UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

IUK 291 – MATEMATIK II
[MATHEMATICS II]

Duration: 3 hours
Masa: [3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer FOUR (4) questions. You may answer the questions either in Bahasa Malaysia or in English.

[Arahan: *Jawab EMPAT (4) soalan. Anda dibenarkan menjawab soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) Find the Taylor series for $f(x) = 5 + 3x + 2x^2$ centred at $a = 3$.

(5 marks)

(b) In what direction is the function defined by $f(x, y) = xe^{3x-y}$ increasing most rapidly at the point $P_0(1,3)$, and what is the maximum rate of increase? In what direction is the function decreasing most rapidly?

(10 marks)

(c) Evaluate the integral by first reversing the order of integration

$$\int_1^4 \int_{\sqrt{x}}^2 (x + 2y)\,dy\,dx$$

(10 marks)

2. (a) Find the local maximum, minimum or saddle points of the function $f(x, y) = e^{-4y}(x^2 + y^2)$. Use the second partial test to classify each point.

(9 marks)

(b) A cylinder with no top is to be constructed from 12π cm^2 of material. Use the method of Lagrange to determine the dimensions of the cylinder if it is to enclose the maximum volume.

(6 marks)

(c) Evaluate $\int_0^1 \int_2^4 \frac{xy^2}{x^2+1}\,dy\,dx$

(5 marks)

(d) Find the volume under the plane $z = 2x + y + 3$ above the region D bounded by the lines $y = 2x$, $y = 3 - x$ and $y = 0$.

(5 marks)

3. (a) Use partial fractions to find Maclaurin series for the function

$$f(x) = \frac{5 + 5x}{x^2 + 3x - 4}$$

(10 marks)

(b) Find the values of x for which the series $\sum_{n=0}^{\infty} 8^n x^n$ converges and find the sum of the series for the value of x.

(5 marks)

(c) Solve $y'' - y' - 2y = x^2 e^x$.

(5 marks)

(d) Evaluate $\iint_R xy \, dA$ over the region R enclosed between $y = \frac{x}{2}$, $y = \sqrt{x}$, $x = 3$ and $x = 2$.

(5 marks)

4. (a) Use the binomial series to obtain the power expansion of $\frac{1}{\sqrt{1-x^3}}$.

(5 marks)

(b) Find the interval of convergence for the power series $\sum_{k=1}^{\infty} k^2 4^k (x+2)^k$.

(5 marks)

(c) Use the Ratio test to test the series $\sum_{k=0}^{\infty} \frac{k!}{k^2 (k+1)^2}$ for convergence.

(5 marks)

(d) Solve the initial value problem using the method of undetermined coefficients.

$$y'' + 2y' + 2y = \cos x \qquad y(0) = 0, \; y'(0) = -4$$

(10 marks)

1. (a) *Dapatkan siri Taylor bagi fungsi* $f(x) = 5 + 3x + 2x^2$ *berpusat di* $a = 3$.

(5 markah)

(b) *Pada arah manakah fungsi* $f(x, y) = xe^{3x-y}$ *meningkat mendadak di titik* $P_0(1,3)$, *dan apakah kadar kenaikan maksima? Pada arah manakah fungsi tersebut menurun mendadak?*

(10 markah)

(c) *Selesaikan kamiran tersebut dengan menyongsangkan dahulu tertibkamiran.*

$$\int_1^4 \int_{\sqrt{x}}^2 (x + 2y)\,dydx$$

(10 markah)

2. (a) *Cari titik maksima setempat, minima setempat atau titik lengkok balas bagi fungsi* $f(x, y) = e^{-4y}(x^2 + y^2)$. *Guna ujian separa kedua untuk mengkelaskan setiap titik.*

(9 markah)

(b) *Sebuah silinder tanpa penutup dibina dari* $12\pi\ sm^2$ *bahan. Guna kaedah pendarab Lagrange utuk menentukan dimensi kotak bagi mendapatkan isipadu maksima.*

(6 markah)

(c) *Selesaikan* $\int_0^1 \int_2^4 \frac{xy^2}{x^2 + 1}\,dydx$

(5 markah)

(d) *Cari isipadu di bawah satah* $z = 2x + y + 3$ *di atas kawasan D yang dibatasi oleh garis-garis* $y = 2x$, $y = 3 - x$ *dan* $y = 0$.

(5 markah)

3. (a) *Guna pecahan separa untuk mencari siri Maclaurin bagi fungsi*

$$f(x) = \frac{5 + 5x}{x^2 + 3x - 4}.$$

(10 markah)

(b) *Dapatkan nilai-nilai* x *di mana siri* $\sum_{n=0}^{\infty} 8^n x^n$ *menumpu dan dapatkan jumlah*
bagi siri tersebut untuk nilai-nilai x.

(5 markah)

(c) *Selesaikan* $y''-y'-2y=x^2e^x$.

(5 markah)

(d) *Selesaikan* $\iint_R xy\,dA$ *di atas kawasan* R *yang ditutupi antara* $y=\frac{x}{2}$, $y=\sqrt{x}$, $x=3$ *and* $x=2$.

(5 markah)

4. (a) *Guna siri binomial untuk mendapatkan kembangan siri kuasa bagi*

$$\frac{1}{\sqrt{1-x^3}}.$$

(5 markah)

(b) *Cari jeda penumpuan bagi siri kuasa* $\sum_{k=1}^{\infty} k^2 4^k (x+2)^k$.

(5 markah)

(c) *Guna Ujian Nisbah untuk menguji siri* $\sum_{k=0}^{\infty} \frac{k!}{k^2(k+1)^2}$ *bagi penumpuan.*

(5 markah)

(d) *Selesaikan masalah nilai awal dengan kaedah koefisien tak ditentukan.*

$$y''+2y'+2y=\cos x \qquad y(0)=0,\ y'(0)=-4$$

(10 markah)