UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama

Sidang Akademik 2003/2004

September/Oktober 2003

# EEE 228E – ISYARAT DAN SISTEM

Masa : 3 Jam

---

**ARAHAN KEPADA CALON:-**

Sila pastikan kertas peperiksaan ini mengandungi **SEMBILAN BELAS (19)** muka surat termasuk **10 Lampiran** bercetak dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan ini.

Jawab **LIMA (5)** soalan.

Agihan markah diberikan di sut sebelah kanan soalan berkenaan.

Pelajar dibenarkan menjawab semua soalan di dalam Bahasa Inggeris ATAU Bahasa Malaysia ATAU kombinasi kedua-duanya.

...2/-

1. (a) Consider the simple FM stereo transmitter shown in Figure 1.

*Pertimbangkan penghantar stereo FM ringkas yang ditunjukkan dalam Rajah 1.*

[i] Sketch the signals L+R and L-R.
*Lakarkan isyarat-isyarat L+R dan L-R.*

[ii] If the outputs of the two adders are added, sketch the resulting waveform.

*Jika keluaran dua penambah ditambahkan, lakarkan hasil gelombang.*

(12%)

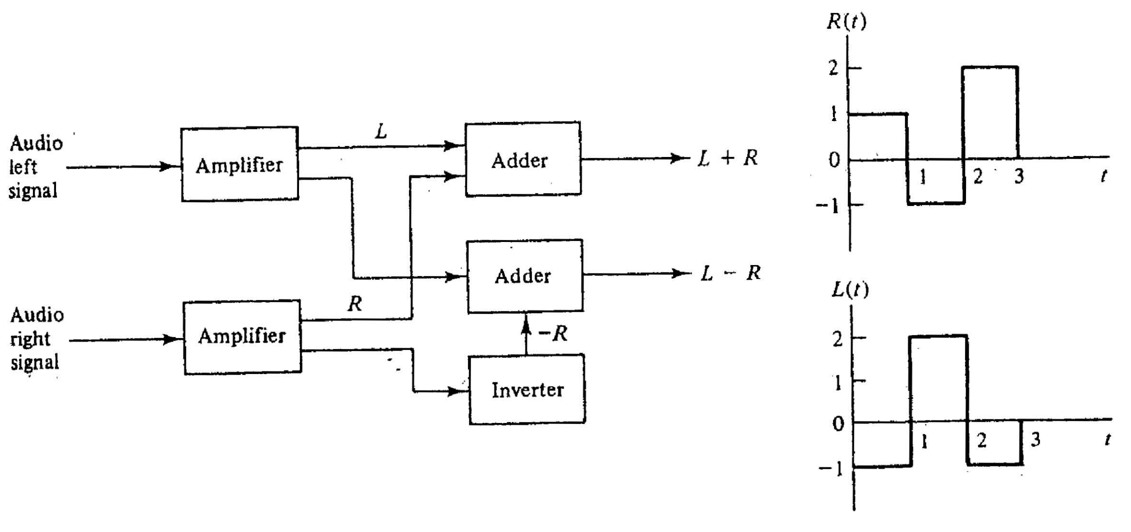


Figure 1
*Rajah 1*

(b) Explain fully the sampling property of the unit impulse function.
*Huraikan dengan lengkap ciri pensampelan fungsi dedenyut unit.*

(8%)

...3/-

2. (a) Consider the system with an input f(t) and output y(t) specified by y(t) = f(t-2) + f(t+2). Determine if the system is causal or non-causal. Justify your answer.

*Pertimbangkan suatu sistem dengan masukan f(t) dan keluaran y(t) yang dispesifikasikan oleh y(t) = f(t-2) + f(t+2). Tentukan sama ada sistem ini adalah kausal atau tidak kausal. Jelaskan jawapan anda.*

(4%)

(b) The signal $x(t) = rect(t/2)$ is transmitted through the atmosphere and is reflected by different objects located at different distances. The received signal is

*Suatu isyarat $x(t) = rect(t/2)$ dihantar melalui atmosfera dan dipantulkan oleh objek-objek berlainan yang berlokasi pada jarak-jarak berlainan. Isyarat yang diterima ialah*

$$y(t) = x(t) + 0.5x\left(t - \frac{T}{2}\right) + 0.25x\,(t - T),\ T >> 2$$

Signal y(t) is processed as shown in Figure 2.
*Isyarat y(t) diproses seperti yang ditunjukkan dalam Rajah 2.*

