UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2010/2011 Academic Session

April/May 2011

EKC 314 – Transport Phenomena
[Fenomena Pengangkutan]

Duration : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains NINE printed pages and THREE printed page of Appendix before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak dan TIGA muka surat Lampiran sebelum anda memulakan peperiksaan ini.]

Instruction: Answer **FIVE (5)** questions. Answer **ALL (3)** questions from Section A. Answer any **TWO (2)** questions from Section B. All questions carry the same marks.

*[**Arahan:** Jawab **LIMA (5)** soalan. Jawab **SEMUA (3)** soalan dari Bahagian A. Jawab mana-mana **DUA (2)** soalan dari Bahagian B. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

<u>Section A</u> : Answer <u>ALL</u> questions.
<u>Bahagian A</u> : Jawab <u>SEMUA</u> soalan.

1. [a] Compute the mean molecular velocity $\bar{u}$ (in cm/s) and the mean free path, λ (in cm) for nitrogen (N_2) at 1 atm and 273.2 K. A reasonable value for d is 2.5 Å. What is the ratio of the mean free path to the molecular diameter under these conditions? What would be the order of magnitude of the corresponding ratio in the liquid state?

Kirakan purata halaju molekul $\bar{u}$ (dalam sm/s) *dan purata laluan bebas,* λ *(dalam sm) bagi nitrogen (N_2) pada 1 atm dan 273.2 K. Suatu nilai* d *yang berpatutan ialah 2.5 Å. Apakah nisbah purata laluan bebas kepada diameter molekul di bawah keadaan-keadaan ini? Apakah tertib magnitud bagi nisbah sepadan pada keadaan cecair?*

[10 marks/markah]

[b] Prove with an aid of diagram that the continuity equation is given by;
Buktikan dengan bantuan gambarajah bagi persamaan keterusan yang diberi sebagai;

$$\frac{\partial \rho}{\partial t} = -(\Delta \bullet \rho v) \qquad (1.1)$$

and that for an incompressible fluid;
dan bagi suatu bendalir ketidakbolehmampatan;

$$(\Delta \bullet v) = 0 \qquad (1.2)$$

[10 marks/markah]

2. A long cylinder nuclear fuel rod is surrounded by annular layer of aluminium cladding as shown in Figure Q.2. Within the fuel rod, heat is generated at a rate of S_n (kJ/m^{3}s) as given by the equation below.

Satu rod silinder bahan api nuklear yang panjang dikelilingi oleh anulus pelapisan aluminum seperti yang ditunjukkan dalam Gambarajah S.2. di bawah. Di dalam rod bahan api, haba dijana pada kadar S_n (kJ/m^{3}s) seperti yang diberi oleh persamaan di bawah

$$S_n = S_{no}\left[1 + b\left(\frac{r}{R_F}\right)^2\right] \qquad (2.1)$$

Here, S_{n0} and b are known constants and r is the radial coordinate measured from the axis of the cylindrical fuel rod. Heat is loss at the outer surface of the fuel rod to a gas stream at constant temperature of T_L (°C) by convective heat transfer with heat transfer coefficient, h_L(W/m^{2}K). The thermal conductivities of the fuel rod and cladding are k_F (W/mK) and k_C (W/mK).

S_{n0} *dan* b *adalah pemalar dan r adalah koordinat jejarian yang diukur dari paksi rod bahan api silinder. Haba di permukaan luar rod bahan api disebar ke aliran gas pada suhu malar* T_L *(°C) oleh pemindahan haba berolak dengan pekali pemindahan haba* h_L*(W/m^{2}K). Pengaliran terma rod bahan api dan pelapisan adalah* k_F *(W/mK) dan* k_C *(W/mK).*

