UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2010/2011

November 2010

EMH 451/3 – Numerical Methods For Engineers
Kaedah Berangka Untuk Jurutera

Duration : 2 hours
Masa : 2 jam

INSTRUCTIONS TO CANDIDATE:
ARAHAN KEPADA CALON:

Please check that this paper contains **SEVEN (7)** printed pages, **ONE (1)** page appendix and **THREE (3)** questions before you begin the examination.
*Sila pastikan bahawa kertas soalan ini mengandungi **TUJUH (7)** mukasurat bercetak, **SATU (1)** mukasurat lampiran dan **TIGA (3)** soalan sebelum anda memulakan peperiksaan.*

Answer **ALL** questions.
*Jawab **SEMUA** soalan.*

Appendix/*Lampiran :*

1. Pseudocode for the LU Method and the CG Method [1 page/mukasurat]

You may answer all questions in **English** OR **Bahasa Malaysia** OR a combination of both.
*Calon boleh menjawab semua soalan dalam **Bahasa Malaysia** ATAU **Bahasa Inggeris** ATAU kombinasi kedua-duanya.*

Answer to each question must begin from a new page.
Jawapan untuk setiap soalan mestilah dimulakan pada mukasurat yang baru.

In the event of any discrepancies, the English version shall be used.
Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.

Q1. [a] Provide <u>BRIEF</u> answers to the following questions:

(i) What is Galerkin's method?

(ii) What is the condition of the basis functions used as finite elements?

(iii) What is the order of convergence in error for Finite Element Method (FEM) with respect to element size *h*?

(iv) Describe briefly about the LU decomposition method and Conjugate Gradient (CG) method including their advantages and disadvantages.

Berikan jawapan-jawapan <u>RINGKAS</u> bagi soalan-soalan berikut:

(i) Apakah kaedah Galerkin?

(ii) Apakah syarat bagi fungsi-fungsi basis yang digunakan sebagai unsur-unsur terhingga?

(iii) Apakah darjah penumpuan di dalam ralat bagi FEM yang berkaitan dengan saiz unsur h?

(iv) Terangkan dengan secara ringkas tentang kaedah penguraian LU dan kaedah kecerunan konjugat (CG) termasuk kebaikan dan kekurangan masing-masing.

(20 marks/*markah*)

[b] Consider a heat conduction problem in a straight metal wire of 1 m length. The temperatures at x = 0 m and x = 1 m are 0 K and 200 K, respectively. A constant heat source of 5 W/m is supplied along the wire. The material has thermal conductivity of 2 W/K/m.

(i) State the strong form of the PDE for the above problem (derivation not necessary).

(ii) Suppose the domain is discretized equally, and the mesh has the element and node numbers as in Figure Q1[b]. Set up the corresponding system matrix K and load vector b. Ignore the effect of the boundary conditions. Show the non-zero values in the matrix and vector with "*". Do not solve the problem.

(iii) Without any calculations, sketch the plot of the expected FEM solution of the temperature $T(x)$ along the wire given by the mesh above.

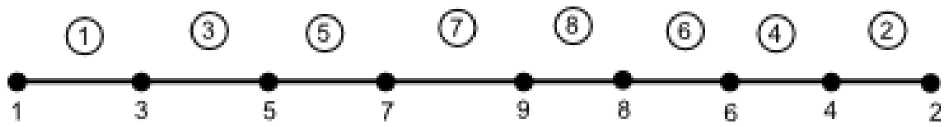

Figure Q1[b]
Rajah S1[b]

Diberikan masalah pengaliran haba di dalam sebatang wayar logam lurus sepanjang 1 m. Suhu pada x = 0 m and x = 1 m adalah masing-masing 0 K dan 200 K. Sumber haba malar 5 W/m diberikan sepanjang wayar. Bahan itu mempunyai kekonduksian haba 2 W/K/m.

(i) Nyatakan bentuk kuat persamaan spara kebezaan bagi masalah di atas (terbitan tidak perlu).