[i] Sketch x(t).
*Lakar x(t).* (3%)

[ii] Sketch y(t) for T = 10.
*Lakar y(t) untuk T = 10.* (6%)

[iii] Sketch z(t) for T = 10.
*Lakar z(t) untuk T = 10.* (7%)

...4/-

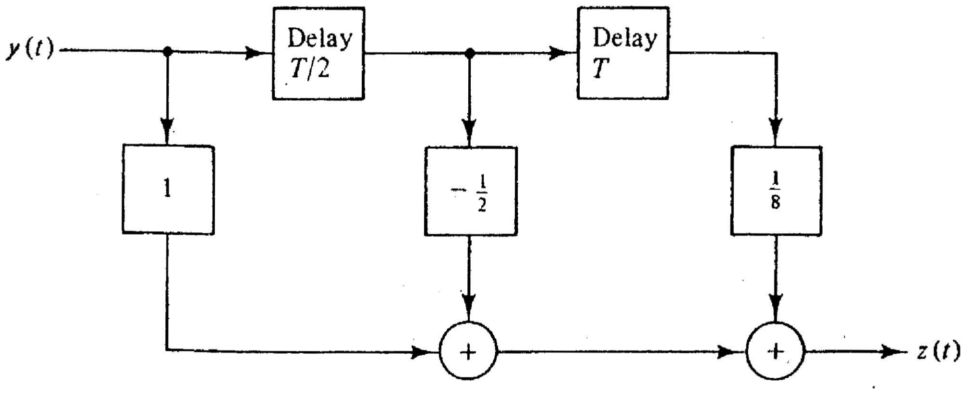


Figure 2
*Rajah 2*

3. (a) State, with reasons, whether the following signals are periodic or aperiodic. For periodic signals, find the fundamental frequency and the period. Also state which of the harmonics are present in the series.

*Nyatakan, dengan sebab, sama ada isyarat-isyarat berikut adalah berkala atau tidak berkala. Untuk isyarat berkala, ciri frekuensi asas dan kalanya. Juga nyatakan harmonik-harmonik yang manakah hadir dalam siri tersebut.*

[i] $x_1(t) = 2\sin 3t + 7\cos \pi t$

[ii] $x_2(t) = 8 + 4\cos\left(\frac{1}{2}t + 20°\right) + 5\cos\left(\frac{2}{3}t + 30°\right) + 9\cos\left(\frac{7}{6}t + 40°\right)$

(10%)

...5/-

(b) Find and sketch the Fourier transform of
*Cari dan lakarkan jelmaan Fourier untuk*

[i] $f(t) = rect\left(\frac{t}{\tau}\right)$

(6%)

[ii] Using the properties of the Fourier transform (refer to Appendix A: Fourier Transform Operations), determine the Fourier transform of the pulse

*Menggunakan ciri-ciri jelmaan Fourier (rujuk kepada lampiran A : operasi-operasi jelmaan Fourier), tentukan jelmaan Fourier untuk denyut*

$$x(t) = \alpha \; rect\left(\alpha t / \tau\right), \; \alpha > 0 .$$

(4%)

4. (a) Sketch the discrete-time signal.
*Lakar isyarat masa-diskret ini.*

$$(-k+8)\{\,u\left[k-6\right]-u\left[k-9\right]\}$$ (4%)

(b) Use iteration to find the first two terms of the difference equation if *f[k]* is the unit step function.

*Gunakan kaedah iterasi untuk mencari dua sebutan yang pertama bagi persamaan bezaan berikut jika f[k] ialah fungsi langkah unit.*

(4%)

$$y[k] + y[k-1] + \frac{1}{4}y[k-2] = f[k], \quad k \geq 0$$

$$y[-1] = 0, \; y[-2] = 1$$

...6/-

(c) Figure 3 shows signals x(t) and g(t).