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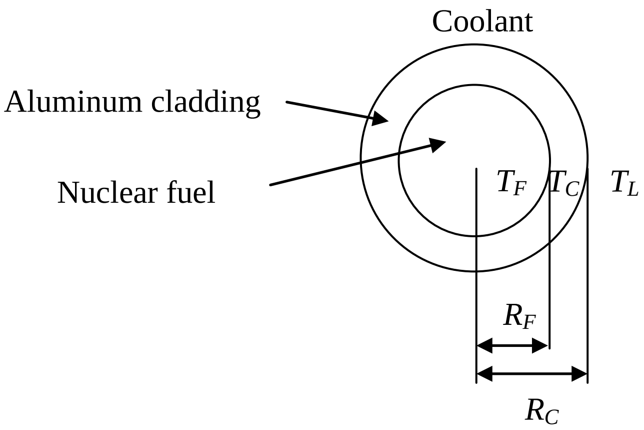

Figure Q.2.
Gambarajah S.2.

[a] Set up the differential equation that governs the temperature distribution *T(r)* radially in the fuel rod by making a shell energy balance.

Terbitkan persamaan pembezaan untuk taburan suhu T(r) *jejarian oleh rod bahan api melalui pengimbangan tenaga kelompang.*

[6 marks/markah]

[b] Show that the temperature profile within the fuel rod is given by:
Tunjukkan bahawa susuk suhu dalam rod bahan api diberi oleh:

$$T_F - T_L = -\frac{S_{n0} r^2}{4k_F}\left(1 + \frac{b}{R_F^2}\frac{r^2}{4}\right) + \frac{S_{n0}R_F^2}{4k_F}\left(1+\frac{b}{4}\right) + \frac{S_{n0}R_F^2}{2k_C}\left(1+\frac{b}{4}\right)\left(\frac{k_C}{R_C h_L} + \ln\frac{R_C}{R_F}\right) \quad (2.2)$$

[10 marks/markah]

[c] What is the limiting form of *T(r)* when the heat transfer coefficient is large?
Apakah bentuk terhad T(r) apabila pekali pemindahan haba adalah besar?

[2 marks/markah]

[d] What is the maximum temperature in the system?
Apakah suhu maksimum sistem?

[2 marks/markah]

3. Chlorine gas (A) is absorbed into pure cyclohexane contained in a beaker. The absorption occurs through the exposed surface of the cyclohexane which is stagnant within the beaker. The depth of cyclohexane in the beaker is L meters and can be regarded as large (i.e. $L \rightarrow \infty$). The reaction of chlorine with cyclohexane in the liquid phase is second order with respect to the concentration of chlorine. C_A is the concentration of chlorine ($kmol/m^3$) at a distance z from the surface, C_{AO} is the concentration of chlorine ($kmol/m^3$) at the surface, k_2 is the second order rate constant and D_{AB} is the diffusion coefficient of chlorine in cyclohexane. The partial pressure of gas above the liquid cyclohexane in the beaker is maintained constant so that C_{AO} could be regarded as a constant. The absorption occurs at steady state.

Gas klorin (A) telah terserap ke dalam sikloheksana tulen yang terkandung dalam sebuah bikar. Penyerapan tersebut berlaku menerusi permukaan terdedah sikloheksana tersebut yang bergenang dalam bikar. Kedalaman sikloheksana dalam bikar ialah L *meter dan boleh dianggap sebagai besar (iaitu* $L \rightarrow \infty$*). Tindak balas klorin dengan sikloheksana dalam fasa cecair ialah tertib kedua terhadap kepekatan klorin.* C_A *ialah kepekatan klorin* ($kmol/m^3$) *pada suatu jarak* z *dari permukaan,* C_{AO} *ialah kepekatan klorin* ($kmol/m^3$) *pada permukaan,* k_2 *ialah pemalar kadar tertib kedua dan* D_{AB} *ialah pekali resapan klorin dalam sikloheksana. Tekanan separa gas di atas sikloheksana cecair dalam bikar dikekalkan malar supaya* C_{AO} *boleh dianggap sebagai malar. Penyerapan tersebut berlaku pada keadaan mantap.*

[a] Show that the concentration of chlorine at a distance z from the surface of liquid in the beaker could be expressed as

Tunjukkan bahawa kepekatan klorin pada suatu jarak z *dari permukaan cecair dalam bikar boleh diungkapkan sebagai*

$$\frac{C_{Ao}}{C_A} = \left[1 + z\sqrt{\frac{k_2 C_{Ao}}{6 D_{AB}}}\right]^2 \tag{3.1}$$

[10 marks/markah]

[b] Evaluate the effective rate of absorption of chlorine at the surface of cyclohexane in $kmol/(m^2.s)$.

