UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSG 368 – Sample Survey and Sampling Technique
[Tinjauan Sampel dan Teknik Pensampelan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TWELVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi DUA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) A Gatexpress Plant is one of six large facilities operated by the Post Office for the handling of parcels. The plant serves a large geographic area in the West Malaysia. All parcels are shipped in containers of two kinds : monotainers (wire cage about 50x42x40 in.) and bags (canvas bags secured with a string). The data shown below are the estimation during one week of operation.

Containers	N_i	W_i	n_i	$\overline{y_i}$	s_i	$\hat{p}_i$
Monotainers	2,500	0.053	100	92	46.26	0.17
Bags	45,000	0.947	450	12	9.28	0.12

(i) Estimate the average number of parcels per container and the total number of parcels.

(ii) Determine the error of estimation for the population characteristics in part (i).

(iii) How large should be next week's sample of containers so that the estimate of the proportion of containers shipped to East Malaysia is within 0.01 of the true value with probability of 95%?

(iv) Assume that it costs RM9 to sample one monotainer and RM1 to sample one bag. The fixed cost is RM100, and the budget is RM2,000. Calculate the total sample size and its allocation subject to the budget constraint.

(b) Determine the variance of stratified random sampling, Var($\hat{p}_{st}$) for

(i) Proportional allocation

(ii) Optimum allocation

(c) The number of members in Staff Association is N, is separated into k groups (stratum) of size N_i , i = 1, 2, ..., k. A survey will be conducted to estimate P, proportion of staff in the population who disagree to work overtime. A simple random sampling of size N_i, i = 1, 2, ..., k. are selected from each stratum. Given that

$$\hat{p}_{st} = \sum_{i=1}^{k} \frac{N_i \hat{p}_i}{N} = \sum_{i=1}^{k} W_i \hat{p}_i$$

is the estimator for P, show that

(i) $\hat{p}_{st}$ is an unbiased estimator for P.

(ii) $Var(\hat{p}_{st}) = \sum_{i=1}^{k} \frac{W_i^2 (N_i - n_i) P_i Q_i}{(N_i - 1) n_i}$,

[100 marks]

1. (a) *Kilang Gatexpress ialah salah satu daripada 6 kemudahan terbesar yang dikelolakan oleh Pejabat Pos untuk menguruskan bungkusan-bungkusan. Kilang tersebut menjalankan perkhidmatan yang luas di kawasan Malaysia Barat. Semua bungkusan akan dikirimkan dengan menggunakan 2 jenis bekas: "monotainer" (sangkar dawai sekitar 50x42x40 inci) dan beg (beg kanvas yang diikat rapi dengan tali). Data yang ditunjukkan di bawah adalah anggaran selama satu minggu operasi.*

Bekas	N_i	W_i	n_i	$\overline{y_i}$	s_i	$\hat{p}_i$
"Monotainers"	*2,500*	*0.053*	*100*	*92*	*46.26*	*0.17*
Begs	*45,000*	*0.947*	*450*	*12*	*9.28*	*0.12*

(i) *Anggarkan purata bilangan bungkusan bagi satu bekas dan jumlah bilangan bekas.*

(ii) *Tentukan batas ralat penganggaran bagi ciri-ciri populasi di bahagian (i).*

(iii) *Berapakah saiz sampel yang sepatutnya diambil untuk minggu depan supaya anggaran kadaran bekas yang dikirimkan ke Malaysia Timur ialah dalam lingkungan ± 0.01 dari nilai sebenar dengan kebarangkalian 0.95?*

(iv) *Andaikan kos untuk mensampelkan satu "monotainer" ialah RM9 dan RM1 untuk mensampelkan satu beg. Kos tetap ialah RM100 dan anggaran belanjawan ialah RM2,000. Hitung jumlah saiz sampel dan setiap peruntukan berdasarkan kekangan belanjawan.*

(b) *Tentukan varians bagi pensampelan rawak berstratum, Var($\hat{p}_{st}$) untuk*

(i) *peruntukan berkadaran*

(ii) *peruntukan optimum*

(c) *Bilangan ahli dalam Kesatuan Staf ialah N, yang telah dibahagi kepada k kumpulan (stratum) bersaiz N_i , i = 1, 2, ..., k. Satu tinjauan akan diadakan untuk menganggarkan P, kadaran staf dalam populasi yang enggan bekerja lebih masa. Satu sampel rawak ringkas bersaiz N_i, i = 1, 2, ..., k. dipilih dari setiap stratum. Diberi bahawa*

$$\hat{p}_{st} = \sum_{i=1}^{k} \frac{N_i \hat{p}_i}{N} = \sum_{i=1}^{k} W_i \hat{p}_i$$

adalah penganggar bagi P, tunjukkan

(i) $\hat{p}_{st}$ *adalah penganggar saksama bagi P.*

(ii) $$Var(\hat{p}_{st}) = \sum_{i=1}^{k} \frac{W_i^2 (N_i - n_i) P_i Q_i}{(N_i - 1) n_i}$$

[100 markah]

2. (a) The accompanying table present a population of 5 supermarkets denoted by A, B, C, D , and E. The total number of workers is given for each supermarket.