*Rajah 3 menunjukkan isyarat-isyarat x(t) dan g(t).*

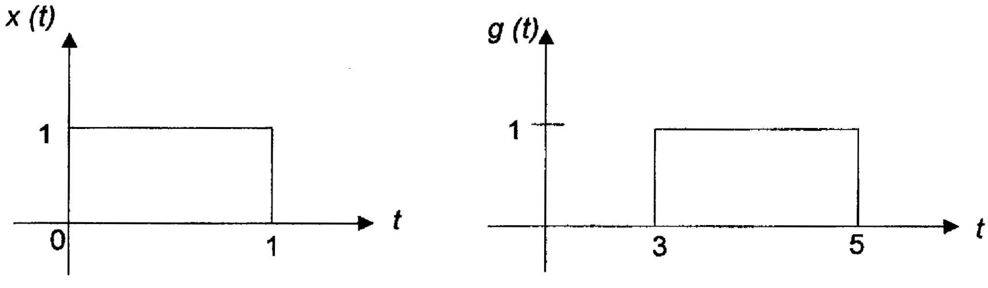


*Figure 3*
Rajah 3

[i] Express each of the signals in Figure 3 by a single expression using the unit gate function, rect(t).

*Ungkapkan setiap isyarat dalam Rajah 3 oleh satu ungkapan menggunakan fungsi get unit, rect (t).*

[ii] The resulting convolution, $c(t) = x(t) * g(t)$. Find and sketch *c(t)*.

*Hasil konvolusi,* $c(t) = x(t) * g(t)$. *Cari dan lakar c(t).*

(12%)

...7/-

5. (a) Consider a system *S* with input *x[k]* and output *y[k]*. The system is obtained through a series interconnection of a system $S_1$ followed by a system $S_2$. The input-output relationships for $S_1$ and $S_2$ are

*Pertimbangkan suatu sistem S dengan masukan x[k] dan keluaran y[k]. Sistem ini diperolehi melalui sambungan siri sistem $S_1$ diikuti oleh sistem $S_2$. Perihalan masukan keluaran untuk $S_1$ dan $S_2$ ialah*

$$S_1 : y_1[k] = 2x_1[k] + 4x_1[k-1]$$
$$S_2 : y_2[k] = x_2[k-2] + \frac{1}{2}x_2[k-3]$$

where $x_1[k]$ and $x_2[k]$ denote input signals.

*yang mana $x_1[k]$ dan $x_2[k]$ mewakili isyarat-isyarat masukan.*

[i] Determine the input-output relationship for system S.

*Tentukan perihalan masukan keluaran untuk sistem S.*

[ii] Does the input-output relationship of system S change if the order in which $S_1$ and $S_2$ are connected in series is reversed (i.e., if $S_2$ follows $S_1$)?

*Adakah perhubungan masukan keluaran sistem S berubah jika susunan $S_1$ dan $S_2$ yang bersambung siri diterbalikkan (yakni, $S_2$ dahulu diikuti $S_1$)?*

(8%)

(b) Compute the four-point DFT and IDFT for the waveform of Figure 4.

*Kira DFT dan IDFT empat-titik untuk gelombang dalam Rajah 4.*

...8/-

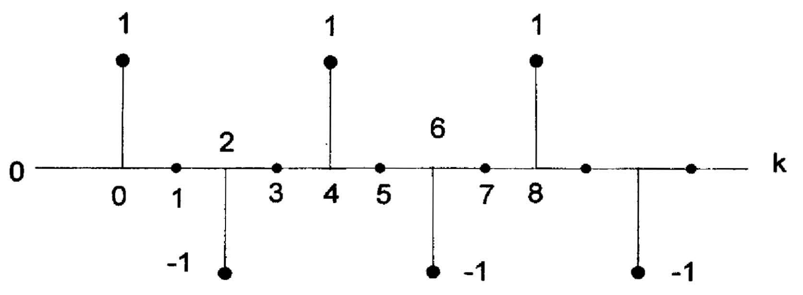


Figure 4
*Rajah 4*

(12%)

6. (a) Find the first five terms of *f[k]* using long division method if

*Cari lima sebutan yang pertama untuk f[k] menggunakan kaedah pembahagian panjang jika*

$$F[z] = \frac{-2z^4 + 5z^3 + z^2 - 6z + 3}{z^2 - z - 2}$$

(7%)

(b) Show that the transfer function of a unit delay is $\frac{1}{z}$ .

*Tunjukkan bahawa fungsi pindah suatu unit lengah ialah* $\frac{1}{z}$.