Taksirkan kadar berkesan penyerapan klorin pada permukaan sikloheksana dalam $kmol/(m^2.s)$.

Given: The diffusion coefficient of chlorine in cyclohexane is 3.3×10^{-10} m^2/s and the second order rate constant is 9.1×10^{-16} $m^3/(kmol.s)$. The equilibrium concentration of chlorine at the surface (C_{A0}) may be assumed to be 0.0225 $kmol/m^3$.

Diberi: Pekali resapan klorin dalam sikloheksana ialah 3.3×10^{-10} m^2/s *dan pemalar kadar tertib kedua ialah* 9.1×10^{-16} $m^3/(kmol.s)$*. Kepekatan keseimbangan klorin pada permukaan* (C_{AO}) *boleh diandaikan sebagai 0.0225* $kmol/m^3$.

[10 marks/markah]

Section B: Answer any TWO questions.
Bahagian B: Jawab mana-mana DUA soalan.

4. A liquid flows over a flat plate in the form of a layer as shown in Figure Q.4. Above the plate is a gas having a component *A* which is soluble in the liquid. At a section *PP* downstream of flow, the thickness of the liquid layer could be assumed constant and equal to δ meters.

Suatu cecair mengalir di atas suatu plat rata dalam bentuk satu lapisan seperti yang ditunjukkan dalam Rajah S.4. Di atas plat ialah suatu gas dengan komponen A *yang boleh larut dalam cecair tersebut. Pada rentasan* PP *di hilir aliran, tebal lapisan cecair tersebut boleh diandaikan sebagai malar dan bersamaan dengan* δ *meter.*

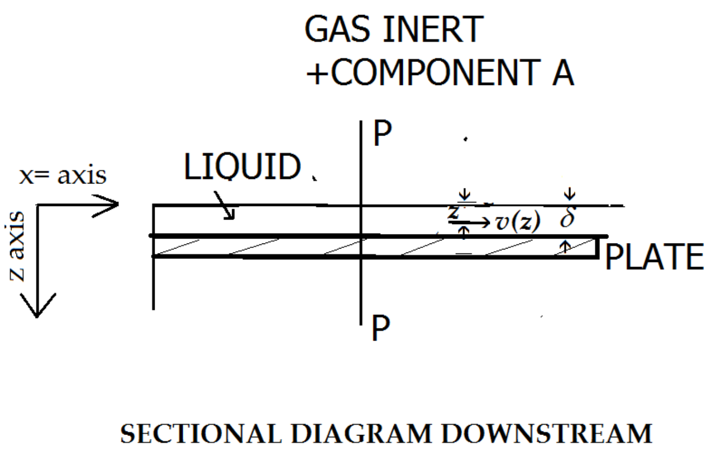

Figure Q.4.
Rajah S.4.

[a] At downstream and at steady state, if the flow within the layer is considered laminar, the velocity profile *V(z)* across the film can be expressed as

Di hiliran pada keadaan mantap, jika aliran dalam lapisan dianggap sebagai laminar, susuk halaju V(z) *merentas saput tersebut boleh diungkapkan sebagai*

$$V(z) = V_{max}\left[1-\left(\frac{z}{\delta}\right)^2\right] \qquad (4.1)$$

where and *z* is distance of a point from the exposed surface and V_{max} is the velocity of liquid at the surface of the film at z = 0. The concentration C_A within the layer varies with the distances *x* and *z* so that $C_A = C_A(x,z)$. Using the three dimensional equations of diffusion and continuity, show that the concentration $C_A(x,z)$ profile across the film could be described by the differential equation.

