UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all five** [5] questions.

*[**Arahan** : Jawab **semua lima** [5] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) (i) Let V be the set of points on a line that does not go through the origin in $\mathbb{R}^2$ with the standard addition and scalar multiplication. Show that V is not a vector space.

(ii) Let V denote the set of real polynomials

$$V = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n.$$

Let W be the set of real polynomials of degree n or less. Is W a subspace of V?

[10 marks]

(b) Let W_1, W_2, and W_3 be the x-axis, the y-axis, and the line $y = -2x$ respectively. Show that

$$\mathbb{R}^2 \neq W_1 \oplus W_2 \oplus W_3.$$

[4 marks]

2. (a) Let

$$A = \begin{pmatrix} -2 & 7 & 2 & 5 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 5 & -3 \\ -1 & 4 & -9 & 3 \end{pmatrix}.$$

Using cofactor expansion, compute the determinant of A.

[5 marks]

(b) Solve the simultaneous equations below using Cramer's Rule:

$$\begin{aligned} x_1 + 2x_3 + 3x_4 &= 5 \\ x_2 + x_3 + 2x_4 &= 4 \\ -4x_1 + x_2 - 6x_3 - 5x_4 &= -13 \\ 2x_2 + 5x_3 + 21x_4 &= 21 \end{aligned}$$

[10marks]

(c) Let

$$B = \begin{pmatrix} 0 & 1+i & 1+2i \\ 1-i & 0 & 2-3i \\ 1-2i & 2+3i & 0 \end{pmatrix}.$$

Using row reduction method, evaluate the determinant of B.

[8 marks]

1. (a) (i) *Biar V suatu set titik pada satu garisan yang tidak melalui titik asal pada* $\mathbb{R}^2$ *dengan penambahan dan pendaraban skalar yang piawai. Tunjukkan bahawa V bukan suatu ruang vektor.*

(ii) *Biar V suatu set polinomial nyata*

$$V = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n.$$

Biar W suatu set polinomial nyata dengan darjah n atau kurang. Adakah W suatu subruang bagi V?

[10 markah]

(b) *Biar* $W_1, W_2,$ *dan* W_3 *adalah masing-masing paksi-x, paksi-y dan garisan* $y = -2x$*. Tunjukkan bahawa*

$$\mathbb{R}^2 \neq W_1 \oplus W_2 \oplus W_3.$$

[4 markah]

2. (a) *Biar*

$$A = \begin{pmatrix} -2 & 7 & 2 & 5 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 5 & -3 \\ -1 & 4 & -9 & 3 \end{pmatrix}.$$

Dengan menggunakan kembangan kofaktor, kira penentu bagi A.

[5 markah]

(b) *Selesaikan persamaan serentak di bawah dengan menggunakan Petua Cramer:*

$$\begin{aligned} x_1 + 2x_3 + 3x_4 &= 5 \\ x_2 + x_3 + 2x_4 &= 4 \\ -4x_1 + x_2 - 6x_3 - 5x_4 &= -13 \\ 2x_2 + 5x_3 + 21x_4 &= 21 \end{aligned}$$

[10 markah]

(c) *Biar*

$$B = \begin{pmatrix} 0 & 1+i & 1+2i \\ 1-i & 0 & 2-3i \\ 1-2i & 2+3i & 0 \end{pmatrix}.$$

Menggunakan kaedah penurunan baris, nilaikan penentu bagi B.

[8 markah]

3. (a) Let $T: M_{2\times 2}(□) \to P_3(□)$ be a linear transformation over □ such that

$$T\begin{pmatrix} a_0 & a_1 \\ a_2 & a_3 \end{pmatrix} = a_0 + a_1x + a_2x^2 + a_3x^3.$$

(i) List down all the bases for $M_{2\times 2}(□)$ over □ .

(ii) List down all the bases for $P_3(□)$ over □ .

(iii) Are $M_{2\times 2}(□)$ and $P_3(□)$ isomorphic over □ ?

[16 marks]

(b) Let $V = span(v_1, v_2, \ldots, v_n)$ be a vector space over □ and let $W = span(v_1, v_2, \ldots, v_n)$ be a vector space over □ . Can V be isomorphic to W ? Verify your answer.

[4 marks]

(c) (i) Show that a linear operator $T: V \to V$ on a vector space of finite dimension is invertible if and only if it is nonsingular.

(ii) Let $T: □^2 \to □^2$ be a linear transformation such that

$$T(x, y) = (x + 2y, 3x + 2y).$$

Using the statement in 3.c.i), show that T is invertible.

[15 marks]

3. (a) *Biar* $T: M_{2\times 2}(\square) \rightarrow P_3(\square)$ *suatu transformasi linear atas* $\square$ *sedemikian*

$$T\begin{pmatrix} a_0 & a_1 \\ a_2 & a_3 \end{pmatrix} = a_0 + a_1 x + a_2 x^2 + a_3 x^3.$$

(i) *Senaraikan semua asas bagi* $M_{2\times 2}(\square)$ *atas* $\square$.