(ii) Andaikan domain itu dibahagikan sama rata, dan jaring itu mempunyai nombor unsur dan nod seperti yang tertera di dalam Rajah S1[b]. Binakan matriks sistem **K** *dan vektor beban* **b**. *Abaikan kesan syarat-syarat sempadan Tunjukkan nilai-nilai bukan sifar di dalam matriks dan vektor dengan simbol "*". Jangan selesaikan masalah ini.*

(iii) Tanpa pengiraan, lakarkan plot bagi suhu yang dijangkakan T(x) dari penyelesaian FEM yang diberikan oleh jaring di atas pada sepanjang wayar itu.

(40 marks/*markah***)**

[c] For a linear system A·x = b, where A = [1 -1 -2; -1 4 0; -2 0 1] and b = [1; 2; 0], use either the LU decomposition or the CG method to solve for x. If the CG method is used, show <u>three</u> iteration steps to find x. Refer to the respective pseudocodes in the Appendix.

Bagi sistem linear **A·x** = **b**, *di mana* **A** = [1 -1 -2; -1 4 0; -2 0 1] *dan* **b** = [1; 2; 0], *gunakan samada kaedah uraian LU atau kaedah kecerunan konjugat (CG) untuk menyelesaikan* **x**. *Jika kaedah CG digunakan, tunjukkan <u>tiga</u> langkah lelaran untuk mencari* **x.** *Rujuk pseudokod yang berkaitan di dalam Lampiran.*

(40 marks/*markah***)**

Q2. [a] State and show that whether the following equations are elliptic, parabolic or hyperbolic.

Nyata dan tunjukkan samada persamaan berikut adalah elip, parabola atau hiperbola.

(i) $3\dfrac{\partial^2 f}{\partial x^2} + 4\dfrac{\partial^2 f}{\partial y^2} = 0$

(ii) $\dfrac{\partial^2 f}{\partial x^2} - \dfrac{\partial^2 f}{\partial y^2} = 0$

(iii) $\dfrac{\partial^2 f}{\partial x^2} + 2\dfrac{\partial^2 f}{\partial y^2} + 4\dfrac{\partial f}{\partial y} = 0$

(iv) $\dfrac{\partial^2 f}{\partial x \partial y} - \dfrac{\partial f}{\partial y} = 0$

(v) $\dfrac{\partial^2 f}{\partial x^2} + 6\dfrac{\partial^2 f}{\partial x \partial y} + 9\dfrac{\partial^2 f}{\partial y^2} = 0$

(30 marks/*markah*)

[b] Discretize the following first and second derivatives using the central differential approximations based on the grid given in Figure Q2[b].

Diskretkan pembezaan pertama dan kedua berikut dengan menggunakan anggaran pembezaan pusat berdasarkan grid yang diberikan di dalam Rajah S2[b].

(i) $\dfrac{\partial T}{\partial x}$ (ii) $\dfrac{\partial T}{\partial y}$ (iii) $\dfrac{\partial^2 T}{\partial x^2}$ (iv) $\dfrac{\partial^2 T}{\partial y^2}$ (v) $\dfrac{\partial^2 T}{\partial x \partial y}$

