3.470	119.	*a*	180.	64.	57.	258.	118.2
N_2O	44.012	3.879	220.	*a*	309.7	71.7	96.3	332.	131.
SO_2	64.065	4.026	363.	*c*	430.7	77.8	122.	411.	98.6
F_2	37.997	3.653	112.	*a*	—	—	—	—	—
Cl_2	70.905	4.115	357.	*a*	417.	76.1	124.	420.	97.0
Br_2	159.808	4.268	520.	*a*	584.	102.	144.	—	—
I_2	253.809	4.982	550.	*a*	800.	—	—	—	—
Hydrocarbons:									
CH_4	16.04	3.780	154.	*b*	191.1	45.8	98.7	159.	158.
$CH{\equiv}CH$	26.04	4.114	212.	*d*	308.7	61.6	112.9	237.	—
$CH_2{=}CH_2$	28.05	4.228	216.	*b*	282.4	50.0	124.	215.	—
C_2H_6	30.07	4.388	232.	*b*	305.4	48.2	148.	210.	203.
$CH_3C{\equiv}CH$	40.06	4.742	261.	*d*	394.8	—	—	—	—
$CH_3CH{=}CH_2$	42.08	4.766	275.	*b*	365.0	45.5	181.	233.	—
C_3H_8	44.10	4.934	273.	*b*	369.8	41.9	200.	228.	—
$n{-}C_4H_{10}$	58.12	5.604	304.	*b*	425.2	37.5	255.	239.	—
$i{-}C_4H_{10}$	58.12	5.393	295.	*b*	408.1	36.0	263.	239.	—
$n{-}C_5H_{12}$	72.15	5.850	326.	*b*	469.5	33.2	311.	238.	—
$i{-}C_5H_{12}$	72.15	5.812	327.	*b*	460.4	33.7	306.	—	—
$C(CH_3)_4$	72.15	5.759	312.	*b*	433.8	31.6	303.	—	—
$n{-}C_6H_{14}$	86.18	6.264	342.	*b*	507.3	29.7	370.	248.	—
$n{-}C_7H_{16}$	100.20	6.663	352.	*b*	540.1	27.0	432.	254.	—
$n{-}C_8H_{18}$	114.23	7.035	361.	*b*	568.7	24.5	492	259.	—
$n{-}C_9H_{20}$	128.26	7.463	351.	*b*	594.6	22.6	548.	265.	—
Cyclohexane	84.16	6.143	313.	*d*	553.	40.0	308.	284.	—
Benzene	78.11	5.443	387.	*b*	562.6	48.6	260.	312.	—
Other organic compounds:									
CH_4	16.04	3.780	154.	*b*	191.1	45.8	98.7	159.	158.
CH_3Cl	50.49	4.151	355.	*c*	416.3	65.9	143.	338.	—
CH_2Cl_2	84.93	4.748	398.	*c*	510.	60.	—	—	—
$CHCl_3$	119.38	5.389	340.	*e*	536.6	54.	240.	410.	—
CCl_4	153.82	5.947	323.	*e*	556.4	45.0	276.	413.	—
C_2N_2	52.034	4.361	349.	*e*	400.	59.	—	—	—
COS	60.076	4.130	336.	*e*	378.	61.	—	—	—
CS_2	76.143	4.483	467.	*e*	552.	78.	170.	404.	—
CCl_2F_2	120.91	5.116	280.	*b*	384.7	39.6	218.	—	—

Table E.1

Collision Integrals for use with the Lennard-Jones Potential for the Prediction of Transport Properties of Gases at Low Densities

$\kappa T/\varepsilon$ or $\kappa T/\varepsilon_{AB}$	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{\mathfrak{D},AB}$ (for diffusivity)	$\kappa T/\varepsilon$ or $\kappa T/\varepsilon_{AB}$	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{\mathfrak{D},AB}$ (for diffusivity)
0.30	2.840	2.649	2.7	1.0691	0.9782
0.35	2.676	2.468	2.8	1.0583	0.9682
0.40	2.531	2.314	2.9	1.0482	0.9588
0.45	2.401	2.182	3.0	1.0388	0.9500
0.50	2.284	2.066	3.1	1.0300	0.9418
0.55	2.178	1.965	3.2	1.0217	0.9340
0.60	2.084	1.877	3.3	1.0139	0.9267
0.65	1.999	1.799	3.4	1.0066	0.9197
0.70	1.922	1.729	3.5	0.9996	0.9131
0.75	1.853	1.667	3.6	0.9931	0.9068
0.80	1.790	1.612	3.7	0.9868	0.9008
0.85	1.734	1.562	3.8	0.9809	0.8952
0.90	1.682	1.517	3.9	0.9753	0.8897
0.95	1.636	1.477	4.0	0.9699	0.8845
1.00	1.593	1.440	4.1	0.9647	0.8796
1.05	1.554	1.406	4.2	0.9598	0.8748
1.10	1.518	1.375	4.3	0.9551	0.8703
1.15	1.485	1.347	4.4	0.9506	0.8659
1.20	1.455	1.320	4.5	0.9462	0.8617
1.25	1.427	1.296	4.6	0.9420	0.8576
1.30	1.401	1.274	4.7	0.9380	0.8537
1.35	1.377	1.253	4.8	0.9341	0.8499
1.40	1.355	1.234	4.9	0.9304	0.8463
1.45	1.334	1.216	5.0	0.9268	0.8428
1.50	1.315	1.199	6.0	0.8962	0.8129
1.55	1.297	1.183	7.0	0.8727	0.7898
1.60	1.280	1.168	8.0	0.8538	0.7711
1.65	1.264	1.154	9.0	0.8380	0.7555
1.70	1.249	1.141	10.0	0.8244	0.7422
1.75	1.235	1.128	12.0	0.8018	0.7202
1.80	1.222	1.117	14.0	0.7836	0.7025
1.85	1.209	1.105	16.0	0.7683	0.6878
1.90	1.198	1.095	18.0	0.7552	0.6751
1.95	1.186	1.085	20.0	0.7436	0.6640
2.00	1.176	1.075	25.0	0.7198	0.6414
2.10	1.156	1.058	30.0	0.7010	0.6235
2.20	1.138	1.042	35.0	0.6854	0.6088
2.30	1.122	1.027	40.0	0.6723	0.5964
2.40	1.107	1.013	50.0	0.6510	0.5763
2.50	1.0933	1.0006	75.0	0.6140	0.5415
2.60	1.0807	0.9890	100.0	0.5887	0.5180

Table E.2