Supermarket	Number of workers (Y)
A	250
B	140
C	400
D	330
E	110

(i) Compute the mean number of workers and the standard deviation of the distribution of number of workers in the population of five supermarkets.

(ii) How many samples of two supermarkets can be from this population and list all possible samples of two supermarkets, and compute the mean number of workers for each sample.

(iii) Assuming each of the samples listed in part (ii) is equally likely , compute $E(\bar{y})$ and $Var(\bar{y})$.

How does $Var(\bar{y})$ compare with the population variance σ^2?

(b) Suppose that a survey is being planned for purposes of estimating the average number of hours spent exercising daily by adults (18 years of age or older) living in a certain community. A list of all individuals living in the town is not available, however, a list of all households is available at the office of the town clerk. Define the following using the above information:

(i) Population
(ii) Elementary unit
(iii) Sampling units
(iv) Sampling frame
(v) Variable

(c) (i) State the purpose of having a pretest in the survey.

(ii) State two types of errors in the survey and briefly describe the factors that contribute to the errors.

(d) A simple random sample will be chosen from a population of N =20,700 farms and information is collected on the number of goats, y on each farm. The following data are obtained:

$$\sum_{i=1}^{N} y_i = 258,810 \qquad \sum_{i=1}^{N} y_i^2 = 5,994,860$$

How many farms should be selected if it is desired to estimate the total number of goats with a bound on the error of estimation equal to 5,000?

[100 marks]

2. (a) *Jadual di bawah memaparkan satu populasi bagi 5 buah pasar raya A, B, C, D, dan E. Jumlah bilangan pekerja bagi setiap pasar raya diberikan seperti yang berikut:*

Pasar raya	***Bilangan Pekerja(Y)***
A	*250*
B	*140*
C	*400*
D	*330*
E	*110*

(i) Hitung purata bilangan pekerja dan sisihan piawai bagi taburan bilangan pekerja dalam populasi yang terdiri daripada 5 buah pasar raya.

(ii) Berapakah bilangan sampel bagi 2 pasar raya yang mungkin boleh diambil dari populasi dan senaraikan semua sampel tersebut dan kira purata bilangan pekerja masing-masing.

(iii) Andaikan setiap sampel yang disenaraikan di bahagian (ii) itu sama boleh jadi, tentukan $E(\bar{y})$ *dan* $Var(\bar{y})$.

Bagaimana $Var(\bar{y})$ *dibandingkan dengan varians populasi* σ^2 *?*

(b) *Andaikan satu tinjauan telah dirancang bagi tujuan menganggar purata bilangan jam yang digunakan untuk bersenam setiap hari oleh orang dewasa (usia 18 tahun ke atas) yang tinggal dalam masyarakat tertentu. Satu senarai semua individu yang tinggal di bandar tidak boleh diperoleh, akan tetapi satu senarai semua isirumah boleh diperoleh dari pejabat seorang pegawai di bandar. Takrifkan yang berikut dengan menggunakan maklumat di atas.*

(i) Populasi
(ii) Unit asas
(iii) Unit pensampelan
(iv) Rangka pensampelan
(v) Pembolehubah

(c) *(i) Nyatakan tujuan mengadakan pra-ujian dalam tinjauan.*

(ii) Nyatakan dua jenis ralat dalam tinjauan dan huraikan secara ringkas faktor-faktor yang menyumbang kepada ralat-ralat tersebut.

(d) *Suatu sampel rawak mudah akan dipilih daripada populasi N = 20,700 ladang dan maklumat bilangan kambing, y dari setiap ladang telah dikumpul. Yang berikut adalah data yang diperoleh:*

$$\sum_{i=1}^{N} y_i = 258{,}810\,, \quad \sum_{i=1}^{N} y_i^2 = 5{,}994{,}860$$

Berapakah ladang yang patut dipilih jika dikehendaki untuk menganggar jumlah bilangan kambing dengan batas ralatnya adalah 5,000?

[100 markah]

3. (a) In a survey to determine the amount of crop yield due to an air pollutant, a simple random sample of $n = 20$ plots were selected from $N = 1{,}000$ in the population. The summary statistics on yield, y_i (in weight) and level of pollutant, x_i (in parts per million) were

$$\bar{y} = 10,\ \bar{x} = 6,\ \sum_{i=1}^{20}(x_i - \bar{x})(y_i - \bar{y}) = -60,$$

$$\sum_{i=1}^{20}(x_i - \bar{x})^2 = 30,\ \sum_{i=1}^{20}(y_i - a - bx_i)^2 = 80$$

The total pollutant level is 5,000.