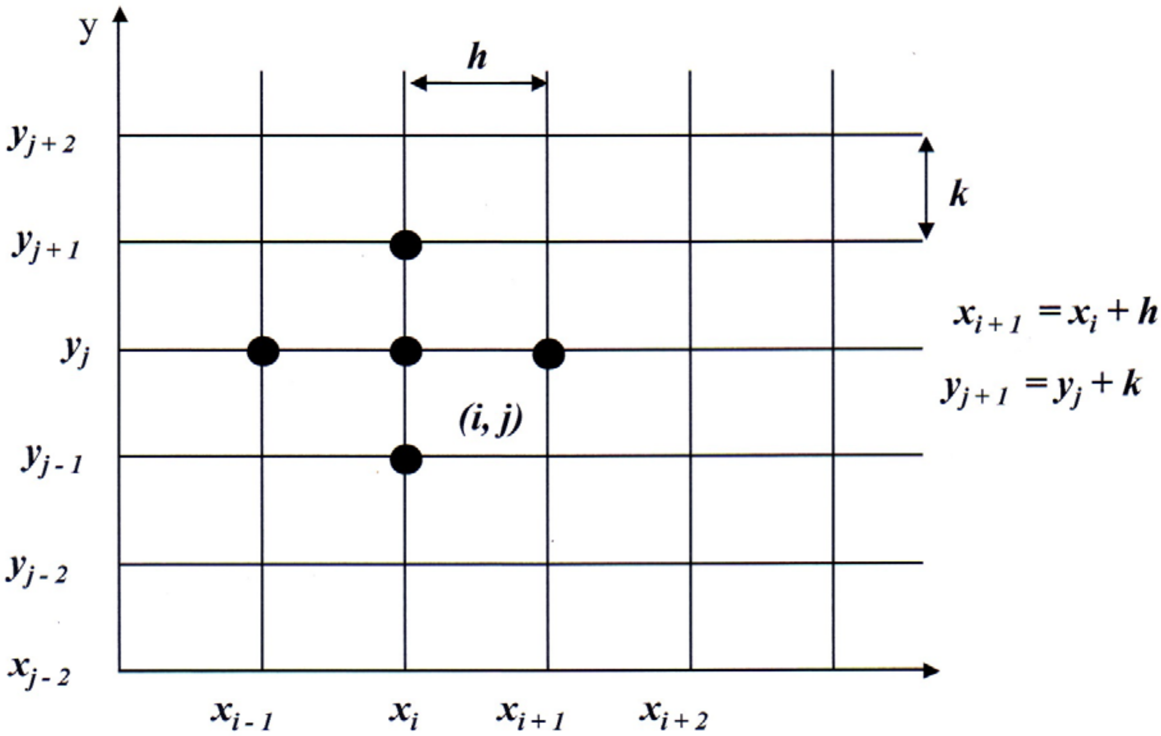

Figure Q2[b]
Rajah S2[b]

(30 marks/*markah*)

[c] **The problem of determining the subsurface temperature fluctuations of rock as the result of daily temperature variations can be predicted using the equation below:**

Masalah menentukan turun naik suhu sub-permukaan bagi batu disebabkan oleh perubahan suhu harian boleh diramalkan dengan menggunakan persamaan di bawah:

$$\frac{\partial T(t,x)}{\partial t} = K\frac{\partial^2 T}{\partial x^2} \quad , \quad 0 \le t \le 24hrs \ \ \textbf{and} \quad 0 \le x \le 100m$$

where *T* is temperature, *x* is the depth from the surface, and *t* is time. *K* is a constant.

di mana T ialah suhu, x ialah kedalaman dari permukaan, dan t ialah masa. K ialah pemalar.

Given the boundary and initial conditions as

Diberikan keadaan sempadan dan awal adalah

$$T(t,0) = 15 - 10\sin\left(\frac{2\pi t}{24}\right), \qquad 0 \le t \le 24hrs$$

$$\frac{\partial T(t,100)}{\partial x} = 0$$

$$T(0,x) = 15.0 \qquad 0 \le x \le 100m$$

Use the explicit formula as

Gunakan formula tersurat sebagai

$$T_i^{l+1} = T_i^l + \lambda(T_{i+1}^l - 2T_i^l + T_{i-1}^l)$$

where $\lambda = K\Delta t/\Delta x^2$ **and K is 1 × 10^{-6} m^2/s. Using FIVE grid points including the boundaries, calculate the temperature of the rock for first four time steps and comment on the results.**

Di sini $\lambda = K\Delta t/\Delta x^2$ *dan K adalah 1 × 10^{-6} m^2/s. Dengan menggunakan, LIMA titik grid termasuk sempadan, kirakan suhu batu bagi empat selang masa yang pertama dan komen ke atas keputusan yang didapati.*

(40 marks/*markah*)

Q3. [a] In your own words, explain the differences between finite volume (FV), finite difference (FD) and finite element (FE) methods. Are these three methods related to each other? If so, please mathematically relate all the three methods.

Dengan menggunakan pemahaman sendiri, terangkan perbezaan antara kaedah FV, FD dan FE. Adakah kaedah-kaedah ini mempunyai kaitan antara satu sama lain? Jika ya, tuliskan persamaan matematik yang mengaitkan ketiga-tiga kaedah tersebut.

