
UNIVERSITI SAINS MALAYSIA

Supplementary Semester Examination
Academic Session 2009/2010

June 2010

IUK 291 – MATHEMATICS II
[MATEMATIK II]

Duration: 3 hours
[Masa: 3 jam]

Please check that the examination paper consists of **FIVE (5)** pages of printed material before you begin this examination.

Answer **FOUR** questions. All questions can be answered in Bahasa Malaysia OR English.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA (5)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

*Jawab **EMPAT** soalan. Semua soalan boleh dijawab dalam Bahasa Malaysia ATAU Bahasa Inggeris.*

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Answer all questions.

1. (i) Find the Taylor series for $f(x) = 6 + 8x + 9x^3$ centred at $a=2$.
(6 marks)
- (ii) In what direction is the function defined by $f(x, y) = xe^{3x-y}$ increasing most rapidly at the point $P_0(1,3)$, and what is the maximum rate of increase?
(9 marks)
- (iii) Evaluate $\int_1^2 \int_1^{y^2} (x+2y) dx dy$.
(10 marks)
2. (i) Find the local maximum, minimum or saddle points of the function $f(x, y) = e^{-4y} (x^2 + y^2)$. Use the second partial test to classify each point.
(9 marks)
- (ii) A cylinder with no top is to be constructed from 24π cm² of material. Use the method of Lagrange to determine the dimensions of the cylinder if it is to enclose the maximum volume. (Hint: $V = \pi r^2 h$ and $S = \pi r^2 + 2\pi r h$)
(6 marks)
- (iii) Find the values of x for which the series $\sum_{n=0}^{\infty} (4x)^n$ converges and find the sum of the series for the value of x .
(5 marks)
- (iv) Show that

$$\int_{-1}^1 \int_0^2 (4 + x^2 - y^2) dy dx = 12$$

(5 marks)

3. (i) Use partial fractions to find Maclaurin series for the function

$$f(x) = \frac{5 + 5x}{x^2 + 3x - 4}$$

(10 marks)

- (ii) Find the volume under the plane $z = 2x + y + 3$ above the region D bounded by the lines $y = 2x$, $y = 3 - x$ and $y = 0$.

(10 marks)

- (iii) Evaluate $\iint_R xy \, dA$ over the region R enclosed between $y = 2$, $y = 4$, $x = 1$ and $x = 3$.

(5 marks)

4. (i) Use the binomial series to obtain the power expansion of $\frac{1}{\sqrt{1-x^2}}$.

(8 marks)

- (ii) Find the interval of convergence for the power series $\sum_{k=1}^{\infty} k^2 4^k (x+2)^k$.

(7 marks)

- (iii) Solve $y'' - y' - 2y = x$.

(10 marks)

Jawab semua soalan.

1. (i) Dapatkan siri Taylor bagi fungsi $f(x) = 6 + 8x + 9x^3$ berpusat di $a=2$.
(6 markah)
- (ii) Pada arah manakah fungsi $f(x, y) = xe^{3x-y}$ meningkat mendadak di titik $P_0(1,3)$, dan apakah kadar kenaikan maksima?
(9 markah)
- (iii) Selesaikan $\int_1^2 \int_1^{y^2} (x+2y) dx dy$.
(10 markah)
2. (i) Cari titik maksima setempat, minima setempat atau titik lengkok balas bagi fungsi $f(x, y) = e^{-4y} (x^2 + y^2)$. Guna ujian separa kedua untuk mengelaskan setiap titik.
(9 markah)
- (ii) Sebuah silinder tanpa penutup dibina dari $24\pi \text{ sm}^2$ bahan. Guna kaedah pendarab Lagrange untuk menentukan dimensi kotak bagi mendapatkan isipadu maksima. (Petua: $V = \pi r^2 h$ and $S = \pi r^2 + 2\pi r h$)
(6 markah)
- (iii) Dapatkan nilai-nilai x di mana siri $\sum_{n=0}^{\infty} (4x)^n$ menumpu dan dapatkan jumlah bagi siri tersebut untuk nilai-nilai x .
(5 markah)
- (iv) Tunjukkan bahawa

$$\int_{-1}^1 \int_0^2 (4 + x^2 - y^2) dy dx = 12$$

(5 markah)

3. (i) Guna pecahan separa untuk mencari siri Maclaurin bagi fungsi

$$f(x) = \frac{5 + 5x}{x^2 + 3x - 4}.$$

(10 markah)

- (ii) Cari isipadu di bawah satah $z = 2x + y + 3$ di atas kawasan D yang dibatasi oleh garis-garis $y = 2x$, $y = 3 - x$ dan $y = 0$.

(10 markah)

- (iii) Selesaikan $\iint_R xy \, dA$ di atas kawasan R yang ditutupi antara $y = 2$, $y = 4$, $x = 1$ dan $x = 3$.

(5 markah)

4. (i) Guna siri binomial untuk mendapatkan kembangan siri kuasa bagi

$$\frac{1}{\sqrt{1-x^2}}.$$

(8 markah)

- (ii) Cari jeda penumpuan bagi siri kuasa $\sum_{k=1}^{\infty} k^2 4^k (x+2)^k$.

(7 markah)

- (iii) Selesaikan $y'' - y' - 2y = x$.

(10 markah)