(ii) *Senaraikan semua asas bagi* $P_3(\square)$ *atas* $\square$.

(iii) *Adakah* $M_{2\times 2}(\square)$ *dan* $P_3(\square)$ *isomorfik atas* $\square$?

[16 markah]

(b) *Biar* $V = rentang(v_1, v_2, \ldots, v_n)$ *suatu ruang vektor atas* $\square$ *dan biar* $W = rentang(v_1, v_2, \ldots, v_n)$ *suatu ruang vektor atas* $\square$. *Adakah V isomorfik dengan W ? Tentusahkan jawapan anda.*

[4 markah]

(c) (i) *Tunjukkan bahawa suatu operator linear* $T: V \rightarrow V$ *bagi suatu ruang vektor yang berdimensi terhingga adalah tersongsangkan jika dan hanya jika ianya tak singular.*

(ii) *Biar* $T: \square^2 \rightarrow \square^2$ *suatu transformasi linear sedemikian*

$$T(x, y) = (x + 2y, 3x + 2y).$$

Dengan menggunakan pernyataan 3.c.i), tunjukkan bahawa T adalah tersongsangkan.

[15 markah]

4. (a) Let

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & -5 \\ 0 & 1 & -2 \end{pmatrix}.$$

Determine whether A can be diagonalised

(i) over □ ?

(ii) over □ ?

[8 marks]

(b) Let

$$B = \begin{pmatrix} i & 0 \\ 1 & i \end{pmatrix}.$$

Find the Jordan canonical form for B. Then, compute

(i) B^{-1}.

(ii) B^k, k is a positive integer.

[14 marks]

(c) Determine whether the following statements are true or false in general, and justify your answers.

(i) Any square matrix is similar to a triangular matrix.

(ii) If a matrix A has exactly k linearly independent eigenvectors, then the Jordan canonical form of A has k Jordan blocks.

(iii) If a matrix A has k distinct eigenvalues, then the Jordan canonical form of A has k Jordan blocks.

[6 marks]

4. (a) Biar

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & -5 \\ 0 & 1 & -2 \end{pmatrix}.$$

Tentukan sama ada A boleh dipepenjurukan

(i) *atas* □ ?

(ii) *atas* □ ?

[8 markah]

(b) *Biar*

$$B = \begin{pmatrix} i & 0 \\ 1 & i \end{pmatrix}.$$

Cari bentuk berkanun Jordan bagi B. Kemudian, hitung

(i) B^{-1}.

(ii) B^k, *k suatu integer positif.*

[14 markah]

(c) *Tentukan sama ada pernyataan berikut adalah benar atau palsu secara am, dan jelaskan jawapan anda.*

(i) *Sebarang matriks segiempat sama adalah serupa dengan suatu matriks segitiga.*

(ii) *Jika suatu matriks A mempunyai k vektor eigen yang tak bersandar linear, maka bentuk berkanun Jordan bagi A mempunyai k blok Jordan.*

(iii) *Jika suatu matriks A mempunyai k nilai eigen yang berbeza, maka bentuk berkanun Jordan bagi A mempunyai k blok Jordan.*

[6 markah]

5. (a) Let T be a linear operator on $\mathbb{C}^3$ defined by

$$T(x,y,z)=(2x+(1-i)y-iz,(3+2i)x+y-4iz,2ix+(4-3i)y-3z).$$

Find $T^*(x,y,z)$.

[5 marks]

(b) Let

$$A=\begin{pmatrix}\frac{1+i}{2} & \frac{1+i}{2}\\ \frac{1-i}{2} & \frac{-1+i}{2}\end{pmatrix}.$$

Show that A is unitary but neither Hermitian nor skew-Hermitian.

[6 marks]

(c) Show that T^*T , TT^* and T^*+T are self-adjoint for any operator T on any vector space V.

[9 marks]

5. (a) *Biar T suatu operator linear pada* $\mathbb{C}^3$ *ditakrifkan sebagai*

$$T(x,y,z)=(2x+(1-i)y-iz,(3+2i)x+y-4iz,2ix+(4-3i)y-3z).$$

Cari $T^*(x,y,z)$.

[5 markah]

(b) *Biar*

$$A=\begin{pmatrix} \frac{1+i}{2} & \frac{1+i}{2} \\ \frac{1-i}{2} & \frac{-1+i}{2} \end{pmatrix}.$$

Tunjukkan bahawa A adalah unitari tetapi tidak Hermitean dan tidak Hermitean pencong.

[6 markah]

(c) *Tunjukkan bahawa* T^*T , TT^* *dan* T^*+T *adalah swadampingan bagi sebarang operator T pada sebarang ruang vektor V.*

[9 markah]

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