di mana z *ialah jarak suatu titik dari permukaan terdedah dan* V_{max} *ialah halaju cecair pada permukaan saput pada* z *= 0. Kepekatan* C_A *dalam lapisan berubah dengan jarak* x *dan jarak* z, *jadi* $C_A = C_A(x,z)$. *Dengan menggunakan persamaan-persamaan tiga dimensi resapan dan keselanjaran, tunjukkan bahawa profil kepekatan* $C_A(x,z)$ *merentasi saput tersebut boleh dihuraikan oleh persamaan pembezaan.*

$$V_{\max}\left[1-\left(\frac{z}{\delta}\right)^2\right]\frac{\partial c_A}{\partial x} = D_{AB}\left[\frac{\partial^2 c_A}{\partial x^2}+\frac{\partial^2 c_A}{\partial z^2}\right] \qquad (4.2)$$

Write down three boundary conditions for the above equation.
Tuliskan tiga keadaan sempadan untuk persamaan di atas.

[10 marks/markah]

[b] If the velocity is increased to the turbulent region when *V* can be regarded as constant and is independent of *z*, and the diffusion flux of component of A in the direction of *x* can be assumed to be negligible compared with the diffusion component in the z direction so that $\frac{\partial^2 c_A}{\partial x^2} << \frac{\partial^2 c_A}{\partial z^2}$,

Jika halaju dinaikkan sehingga ke kawasan bergelora di mana V *boleh dianggap sebagai malar dan tidak bersandar terhadap* z, *dan kadar resapan komponen* A *pada arah* x *boleh diandaikan sebagai terabai berbanding dengan resapan komponen pada arah* z *iaitu* $\frac{\partial^2 c_A}{\partial x^2} << \frac{\partial^2 c_A}{\partial z^2}$

[i] Show that
Tunjukkan yang

$$V.\frac{\partial C_A}{\partial x} = D_{AB}\frac{\partial^2 C_A}{\partial z^2} \qquad (4.3)$$

[ii] Obtain a general solution for C_A in terms of *x* and *z.*
Dapatkan suatu penyelesaian umum untuk C_A *menurut* x *dan z.*

[10 marks/markah]

5. A 2 m x 2 m vertical plate is exposed on one side to saturated steam at atmospheric pressure and on the other side to cooling water that maintains the plate temperature of 60 °C.

Satu plat tegak 2 m x *2 m didedahkan ke stim tepu pada tekanan atmosfera pada satu permukaan dan pada permukaan bersebelahannya kepada air sejuk yang mengekalkan plat tersebut pada suhu 60 °C.*

[a] Derive an equation for the rate of heat transfer coefficient ($h_{L(vert)}$) to the coolant. The liquid velocity profile of the film condensate is given below.

Terbitkan satu persamaan untuk kadar pekali pemindahan haba ($h_{L(vert)}$) kepada air penyejuk. Profil halaju cecair kondensasi filem adalah seperti berikut.

$$v_x = \frac{g(\rho_l - \rho_v)}{\mu}\left(\delta y - \frac{y^2}{2}\right) \tag{5.1}$$

[10 marks/markah]

[b] What is the rate at which steam condenses on the plate?
Apakah kadar kondensasi stim pada plat tersebut?

