
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

IUK 291 – MATEMATIK II
[MATHEMATICS II]

Duration: 3 hours
Masa: [3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer FOUR (4) questions. You may answer the questions either in Bahasa Malaysia or in English.

Arahan: Jawab EMPAT (4) soalan. Anda dibenarkan menjawab soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) Find the Taylor series for $f(x) = 5 + 3x + 2x^2$ centred at $a = 3$.

(5 marks)

- (b) In what direction is the function defined by $f(x, y) = xe^{3x-y}$ increasing most rapidly at the point $P_0(1,3)$, and what is the maximum rate of increase? In what direction is the function decreasing most rapidly?

(10 marks)

- (c) Evaluate the integral by first reversing the order of integration

$$\int_1^4 \int_{\sqrt{x}}^2 (x+2y) dy dx$$

(10 marks)

2. (a) Find the local maximum, minimum or saddle points of the function $f(x, y) = e^{-4y}(x^2 + y^2)$. Use the second partial test to classify each point.

(9 marks)

- (b) A cylinder with no top is to be constructed from 12π cm² of material. Use the method of Lagrange to determine the dimensions of the cylinder if it is to enclose the maximum volume.

(6 marks)

- (c) Evaluate $\int_0^1 \int_2^4 \frac{xy^2}{x^2+1} dy dx$

(5 marks)

- (d) Find the volume under the plane $z = 2x + y + 3$ above the region D bounded by the lines $y = 2x$, $y = 3 - x$ and $y = 0$.

(5 marks)

3. (a) Use partial fractions to find Maclaurin series for the function

$$f(x) = \frac{5+5x}{x^2+3x-4}$$

(10 marks)

- (b) Find the values of x for which the series $\sum_{n=0}^{\infty} 8^n x^n$ converges and find the sum of the series for the value of x .
(5 marks)
- (c) Solve $y'' - y' - 2y = x^2 e^x$.
(5 marks)
- (d) Evaluate $\iint_R xy \, dA$ over the region R enclosed between $y = \frac{x}{2}$, $y = \sqrt{x}$, $x = 3$ and $x = 2$.
(5 marks)
4. (a) Use the binomial series to obtain the power expansion of $\frac{1}{\sqrt{1-x^3}}$.
(5 marks)
- (b) Find the interval of convergence for the power series $\sum_{k=1}^{\infty} k^2 4^k (x+2)^k$.
(5 marks)
- (c) Use the Ratio test to test the series $\sum_{k=0}^{\infty} \frac{k!}{k^2 (k+1)^2}$ for convergence.
(5 marks)
- (d) Solve the initial value problem using the method of undetermined coefficients.
$$y'' + 2y' + 2y = \cos x \quad y(0) = 0, \quad y'(0) = -4$$

(10 marks)

1. (a) Dapatkan siri Taylor bagi fungsi $f(x) = 5 + 3x + 2x^2$ berpusat di $a = 3$.
(5 markah)
- (b) Pada arah manakah fungsi $f(x, y) = xe^{3x-y}$ meningkat mendadak di titik $P_0(1,3)$, dan apakah kadar kenaikan maksima? Pada arah manakah fungsi tersebut menurun mendadak?
(10 markah)
- (c) Selesaikan kamiran tersebut dengan menyongsangkan dahulu tertibkamiran.

$$\int_1^4 \int_{\sqrt{x}}^2 (x+2y) dy dx$$

(10 markah)

2. (a) Cari titik maksima setempat, minima setempat atau titik lengkok balas bagi fungsi $f(x, y) = e^{-4y}(x^2 + y^2)$. Guna ujian separa kedua untuk mengelaskan setiap titik.
(9 markah)
- (b) Sebuah silinder tanpa penutup dibina dari $12\pi \text{ sm}^2$ bahan. Guna kaedah pendarab Lagrange untuk menentukan dimensi kotak bagi mendapatkan isipadu maksima.
(6 markah)
- (c) Selesaikan $\int_0^1 \int_2^4 \frac{xy^2}{x^2+1} dy dx$
(5 markah)
- (d) Cari isipadu di bawah satah $z = 2x + y + 3$ di atas kawasan D yang dibatasi oleh garis-garis $y = 2x$, $y = 3 - x$ dan $y = 0$.
(5 markah)
3. (a) Guna pecahan separa untuk mencari siri Maclaurin bagi fungsi

$$f(x) = \frac{5+5x}{x^2+3x-4}$$

(10 markah)

- (b) Dapatkan nilai-nilai x di mana siri $\sum_{n=0}^{\infty} 8^n x^n$ menumpu dan dapatkan jumlah bagi siri tersebut untuk nilai-nilai x .

(5 markah)

- (c) Selesaikan $y'' - y' - 2y = x^2 e^x$.

(5 markah)

- (d) Selesaikan $\iint_R xy \, dA$ di atas kawasan R yang ditutupi antara $y = \frac{x}{2}$, $y = \sqrt{x}$, $x = 3$ and $x = 2$.

(5 markah)

4. (a) Guna siri binomial untuk mendapatkan kembangan siri kuasa bagi

$$\frac{1}{\sqrt{1-x^3}}.$$

(5 markah)

- (b) Cari jeda penumpuan bagi siri kuasa $\sum_{k=1}^{\infty} k^2 4^k (x+2)^k$.

(5 markah)

- (c) Guna Ujian Nisbah untuk menguji siri $\sum_{k=0}^{\infty} \frac{k!}{k^2(k+1)^2}$ bagi penumpuan.

(5 markah)

- (d) Selesaikan masalah nilai awal dengan kaedah koefisien tak ditentukan.

$$y'' + 2y' + 2y = \cos x \quad y(0) = 0, \quad y'(0) = -4$$

(10 markah)