(4%)

...9/-

(c) An LTID (linear, time-invariant, discrete-time) system is described by the difference equation

*Suatu system LTID (masa-diskret lelurus, tak ubah masa) diperihalkan oleh persamaan bezaan*

$$y[k]+3y[k-1]=f[k]$$

with the input $f[k]=\left(\frac{1}{2}\right)^k u[k]$ and initial condition $y[-1]=1$.

*dengan masukan* $f[k]=\left(\frac{1}{2}\right)^k u[k]$ *dan keadaan awalan* $y[-1]=1$.

[i] Find the z-transform of *f[k]*.

*Cari jelmaan z untuk f[k].*

[ii] Find the zero-state response *y[k]*, by using the unilateral z-transform.

*Cari sambutan keadaan kosong, y[k] menggunakan jelmaan-z unilateral.*

(9%)

- ooOoo -

**A Short Table of Fourier Transforms**

| | $f(t)$ | $F(\omega)$ | |
|---|---|---|---|
| 1 | $e^{-at}u(t)$ | $\dfrac{1}{a+j\omega}$ | $a > 0$ |
| 2 | $e^{at}u(-t)$ | $\dfrac{1}{a-j\omega}$ | $a > 0$ |
| 3 | $e^{-a\|t\|}$ | $\dfrac{2a}{a^2+\omega^2}$ | $a > 0$ |
| 4 | $te^{-at}u(t)$ | $\dfrac{1}{(a+j\omega)^2}$ | $a > 0$ |
| 5 | $t^n e^{-at}u(t)$ | $\dfrac{n!}{(a+j\omega)^{n+1}}$ | $a > 0$ |
| 6 | $\delta(t)$ | $1$ | |
| 7 | $1$ | $2\pi\delta(\omega)$ | |
| 8 | $e^{j\omega_0 t}$ | $2\pi\delta(\omega-\omega_0)$ | |
| 9 | $\cos\omega_0 t$ | $\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$ | |
| 10 | $\sin\omega_0 t$ | $j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$ | |
| 11 | $u(t)$ | $\pi\delta(\omega)+\frac{1}{j\omega}$ | |
| 12 | $\operatorname{sgn} t$ | $\frac{2}{j\omega}$ | |
| 13 | $\cos\omega_0 t\, u(t)$ | $\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$ | |
| 14 | $\sin\omega_0 t\, u(t)$ | $\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$ | |
| 15 | $e^{-at}\sin\omega_0 t\, u(t)$ | $\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$ | $a > 0$ |
| 16 | $e^{-at}\cos\omega_0 t\, u(t)$ | $\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$ | $a > 0$ |
| 17 | $\operatorname{rect}\left(\frac{t}{\tau}\right)$ | $\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$ | |
| 18 | $\frac{W}{\pi}\operatorname{sinc}(Wt)$ | $\operatorname{rect}\left(\frac{\omega}{2W}\right)$ | |
| 19 | $\Delta\left(\frac{t}{\tau}\right)$ | $\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$ | |
| 20 | $\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$ | $\Delta\left(\frac{\omega}{2W}\right)$ | |
| 21 | $\sum_{n=-\infty}^{\infty}\delta(t-nT)$ | $\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$ | $\omega_0=\frac{2\pi}{T}$ |
| 22 | $e^{-t^2/2\sigma^2}$ | $\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$ | |

**Fourier Transform Operations**

| Operation | $f(t)$ | $F(\omega)$ |
|---|---|---|
| Addition | $f_1(t) + f_2(t)$ | $F_1(\omega) + F_2(\omega)$ |
| Scalar multiplication | $kf(t)$ | $kF(\omega)$ |
| Symmetry | $F(t)$ | $2\pi f(-\omega)$ |
| Scaling ($a$ real) | $f(at)$ | $\frac{1}{\|a\|}F\left(\frac{\omega}{a}\right)$ |
| Time shift | $f(t - t_0)$ | $F(\omega)e^{-j\omega t_0}$ |
| Frequency shift ($\omega_0$ real) | $f(t)e^{j\omega_0 t}$ | $F(\omega - \omega_0)$ |
| Time convolution | $f_1(t) * f_2(t)$ | $F_1(\omega)F_2(\omega)$ |
| Frequency convolution | $f_1(t)f_2(t)$ | $\frac{1}{2\pi}F_1(\omega) * F_2(\omega)$ |
| Time differentiation | $\frac{d^n f}{dt^n}$ | $(j\omega)^n F(\omega)$ |
| Time integration | $\int_{-\infty}^{t} f(x)\,dx$ | $\frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$ |

