UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSG389 – Engineering Computation II
*[*Pengiraan Kejuruteraan II*]*

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all four** [4] questions.

*[**Arahan** : Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Solve the following system by using the following methods

$$\begin{pmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 2 & 5 \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

(i) Jacobi iteration method
(ii) Gauss-Seidel method
(iii) SOR method with $\omega = 1.25$

with initial value $\{x_1, x_2, x_3\} = \{0, 0, 0\}$ and up to 6 iterations for each method.

[50 marks]

(b) A tank initially contains 200 gallons of fresh water. Then salt water containing 4 Ibs of salt per gallon is pumped in at a rate of 8 gallons per minute, and the well-mixed mixture is allowed to leave at the same rate. How many pounds of salt there will be in the tank after 50 min?

[50 marks]

2. (a) Solve the following equation

$$y' = xy + x, 0 \le x \le 1, y(0) = 0, h = 0.2$$

by using the following methods

(i) Runge-Kutta method of order 2
(ii) Find the approximate value of $y(1)$ using the Adams-Bashforth two-step method
(iii) Find the approximate value of $y(1)$ using the Adams-Bashforth three-step method and the Adams-Moulton two-step method

Compare your approximate solution with the exact solution $y(x) = -1 + e^{x^2/2}$ for each method.

[30 marks]

(b) Use the linear shooting method to solve the following linear boundary value-problem

$$y'' = -xy' + y + 2x + \frac{2}{x}, y(1) = 0, y(2) = 4\ln 2, h = 0.2$$

[40 marks]

(c) Use the finite-difference method to solve the problem

$$y'' = y + x(x-4) \qquad 0 \le x \le 4$$

with $y(0) = y(4) = 0, n = 4$ subintervals.

[30 marks]

1. (a) *Selesaikan sistem berikut dengan menggunakan*

$$\begin{pmatrix} 3 & 1 & -1 \\ 2 & 4 & 1 \\ -1 & 2 & 5 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

(i) *kaedah lelaran Jacobi*
(ii) *kaedah lelaran Gauss-Seidel*
(iii) *kaedah SOR dengan* $\omega = 1.25$

Dengan nilai awal $\{x_1, x_2, x_3\} = \{0,0,0\}$ *dan sehingga 6 lelaran untuk setiap kaedah.*

[50 markah]

(b) *Sebuah tangki mengandungi 200 gelan air bersih. Kemudiannya, air garam yang mengandungi 4 Ibs garam dipamkan masuk dengan kadar 8 gelan per minit, dan campuran sekata dibenarkan keluar dari tangki dengan kadar yang sama. Berapa banyak paun garam yang akan tertinggal dalam tangki selepas 50 minit?*

[50 markah]

2. (a) *Selesaikan persamaan di bawah*

$$y' = xy + x, 0 \le x \le 1, y(0) = 0, h = 0.2$$

dengan menggunakan kaedah berikut

(i) *kaedah Runge-Kutta tertib 2*
(ii) *dapatkan nilai* $y(1)$ *dengan menggunakan kaedah Adams-Bashforth peringkat kedua*
(iii) *dapatkan nilai* $y(1)$ *dengan menggunakan kaedah Adams-Bashforth peringkat tiga dan Adams-Moulton peringkat dua.*

Bandingkan jawapan anda dengan jawapan sebenar bagi $y(x) = -1 + e^{x^2/2}$ *untuk setiap kaedah.*

[30 markah]

(b) *Dengan menggunakan kaedah tembakan linear, selesaikan persamaan sempadan linear berikut*

$$y'' = -xy' + y + 2x + \frac{2}{x}, y(1) = 0, y(2) = 4\ln 2, h = 0.2$$

[40 markah]

(c) *Gunakan kaeadah beza terhingga bagi menyelesaikan*

$$y'' = y + x(x-4) \qquad 0 \le x \le 4$$

dengan $y(0) = y(4) = 0, n = 4$ *subselang.*

[30 markah]

3. (a) Find the solution of the equation

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial t^2}, 0 \le x \le 1$$

with the boundary conditions

$$\phi(0,t) = 0, t > 0$$
$$\phi(1,t) = 0, t > 0$$

and the initial conditions

$$\phi(x,0) = \sin \pi x, 0 \le x \le 1$$

and $\frac{\partial \phi}{\partial t}(x,0) = 0, 0 \le x \le 1$

Use $\Delta x = 0.25$ and $\Delta t = 0.125$

[80 marks]

(b) Replace the Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0, 0 \le x \le 1, 0 \le y \le 1$$

by a finite difference approximation. If the boundary values of $T(x,y)$ are assigned on all four sides of the square, show how a linear algebraic system is encountered.

[20 marks]

4. (a) A metal rod of length 1m is initially at $70^o C$. The steady state temperature of the left and right ends of the rod are given by $50^o C$ and $20^o C$, respectively. Using $\alpha^2 = 0.1 m^2 / min$, $\Delta x = 0.2 m$ and $\Delta t = 0.1 min$, determine the temperature distribution in the rod for $0 \le t \le 0.3 min$.

(b) Solve the same problem by using Crank-Nicholson method with $\Delta x = 0.2 m$ and $\Delta t = 0.3 min$.

[100 marks]

3. (a) *Cari penyelesaian bagi persamaan*

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \phi}{\partial t^2}, 0 \leq x \leq 1$$

dengan syarat-syarat sempadan

$$\phi(0,t) = 0, t > 0$$
$$\phi(1,t) = 0, t > 0$$

dan syarat awal

$$\phi(x,0) = \sin \pi x, 0 \leq x \leq 1$$

dan $\dfrac{\partial \phi}{\partial t}(x,0) = 0, 0 \leq x \leq 1$

Gunakan $\Delta x = 0.25$ *dan* $\Delta t = 0.125$

[80 markah]

(b) *Gantikan persamaan Laplace*

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0, 0 \leq x \leq 1, 0 \leq y \leq 1$$

dengan penganggaran beza terhingga. Sekiranya syarat sempadan bagi $T(x,y)$ *diberikan kepada kesemua empat sisi segi empat, tunjukkan bagaimana mendapatkan satu system algebra linear.*

[20 markah]

4. (a) *Sebatang rod besi panjang 1m pada awalnya suhu* $70^o C$. *Suhu sekata pada kiri dan kanan rod diberikan sebagai* $50^o C$ *dan* $20^o C$. *Dengan menggunakan* $\alpha^2 = 0.1 m^2 / min$, $\Delta x = 0.2 m$ *dan* $\Delta t = 0.1 min$, *dapatkan taburan suhu dalam rod bagi selang* $0 \leq t \leq 0.3 min$.

(b) *Selesaikan masalah yang sama dengan menggunakan Kaedah Crank-Nicholson dengan* $\Delta x = 0.2 m$ *dan* $\Delta t = 0.3 min$.

[100 markah]

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