Given that
Diberikan

$\rho_v = 0.596 kg/m^3$, $\rho_l = 983 kg/m^3$, h_{fg} = 2358 kJ/m^3,
k_l = 0.668 W/mK, $\mu = 0.4688 x 10^{-3} kg/ms$ and g = 9.81 m/s^2

[5 marks/markah]

[c] For plates inclined at an angle θ from the vertical, the average heat transfer coefficient of the inclined plate, $h_{L(incl)}$ can be approximated by

Untuk plat yang condong pada sudut θ dari tegak, pekali pemindahan haba purata plat condong tersebut $h_{L(incl)}$ boleh dianggarkan oleh bentuk ungkapan

$$h_{L(incl)} = (\cos\theta)^{1/4} h_{L(vert)} \tag{5.2}$$

where $h_{L(vert)}$ is the average heat transfer coefficient for the vertical orientation. If the 2 m x 2 m plate is inclined at 45° from the normal, what is the percentage of reduction in the steam condensation rate?

di mana $h_{L(vert)}$ adalah purata pekali pemindahan haba pada orientasi tegak. Sekiranya plat 2 m x *2 m dicondongkan pada sudut 45° dari normal, apakah peratusan pengurangan kadar kondensasi stim?*

[5 marks/markah]

6. [a] The Navier-Stokes equation may be derived by considering a momentum balance on an element of volume $\Delta x \Delta y \Delta z$ as shown in Figure Q.6.

Persamaan Navier-Stokes mungkin boleh diterbitkan dengan mengambilkira isipadu $\Delta x \Delta y \Delta z$ seperti yang diberikan di dalam Gambarajah S.6.

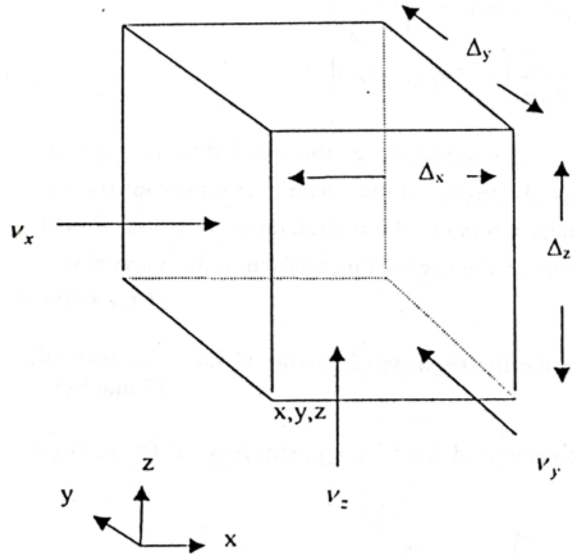

Figure Q.6.
Gambarajah S.6.

The rate of momentum accumulation in the volume element is given by the balance between the rates at which momentum is transferred into and out of the box and the sum of the forces (pressure and gravitational) on the box. Momentum balances are written in each of the three directions *x*, *y* and *z* as shown in the figure.

Kadar penumpukan momentum di dalam isipadu elemen diberikan oleh keseimbangan di antara kadar-kadar di mana momentum dipindahkan ke dalam dan ke luar daripada kotak serta jumlah daya-daya (tekanan dan graviti) ke atas kotak. Keseimbangan-keseimbangan momentum boleh ditulis bagi setiap arah x, y *dan* z *sebagaimana yang ditunjukkan di dalam gambarajah.*

Considering a momentum balance in the *x*-direction, momentum is transferred to the box by convection at the following rate:

*Dengan mengambilkira keseimbangan momentum pada arah-*x*, momentum dipindahkan ke dalam kotak secara perolakan pada kadar berikut:*

$$\Delta y \Delta z\left[\left(\rho v_x v_x\right)\Big|_x - \left(\rho v_x v_x\right)\Big|_{x+\Delta x}\right] + \Delta x \Delta z\left[\left(\rho v_y v_x\right)\Big|_y - \left(\rho v_y v_x\right)\Big|_{y+\Delta y}\right]$$
$$+ \Delta x \Delta y\left[\left(\rho v_z v_x\right)\Big|_z - \left(\rho v_z v_x\right)\Big|_{z+\Delta z}\right] \qquad (6.1)$$

where v_x, v_y and v_z are the velocities in directions x, y and z respectively.
Di mana v_x, v_y *dan* v_z *adalah halaju-halaju pada arah-arah* x, y *dan* z *masing-masing.*

