UNIVERSITI SAINS MALAYSIA

Supplementary Semester Examination
Academic Session 2009/2010

June 2010

IUK 291 – MATHEMATICS II
[MATEMATIK II]

Duration: 3 hours
[Masa: 3 jam]

Please check that the examination paper consists of **FIVE (5)** pages of printed material before you begin this examination.

Answer **FOUR** questions. All questions can be answered in Bahasa Malaysia OR English.

In the event of any discrepancies, the English version shall be used.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA (5)** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **EMPAT** soalan. Semua soalan boleh dijawab dalam Bahasa Malaysia ATAU Bahasa Inggeris.*

Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

Answer all questions.

1. (i) Find the Taylor series for $f(x) = 6 + 8x + 9x^3$ centred at a=2.

(6 marks)

(ii) In what direction is the function defined by $f(x, y) = xe^{3x-y}$ increasing most rapidly at the point $P_0(1,3)$, and what is the maximum rate of increase?

(9 marks)

(iii) Evaluate $\int_1^2 \int_1^{y^2} (x + 2y)dxdy$.

(10 marks)

2. (i) Find the local maximum, minimum or saddle points of the function $f(x, y) = e^{-4y}\left(x^2 + y^2\right)$. Use the second partial test to classify each point.

(9 marks)

(ii) A cylinder with no top is to be constructed from 24π cm^2 of material. Use the method of Lagrange to determine the dimensions of the cylinder if it is to enclose the maximum volume. (Hint: $V = \pi r^2 h$ and $S = \pi r^2 + 2\pi rh$)

(6 marks)

(iii) Find the values of x for which the series $\sum_{n=0}^{\infty} (4x)^n$ converges and find the sum of the series for the value of x.

(5 marks)

(iv) Show that

$$\int_{-1}^{1} \int_{0}^{2} (4 + x^2 - y^2)dydx = 12$$

(5 marks)

3. (i) Use partial fractions to find Maclaurin series for the function

$$f(x) = \frac{5+5x}{x^2+3x-4}$$

(10 marks)

(ii) Find the volume under the plane $z = 2x + y + 3$ above the region D bounded by the lines $y = 2x$, $y = 3 - x$ and $y = 0$.

(10 marks)

(iii) Evaluate $\iint_R xy\, dA$ over the region R enclosed between $y = 2$, $y = 4$, $x = 1$ and $x = 3$.

(5 marks)

4. (i) Use the binomial series to obtain the power expansion of $\frac{1}{\sqrt{1-x^2}}$.

(8 marks)

(ii) Find the interval of convergence for the power series $\sum_{k=1}^{\infty} k^2 4^k (x+2)^k$.

(7 marks)

(iii) Solve $y'' - y' - 2y = x$.

(10 marks)

Jawab semua soalan.

1. (i) *Dapatkan siri Taylor bagi fungsi* $f(x) = 6 + 8x + 9x^3$ *berpusat di a=2.*

(6 markah)

(ii) *Pada arah manakah fungsi* $f(x, y) = xe^{3x-y}$ *meningkat mendadak di titik* $P_0(1,3)$, *dan apakah kadar kenaikan maksima?*

(9 markah)

(iii) *Selesaikan* $\int_1^2 \int_1^{y^2} (x + 2y)dxdy$.

(10 markah)

2. (i) *Cari titik maksima setempat, minima setempat atau titik lengkok balas bagi fungsi* $f(x, y) = e^{-4y}\left(x^2 + y^2\right)$. *Guna ujian separa kedua untuk mengkelaskan setiap titik.*

(9 markah)

(ii) *Sebuah silinder tanpa penutup dibina dari* $24\pi\, sm^2$ *bahan. Guna kaedah pendarab Lagrange utuk menentukan dimensi kotak bagi mendapatkan isipadu maksima. (Petua:* $V = \pi r^2 h$ *and* $S = \pi r^2 + 2\pi rh$*)*

(6 markah)

(iii) *Dapatkan nilai-nilai* x *di mana siri* $\sum_{n=0}^{\infty} (4x)^n$ *menumpu dan dapatkan jumlah bagi siri tersebut untuk nilai-nilai* x.

(5 markah)

(iv) *Tunjukkan bahawa*

$$\int_{-1}^{1} \int_{0}^{2} (4 + x^2 - y^2)dydx = 12$$

(5 markah)

3. (i) *Guna pecahan separa untuk mencari siri Maclaurin bagi fungsi*

$$f(x) = \frac{5+5x}{x^2+3x-4}.$$

(10 markah)

(ii) *Cari isipadu di bawah satah* $z = 2x + y + 3$ *di atas kawasan D yang dibatasi oleh garis-garis* $y = 2x$, $y = 3 - x$ *dan* $y = 0$.

(10 markah)

(iii) *Selesaikan* $\iint_R xy\, dA$ *di atas kawasan R yang ditutupi antara* $y = 2$, $y = 4$, $x = 1$ *dan* $x = 3$.

(5 markah)

4. (i) *Guna siri binomial untuk mendapatkan kembangan siri kuasa bagi*

$$\frac{1}{\sqrt{1-x^2}}.$$

(8 markah)

(ii) *Cari jeda penumpuan bagi siri kuasa* $\sum_{k=1}^{\infty} k^2 4^k (x+2)^k$.

(7 markah)

(iii) *Selesaikan* $y'' - y' - 2y = x$.

(10 markah)

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