(30 marks/*markah***)**

[b] Using Taylor series, in one dimension, show that the difference between the finite volume (FV) method from a finite difference (FD) method is O(Δx^2). State any conclusion that can be made.

Dengan menggunakan siri Taylor dalam satu dimensi, tunjukkan perbezaan antara kaedah FD dan FV adalah O(Δx^2). Nyatakan apakah kesimpulan yang boleh dibuat.

(30 marks/*markah***)**

[c] The temperature of a square conducting plate subjected to a fixed boundary conditions (BC) has been computed using an FV method on a uniform grid of 30 × 30 interval and the value is 725 K. The computation is repeated on a 15 × 15 interval, giving a temperature of 743 K.

Sekeping plat yang mempunyai bentuk segiempat sama dan mempunyai keadaan sempadan tetap telah dikira dengan menggunakan kaedah FV dan mempunyai catatan 725 K apabila menggunakan grid sekata 30 × 30. Pengiraan diulang dengan menggunakan grid 15 × 15 memberikan suhu 743 K.

Consider the two cases below.

Kaji dua kes di bawah.

(i) Assume the FV method is first order accurate. Estimate the temperature of the plate using 60 × 60 interval. If the grid size approaches zero, what is the best temperature prediction for this method?

Anggap kaedah FV di atas mempunyai ketepatan tahap satu. Dengan menggunakan grid 60 × 60, anggarkan suhu plat tersebut. Jika saiz grid menghampiri sifar, anggarkan suhu terbaik dengan menggunakan kaedah ini.

(20 marks/*markah***)**

(ii) **Now assume the FV method is second order accurate. If the grid size approaches zero, what is the best temperature prediction for this method?**

Anggap kaedah FV di atas mempunyai ketepatan tahap dua. Jika saiz grid menghampiri sifar, anggarkan suhu terbaik dengan menggunakan kaedah ini.

(20 marks/*markah*)

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APPENDIX 1/*LAMPIRAN 1*

Pseudocode for the LU Method

The algorithm creates $\mathbf{U}$ with unit diagonals:

$$\begin{bmatrix} 1 & u_{12} & u_{1n} \\ 0 & 1 & u_{2n} \\ 0 & 0 & 1 \end{bmatrix}$$

Algorithm:

$$l_{i,1} = a_{i,1} \qquad \text{for } i = 1, 2, \ldots, n$$

$$u_{1j} = \frac{a_{1j}}{l_{11}} \qquad \text{for } j = 2, 3, \ldots, n$$

For $j = 2, 3, \ldots, n-1$

$$l_{ij} = a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj} \qquad \text{for } i = j, j+1, \ldots, n$$

$$u_{jk} = \frac{a_{jk} - \sum_{i=1}^{j-1} l_{ji} u_{ik}}{l_{jj}} \qquad \text{for } k = j+1, j+2, \ldots, n$$

For the entry l_{nn}:

$$l_{nn} = a_{nn} - \sum_{k=1}^{n-1} l_{nk} u_{kn}$$

Then solve **Ly** = **b**; **Ux** = **y**

Pseudocode for the Conjugate Gradient (CG) Method

Start with some $\boldsymbol{x}_0$. Set $\boldsymbol{p}_0 = \boldsymbol{r}_0 = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}_0$.

For $k = 0, 1, 2, \ldots$

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha_k \boldsymbol{p}_k, \quad \alpha_k = \frac{\boldsymbol{r}_k^T \boldsymbol{r}_k}{\boldsymbol{p}_k^T \boldsymbol{A} \boldsymbol{p}_k}$$

$$\boldsymbol{r}_{k+1} = \boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}_{k+1} = \boldsymbol{r}_k - \alpha_k \boldsymbol{A}\boldsymbol{p}_k$$

$$\boldsymbol{p}_{k+1} = \boldsymbol{r}_{k+1} + \beta_k \boldsymbol{p}_k, \quad \beta_k = \frac{\boldsymbol{r}_{k+1}^T \boldsymbol{r}_{k+1}}{\boldsymbol{r}_k^T \boldsymbol{r}_k}$$