Momentum is also transferred to the box by molecular processes (viscosity) arising from the gradient of velocity. The net momentum transfer by these processes is given by;

Momentum juga boleh dipindahkan ke dalam kotak secara proses-proses molekul (kepekatan) yang terbit daripada kecerunan halaju. Perpindahan momentum bersih bagi proses-proses ini diberikan oleh;

$$\Delta y \Delta z \left[\left(-\mu \frac{\partial v_x}{\partial x} \right)\bigg|_x - \left(-\mu \frac{\partial v_x}{\partial x} \right)\bigg|_{x+\Delta x} \right] + \Delta x \Delta z \left[\left(-\mu \frac{\partial v_x}{\partial y} \right)\bigg|_y - \left(-\mu \frac{\partial v_x}{\partial y} \right)\bigg|_{y+\Delta y} \right] + \Delta x \Delta y \left[\left(-\mu \frac{\partial v_x}{\partial x} \right)\bigg|_z - \left(-\mu \frac{\partial v_x}{\partial x} \right)\bigg|_{z+\Delta z} \right] \quad (6.2)$$

where μ is the viscosity of the fluid. Using the above relationships, derive the Navier- Stokes equation for the x-direction as follows:

di mana μ *adalah kelikatan bendalir. Dengan menggunakan perkaitan di atas, terbitkan persamaan Navier-Stokes bagi arah-x seperti berikut:*

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + vz \frac{\partial v_x}{\partial z} \right) = \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) - \frac{\partial \rho}{\partial x} + \rho g_x \quad (6.3)$$

where ρ is the fluid density and g_x the acceleration due to gravity in the direction of x. Note that you should use the continuity expression of the form given by;

di mana ρ *adalah ketumpatan bendalir dan* g_x *adalah pecutan yang disebabkan oleh graviti pada arah*-x. *Anda perlu menggunakan terbitan keterusan yang diberi dalam bentuk*

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 \quad (6.4)$$

[14 marks/markah]

[b] Write down the equivalent forms of equation (6.3) for the y and z-directions.

Tuliskan persamaan-persamaan yang setara dengan persamaan (6.3) bagi arah-arah y *dan* z.

[6 marks/markah]

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Appendix

SOLUTIONS OF DIFFERENTIAL EQUATIONS:

In the following equations E is the dependant variable and F and G are the independent variables.

DEFINITIONS

$$\int_0^{\theta} exp(-\eta^2)d\eta = \frac{\sqrt{\pi}}{2} erf(\theta)$$

and

$$\int_0^{\infty} exp(-\eta^2)d\eta = \frac{\sqrt{\pi}}{2}$$

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1. Differential equation: Solution

$$\frac{\partial E}{\partial F} = D \frac{\partial^2 E}{\partial G} \qquad E = A + B \int_0^{G/\sqrt{4DF}} \exp(-\eta^2)d\eta$$

where A and B are constants

2. . Differential equation: $$\frac{\partial^2 E}{\partial F^2} = \alpha^2 E$$

Solution $$E = K_1 . \sinh \alpha F + K_2 \cosh \alpha F.$$

where K_1 and K_2 are constants

3 Differential equation: $$\frac{\partial^2 E}{\partial G^2} = 0$$

Solution $$\frac{\partial E}{\partial G} = cons\tan t = K'$$

4. Differential equation: $$\frac{\partial E}{\partial F} = K / F^2$$

Solution: $$E = -K / F + K_1$$

Where K_1 is a constant

5. Differential equation: $$\frac{1}{F^2}\frac{\partial}{\partial F}\left(F^2 \frac{\partial E}{\partial F}\right) = \alpha^2 E$$

Solution: $$\frac{E}{E'} = \frac{C_1}{F}\cosh \alpha F + \frac{C_2}{F}\sinh \alpha F$$

