UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSS 211 – Modern Algebra
[Aljabar Moden]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of THREE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all six** [6] questions.

*[**Arahan** : Jawab **semua enam** [6] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Given A, B and C are sets, prove or disprove the following:

   (a) $A \subseteq B \subseteq C \Leftrightarrow A \times B \subseteq B \times C$.

   (b) $A \cap B \subseteq A \cup B$.

[16 marks]

2. Given $A = 1,2,3,4$ and $H = (1,2),(2,3),(3,1),(2,2),(3,3),(4,4)$, find the smallest possible set K such that $H \cup K$ is an equivalence relation on A. Hence, obtain the set of equivalence classes for $H \cup K$ on A.

[16 marks]

3. Given the bijective functions $f : \square - \{-3\} \rightarrow \square - \{0\}$ and $g : \square - \{0\} \rightarrow \square - \{0\}$ where $(x)f = \dfrac{2}{3+x}$ and $(x)g = \dfrac{1}{x}$, obtain $f \circ g$ and state the reason why $g \circ f$ cannot be defined. Find the inverse functions of f, g and $f \circ g$, and verify that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

[18 marks]

4. Given that $\langle S, * \rangle$ is a group where $S = \{e, a, b, c\}$ and e is its identity element. If $a^2 = b$, obtain the Cayley table for $\langle S, * \rangle$.

[16 marks]

5. List all the elements of the alternating group A_4 and obtain a subgroup of order 4 in it. Show that this is a normal subgroup of A_4. Is A_4 isomorphic to the group $S_3 \times \square_2$?

[20 marks]

6. Given the ring $\langle \square, \oplus, \otimes \rangle$ where $x \oplus y = x + y - 1$ and $x \otimes y = x + y - xy$, obtain the identity element for the abelian group $\langle \square, \oplus \rangle$ and show that $\otimes$ is commutative on $\square$. Hence show that $\langle \square, \oplus, \otimes \rangle$ is an integer domain.

[14 marks]

1. *Diberikan* A, B *dan* C *adalah set, buktikan atau sangkalkan yang berikut:*
   (a) $A \subseteq B \subseteq C \Leftrightarrow A \times B \subseteq B \times C$.
   (b) $A \cap B \subseteq A \cup B$.

   [16 markah]

2. *Diberikan* $A = 1,2,3,4$ *dan* $H = (1,2),(2,3),(3,1),(2,2),(3,3),(4,4)$, *cari set terkecil mungkin* K *supaya* $H \cup K$ *merupakan suatu hubungan kesetaraan atas* A. *Dengan demikian, dapatkan set kelas kesetaraan bagi* $H \cup K$ *atas* A.

   [16 markah]

3. *Diberikan fungsi bijektif* $f : \square - \{-3\} \to \square - \{0\}$ *dan* $g : \square - \{0\} \to \square - \{0\}$ *dengan* $(x)f = \dfrac{2}{3+x}$ *dan* $(x)g = \dfrac{1}{x}$, *dapatkan* $f \circ g$ *dan nyatakan sebab mengapa* $g \circ f$ *tidak dapat ditakrifkan. Cari songsangan bagi setiap fungsi* f, g dan $f \circ g$, *dan tentusahkan bahawa* $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

   [18 markah]

4. *Diberikan bahawa* $\langle S, * \rangle$ *merupakan suatu kumpulan dengan* $S = \{e, a, b, c\}$ *dan* e *merupakan unsur identitinya. Jika* $a^2 = b$, *dapatkan sifir Cayley bagi* $\langle S, * \rangle$.

   [16 markah]

5. *Senaraikan semua unsur kumpulan selang-seli* A_4 *dan dapatkan suatu subkumpulan berperingkat 4 di dalamnya. Tunjukkan bahawa ini merupakan suatu subkumpulan normal bagi* A_4. *Adakah kumpulan* A_4 *berisomorfik dengan kumpulan* $S_3 \times \square_2$?

   [20 markah]

6. *Diberikan gelanggang* $\langle \square, \oplus, \otimes \rangle$ *dengan* $x \oplus y = x + y - 1$ *dan* $x \otimes y = x + y - xy$, *dapatkan unsur identiti bagi kumpulan abelan* $\langle \square, \oplus \rangle$ *dan tunjukkan bahawa* $\otimes$ *merupakan kalis tukar-tertib atas* $\square$. *Dengan demikian tunjukkan bahawa* $\langle \square, \oplus, \otimes \rangle$ *merupakan suatu domain integer.*

   [14 markah]

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