## B.7 Miscellaneous

### B.7-1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form $0/0$ or $\infty/\infty$, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)}$$

### B.7-2 The Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!}\dot{f}(a) + \frac{(x-a)^2}{2!}\ddot{f}(a) + \cdots$$

$$f(x) = f(0) + \frac{x}{1!}\dot{f}(0) + \frac{x^2}{2!}\ddot{f}(0) + \cdots$$

### B.7-3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{k}x^k + \cdots + x^n$$

$$\approx 1 + nx \qquad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

### B.7-4 Sums

$$\sum_{m=0}^{k} r^m = \frac{r^{k+1}-1}{r-1} \qquad r \neq 1$$

$$\sum_{m=M}^{N} r^m = \frac{r^{N+1}-r^M}{r-1} \qquad r \neq 1$$

$$\sum_{m=0}^{k} \left(\frac{a}{b}\right)^m = \frac{a^{k+1}-b^{k+1}}{b^k(a-b)} \qquad a \neq b$$

### B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2 + b^2}, \qquad \theta = \tan^{-1}\left(\tfrac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

### B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \tfrac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \tfrac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos\left(x \pm \tfrac{\pi}{2}\right) = \mp\sin x$$

$$\sin\left(x \pm \tfrac{\pi}{2}\right) = \pm\cos x$$

$$2\sin x\cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \tfrac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \tfrac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

$$\sin x\sin y = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x\cos y = \tfrac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x\cos y = \tfrac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$a\cos x + b\sin x = C\cos(x + \theta)$$

in which $C = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\dfrac{-b}{a}\right)$

## B.7-7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2x^2\sin ax)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}\,dx = \frac{1}{2}\ln(x^2+a^2)$$

### B.7-8 Differentiation Table

| | |
|---|---|
| $\frac{d}{dx}f(u) = \frac{d}{du}f(u)\frac{du}{dx}$ | $\frac{d}{dx}a^{bx} = b(\ln a)a^{bx}$ |
| $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ | $\frac{d}{dx}\sin ax = a\cos ax$ |
| $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ | $\frac{d}{dx}\cos ax = -a\sin ax$ |
| $\frac{dx^n}{dx} = nx^{n-1}$ | $\frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$ |
| $\frac{d}{dx}\ln(ax) = \frac{1}{x}$ | $\frac{d}{dx}(\sin^{-1} ax) = \frac{a}{\sqrt{1-a^2x^2}}$ |
| $\frac{d}{dx}\log(ax) = \frac{\log e}{x}$ | $\frac{d}{dx}(\cos^{-1} ax) = \frac{-a}{\sqrt{1-a^2x^2}}$ |
| $\frac{d}{dx}e^{bx} = be^{bx}$ | $\frac{d}{dx}(\tan^{-1} ax) = \frac{a}{1+a^2x^2}$ |

### B.7-9 Some Useful Constants

$\pi \approx 3.1415926535$

$e \approx 2.7182818284$

$\frac{1}{e} \approx 0.3678794411$

$\log_{10} 2 = 0.30103$

$\log_{10} 3 = 0.47712$

### B.7-10 Solution of Quadratic and Cubic Equations

Any **quadratic** equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**TABLE 2.1: Convolution Table**