(i) Estimate the mean yield for the population using regression estimation.
(ii) Estimate the variance in part (i) and place a bound on the error of estimation.
(iii) Explain your results obtained in part (i) and (ii).
(iv) Predict the yield on a plot in which the pollutant level is $x_i = 4$.

(b) A simple random sample of 48 cities is selected from a population of 200 cities and the present population y of each city is found through fieldwork. Information on the population of these cities at the time of the previous census x is available, the average population per city being 116.94×10^3. The sample gives

$$\sum y_i = 6187 \times 10^3,\quad \sum x_i = 5006 \times 10^3,\quad \sum y_i^2 = 1522257 \times 10^6$$

$$\sum x_i^2 = 1042200 \times 10^6,\quad \sum x_i y_i = 1248030 \times 10^6$$

(i) Estimate the rate of growth of the cities and find its standard deviation.
(ii) Determine the sample size required to estimate the rate of growth of the cities with a bound on the error of estimation of 0.05.
(iii) By using a difference estimator, estimate the average population per city μ_y
(iv) Estimate the variance of $\hat{\mu}_{yD}$.
(v) Compute the estimated relative efficiencies of ratio estimation to difference estimation. Hence which method do you recommend and why?

(c) Describe the three types of population in order to choose between systematic and simple random sampling.

[100 marks]

3. (a) *Dalam tinjauan bagi menentukan amaun hasil tanaman yang disebabkan oleh pencemaran udara, suatu sampel rawak mudah , n = 20 petak telah dipilih daripada populasi , N = 1,000. Ringkasan statistik terhadap hasil, y_i (dalam berat) dan aras pencemaran, x_i (dalam pecahan per juta) adalah:*

$$\bar{y} = 10,\ \bar{x} = 6,\ \sum_{i=1}^{20}(x_i - \bar{x})(y_i - \bar{y}) = -60,$$

$$\sum_{i=1}^{20}(x_i - \bar{x})^2 = 30,\ \ \sum_{i=1}^{20}(y_i - a - bx_i)^2 = 80$$

Jumlah aras pencemaran ialah 5,000.

(i) *Anggarkan purata hasil bagi populasi dengan menggunakan penganggaran regresi.*
(ii) *Anggarkan varians di bahagian(i) dan tentukan batas ralat penganggarannya.*
(iii) *Tafsirkan keputusan yang diperoleh di bahagian (i) dan (ii).*
(iv) *Ramalkan hasil dalam petak apabila aras pencemaran ialah $x_i = 4$.*

(b) *Suatu sampel rawak mudah dengan 48 bandar raya telah dipilih daripada populasi sebanyak 200 bandar raya dan populasi terkini, y bagi setiap bandar raya telah didapati melalui kajian yang dijalankan. Maklumat terhadap populasi bandar raya-bandar raya daripada banci terdahulu, x boleh diperoleh. Purata populasi bagi satu bandar raya ialah 116.94×10^3. Berikut adalah maklumat daripada sampel :*

$$\sum y_i = 6187 \times 10^3,\qquad \sum x_i = 5006 \times 10^3,\ \sum y_i^2 = 1522257 \times 10^6$$

$$\sum x_i^2 = 1042200 \times 10^6,\ \sum x_i y_i = 1248030 \times 10^6$$

(i) *Angggarkan kadar pertumbuhan bagi bandar raya-bandar raya dan cari sisihan piawainya.*
(ii) *Tentukan saiz sampel yang diperlukan untuk menganggarkan kadar pertumbuhan bandar raya dengan batas ralat penganggaran ialah 0.05.*
(iii) *Dengan menggunakan penganggaran beza, anggarkan purata populasi bagi satu bandar raya, μ_y.*
(iv) *Anggarkan varians bagi $\hat{\mu}_{yD}$.*
(v) *Kira anggaran kecekapan relatif penganggaran nisbah kepada penganggaran beza. Kaedah manakah yang anda syorkan dan kenapa?*

(c) *Huraikan tiga jenis populasi supaya dapat menentukan pilihan antara pensampelan sistematik dan rawak mudah?*

[100 markah]

4. (a) A student wants to estimate the average grade point average (GPA) of students in his dormitory. Instead of obtaining a listing of all students in the dorm and conducting a simple random sampel, he notices that the dorm consists of 100 suites, each with four students; he chooses 5 of those suites at random and asks every person in the 5 suites what her or his GPA is. The results are as follows:

Person Number	Suite				
	1	2	3	4	5
1	3.08	2.36	2.00	3.00	2.68
2	2.60	3.04	2.56	2.88	1.92
3	3.44	3.28	2.52	3.44	3.28
4	3.04	2.68	1.88	3.64	3.20

(i) For this sample, what are the clusters and what are the listing units?
(ii) Estimate the sum of all GPAs of all students in the dorm.
(iii) Find the standard deviation of the estimates in part (ii).
(iv) Construct 95% confidence interval for the average of GPAs students in the dorm.