6. Differential equation: $$\frac{\partial^2 E}{\partial F^2} = \alpha \, E^2$$ Solution: $$E = \frac{6\gamma^2}{\alpha(1+\gamma F)^2}$$

where γ is a constant

Useful formulae for Mass transport analysis

NOTATION AND DEFINITIONS

A. Notation

- c_A= concentration of A (kmol of A/m3)
- D_{AB}=Diffusion coefficient of A in B (m^2/s)
- *i,j,k* are unit vectors in three perpendicular directions associated with the system considered.
- J_A=Diffusional flux in kmol/(m^2.s) = $J_x.i+J_y.j+J_z.k$ (vector)
- *Jx,Jy,Jz*= Diffusional flux in kmol/(m^2.s)in the directions *i,j,k* respectively
- n_A= Overall mass transfer flux in kg/(m^2.s)= *Nx.i+Ny.j+Nz.k* (vector)
- n_x,n_y,n_z =Overall mass transfer fluxes in kg/(m^2.s)in directions of the unit vectors *i,j,k*
- N_A= Overall mass transfer flux in kmol/(m^2.s)= *Nx.i+Ny.j+Nz.k* (vector)
- *Nx,Ny,Nz* =Overall mass transfer fluxes in kmol/(m^2.s) in directions of the unit vectors *i,j,k*
- r_A= rate of reaction in kg/(m^3.s)
- R_A=rate of reaction in kmoles/(m^3.s)
- *s*= scalar
- w_A=weight fraction of component A (kg of A /total kg) w_A=weight fraction of component A (kg of A /total kg)
- *x,y,z* = distances (m) in directions of the unit vectors *i,j,k*
- *u,v,w*= velocities (m/s) in i, j,k directions represented by *v*=u.i+v.j+w.k* (vector)
- $\tilde{v}$ = a vector
- ρ =*density of medium kg/m³*

B. Definition of the operator ∇ Cartesian Co-ordinates

VECTOR

$$\nabla . v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

SCALAR

$$\nabla s = \frac{\partial s}{\partial x} i + \frac{\partial s}{\partial y} j + \frac{\partial s}{\partial z} k$$

C Definition of the operator ∇ Cylindrical co-ordinates

$$\nabla . \tilde{v} = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\nabla s = \frac{\partial s}{\partial r} i + \frac{1}{r}\frac{\partial s}{\partial \theta} j + \frac{\partial s}{\partial z} k$$

D. **Definition of the operator ∇** Spherical co-ordinates

$$\nabla.v = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r.Sin\theta}\frac{\partial(v_\theta.Sin\theta)}{\partial\theta} + \frac{1}{rSin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\nabla s = \frac{\partial s}{\partial r}i + \frac{1}{r}\frac{\partial s}{\partial\theta}j + \frac{1}{rSin\theta}\frac{\partial s}{\partial\phi}k$$

E. **Definition of the operator ∇^2** Cartesian Co-ordinates

$$\nabla^2 s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

F. **Definition of the operator ∇^2** Cylindrical co-ordinates

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial\theta^2} + \frac{\partial^2 s}{\partial z^2}$$

G. **Definition of the operator ∇^2** Spherical co-ordinates

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2.Sin\theta}\frac{\partial}{\partial\theta}\left(Sin\theta\frac{\partial s}{\partial\theta}\right) + \frac{1}{r^2 Sin^2\theta}\frac{\partial^2 s}{\partial\phi^2}$$

DIFFERENTIAL EQUATIONS FOR MASS TRANSPORT ANALYSIS

A. Equivalent forms of Fick's law of binary diffusion and Mass/Molar flux equations

(mass flux)

$$n_A = \rho w_A.v - D_{AB}\nabla.(\rho w_A)$$

(molar flux)

$$N_A = c_A.v^* - D_{AB}\nabla.c_A$$

B. The equations of continuity for a multi-component mixture

(mass flux)

$$\frac{\partial(\rho w_A)}{\partial t} = -(\nabla.n_A) + r_A$$

(molar flux)

$$\frac{\partial c_A}{\partial t} = -(\nabla.N_A) + R_A$$