| No | $f_1(t)$ | $f_2(t)$ | $f_1(t) * f_2(t) = f_2(t) * f_1(t)$ |
|---|---|---|---|
| 1 | $f(t)$ | $\delta(t-T)$ | $f(t-T)$ |
| 2 | $e^{\lambda t}u(t)$ | $u(t)$ | $\frac{1-e^{\lambda t}}{-\lambda}u(t)$ |
| 3 | $u(t)$ | $u(t)$ | $tu(t)$ |
| 4 | $e^{\lambda_1 t}u(t)$ | $e^{\lambda_2 t}u(t)$ | $\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t) \quad \lambda_1 \neq \lambda_2$ |
| 5 | $e^{\lambda t}u(t)$ | $e^{\lambda t}u(t)$ | $te^{\lambda t}u(t)$ |
| 6 | $te^{\lambda t}u(t)$ | $e^{\lambda t}u(t)$ | $\frac{1}{2}t^2e^{\lambda t}u(t)$ |
| 7 | $t^n u(t)$ | $e^{\lambda t}u(t)$ | $\frac{n!\,e^{\lambda t}}{\lambda^{n+1}}u(t) - \sum_{j=o}^{n}\frac{n!\,t^{n-j}}{\lambda^{j+1}(n-j)!}u(t)$ |
| 8 | $t^m u(t)$ | $t^n u(t)$ | $\frac{m!n!}{(m+n+1)!}t^{m+n+1}u(t)$ |
| 9 | $te^{\lambda_1 t}u(t)$ | $e^{\lambda_2 t}u(t)$ | $\frac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$ |
| 10 | $t^m e^{\lambda t}u(t)$ | $t^n e^{\lambda t}u(t)$ | $\frac{m!\,n!}{(n+m+1)!}t^{m+n+1}e^{\lambda t}u(t)$ |
| 11 | $t^m e^{\lambda_1 t}u(t)$ | $t^n e^{\lambda_2 t}u(t)$ | $\sum_{j=0}^{m}\frac{(-1)^j m!(n+j)!\,t^{m-j}e^{\lambda_1 t}}{j!(m-j)!(\lambda_1-\lambda_2)^{n+j+1}}u(t)$ |
| | $\lambda_1 \neq \lambda_2$ | | $+\sum_{k=0}^{n}\frac{(-1)^k n!(m+k)!\,t^{n-k}e^{\lambda_2 t}}{k!(n-k)!(\lambda_2-\lambda_1)^{m+k+1}}u(t)$ |
| 12 | $e^{-\alpha t}\cos(\beta t+\theta)u(t)$ | $e^{\lambda t}u(t)$ | $\frac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$ |
| | | | $\phi = \tan^{-1}[-\beta/(\alpha+\lambda)]$ |
| 13 | $e^{\lambda_1 t}u(t)$ | $e^{\lambda_2 t}u(-t)$ | $\frac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1} \quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$ |
| 14 | $e^{\lambda_1 t}u(-t)$ | $e^{\lambda_2 t}u(-t)$ | $\frac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$ |

**TABLE 9.1: Convolution Sums**

| No. | $f_1[k]$ | $f_2[k]$ | $f_1[k] * f_2[k] = f_2[k] * f_1[k]$ | |
|---|---|---|---|---|
| 1 | $\delta[k-j]$ | $f[k]$ | $f[k-j]$ | |
| 2 | $\gamma^k u[k]$ | $u[k]$ | $\left[\frac{1-\gamma^{k+1}}{1-\gamma}\right] u[k]$ | |
| 3 | $u[k]$ | $u[k]$ | $(k+1)u[k]$ | |
| 4 | $\gamma_1^k u[k]$ | $\gamma_2^k u[k]$ | $\left[\frac{\gamma_1^{k+1}-\gamma_2^{k+1}}{\gamma_1-\gamma_2}\right] u[k] \quad \gamma_1 \neq \gamma_2$ | |
| 5 | $\gamma_1^k u[k]$ | $\gamma_2^k u[-(k+1)]$ | $\frac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^k u[k] + \frac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^k u[-(k+1)]$ | $\lvert\gamma_2\rvert > \lvert\gamma_1\rvert$ |
| 6 | $k\gamma_1^k u[k]$ | $\gamma_2^k u[k]$ | $\frac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left[\gamma_2^k - \gamma_1^k + \frac{\gamma_1-\gamma_2}{\gamma_2}k\gamma_1^k\right] u[k]$ | $\gamma_1 \neq \gamma_2$ |
| 7 | $ku[k]$ | $ku[k]$ | $\frac{1}{6}k(k-1)(k+1)u[k]$ | |
| 8 | $\gamma^k u[k]$ | $\gamma^k u[k]$ | $(k+1)\gamma^k u[k]$ | |
| 9 | $\gamma^k u[k]$ | $ku[k]$ | $\left[\frac{\gamma(\gamma^k-1)+k(1-\gamma)}{(1-\gamma)^2}\right] u[k]$ | |
| 10 | $\lvert\gamma_1\rvert^k \cos(\beta k+\theta)u[k]$ | $\gamma_2^k u[k]$ | $\frac{1}{R}\left[\lvert\gamma_1\rvert^{k+1}\cos[\beta(k+1)+\theta-\phi] - \gamma_2^{k+1}\cos(\theta-\phi)\right] u[k]$ | $\gamma_2$ real |
| | | | $R = \left[\lvert\gamma_1\rvert^2 + \gamma_2^2 - 2\lvert\gamma_1\rvert\gamma_2\cos\beta\right]^{1/2}$ | |
| | | | $\phi = \tan^{-1}\left[\frac{(\lvert\gamma_1\rvert\sin\beta)}{(\lvert\gamma_1\rvert\cos\beta - \gamma_2)}\right]$ | |

**Table 11.1: (Unilateral) $z$-Transform Pairs**

| | $f[k]$ | | $F[z]$ |
|---|---|---|---|
| 1 | $\delta[k-j]$ | | $z^{-j}$ |
| 2 | $u[k]$ | | $\frac{z}{z-1}$ |
| 3 | $ku[k]$ | | $\frac{z}{(z-1)^2}$ |
| 4 | $k^2u[k]$ | | $\frac{z(z+1)}{(z-1)^3}$ |
| 5 | $k^3u[k]$ | | $\frac{z(z^2+4z+1)}{(z-1)^4}$ |
| 6 | $\gamma^{k-1}u[k-1]$ | | $\frac{1}{z-\gamma}$ |
| 7 | $\gamma^k u[k]$ | | $\frac{z}{z-\gamma}$ |
| 8 | $k\gamma^k u[k]$ | | $\frac{\gamma z}{(z-\gamma)^2}$ |
| 9 | $k^2\gamma^k u[k]$ | | $\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$ |
| 10 | $\frac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$ | | $\frac{z}{(z-\gamma)^{m+1}}$ |
| 11$a$ | $\lvert\gamma\rvert^k \cos \beta k\, u[k]$ | | $\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$ |
| 11$b$ | $\lvert\gamma\rvert^k \sin \beta k\, u[k]$ | | $\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$ |
| 12$a$ | $r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$ | | $\frac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$ |
| 12$b$ | $r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$ | $\gamma = \lvert\gamma\rvert e^{j\beta}$ | $\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$ |
| 12$c$ | $r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$ <br> $r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ <br> $\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$ | | $\frac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$ |

Table 11.2

$\mathcal{Z}$- Transform Operations

| Operation | $f[k]$ | $F[z]$ |
|---|---|---|
| Addition | $f_1[k] + f_2[k]$ | $F_1[z] + F_2[z]$ |
| Scalar multiplication | $af[k]$ | $aF[z]$ |
| Right-shift | $f[k-m]u[k-m]$ | $\frac{1}{z^m}F[z]$ |
| | $f[k-m]u[k]$ | $\frac{1}{z^m}F[z] + \frac{1}{z^m}\sum_{k=1}^{m} f[-k]z^k$ |
| | $f[k-1]u[k]$ | $\frac{1}{z}F[z] + f[-1]$ |
| | $f[k-2]u[k]$ | $\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$ |
| | $f[k-3]u[k]$ | $\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$ |
| Left-shift | $f[k+m]u[k]$ | $z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$ |
| | $f[k+1]u[k]$ | $zF[z] - zf[0]$ |
| | $f[k+2]u[k]$ | $z^2F[z] - z^2f[0] - zf[1]$ |
| | $f[k+3]u[k]$ | $z^3F[z] - z^3f[0] - z^2f[1] - zf[2]$ |
| Multiplication by $\gamma^k$ | $\gamma^k f[k]u[k]$ | $F\left[\frac{z}{\gamma}\right]$ |
| Multiplication by $k$ | $kf[k]u[k]$ | $-z\frac{d}{dz}F[z]$ |
| Time Convolution | $f_1[k] * f_2[k]$ | $F_1[z]F_2[z]$ |
| Frequency Convolution | $f_1[k]f_2[k]$ | $\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}\,du$ |
| Initial value | $f[0]$ | $\lim_{z\to\infty} F[z]$ |
| Final value | $\lim_{N\to\infty} f[N]$ | $\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle. |