(b) TORELEC is a company selling electrical equipment and supplies to electricians, contractors, and other wholesale customers. The items are stored in stack which are like large bookcases except that they have bins instead of shelves. The TORELEC's warehouse contains 500 stacks of the same shape and size, and there are about 15,000 bins. Suppose that five stacks are selected at random. The relevant information is shown in the following table.

Selected Stack	Total Number of Bins	Number of Selected Bins and Items	Number of Items with a Discrepancy	Total Discrepancy (RM)
051	30	3	3	142
105	20	2	0	0
252	25	2	2	-85
343	40	4	1	30
417	32	3	2	-210

(i) Estimate the total number of items in the warehouse with a discrepancy.
(ii) What would be the standard deviation of the estimated total number of items in the warehouse with a discrepancy?

(c) A number of industrial establishments will be employing 20 or more persons in the town of Bayan Baru in Pulau Pinang. The following are the employment figures based on a 1-in-5 systematic sample.

35	88	35	36	156	25	24	237	80
468	22	139	163	37	37	27	25	20
38	24	62	331	28	31	81	121	49
23	34	23	22	53	50	50		

State the appropriate assumption.
Estimate the total employment with an appropriate margin of error.

[100 marks]

4. (a) *Seorang pelajar ingin menganggar min bagi purata bagi nilai gred (PNG) pelajar-pelajar di bilik asramanya. Selain daripada memperoleh senarai semua pelajar di asrama dan menjalankan sampel rawak mudah, pelajar tersebut mendapati bahawa asrama terdiri daripada 100 rangkaian bilik, setiapnya mengandungi empat orang pelajar; dia memilih 5 daripada rangkaian bilik secara rawak dan bertanya setiap pelajar dalam 5 bilik tersebut apakah PNG mereka. Keputusannya adalah seperti yang berikut:*

Bilangan Pelajar	***Rangkaian***				
	1	*2*	*3*	*4*	*5*
1	*3.08*	*2.36*	*2.00*	*3.00*	*2.68*
2	*2.60*	*3.04*	*2.56*	*2.88*	*1.92*
3	*3.44*	*3.28*	*2.52*	*3.44*	*3.28*
4	*3.04*	*2.68*	*1.88*	*3.64*	*3.20*

(i) Untuk sampel ini, apakah kelompok-kelompok dan apakah senarai unit?
(ii) Anggarkan jumlah kesemua PNG bagi semua pelajar di asrama.
(iii) Cari sisihan piawai bagi anggaran di bahagian (ii).
(iv) Binakan selang keyakinan 95% bagi purata PNG pelajar di asrama.

(b) *TORELEC ialah sebuah syarikat yang menjual alat-alat eletrik dan membekalkan kepada juru eletrik, kontraktor dan pembeli lain yang membeli secara borong. Barang-barang disimpan dalam susunan seperti rak buku kecuali ia mempunyai bekas yang besar dan bukannya rak. Gudang TORELEC mengandungi 500 susunan yang sama saiz dan bentuk, dan terdapat sebanyak 15,000 bekas besar. Andaikan 5 susunan dipilih secara rawak. Informasi yang berkaitan ditunjukkan dalam jadual yang berikut:*

Susunan Terpilih	**Jumlah Bekas Besar**	**Bilangan Bekas Besar Terpilih dan Item**	**Bilangan Item yang Berbeza**	**Jumlah Perbezaan (RM)**
051	30	3	3	142
105	20	2	0	0
252	25	2	2	-85
343	40	4	1	30
417	32	3	2	-210

(i) Anggarkan jumlah bilangan item dalam gudang yang berbeza.
(ii) Apakah nilai sisihan piawai dari anggaran jumlah bilangan item dalam gudang yang berbeza?

(c) *Sebilangan kumpulan perindustrian akan mengambil pekerja seramai 20 orang atau lebih dari bandar Bayan Baru di Pulau Pinang. Berikut adalah bilangan pekerjaan berdasarkan 1-dalam-5 sampel sistematik.*

35	*88*	*35*	*36*	*156*	*25*	*24*	*237*	*80*
468	*22*	*139*	*163*	*37*	*37*	*27*	*25*	*20*
38	*24*	*62*	*331*	*28*	*31*	*81*	*121*	*49*
23	*34*	*23*	*22*	*53*	*50*	*50*		

Nyatakan andaian yang bersesuaian. Anggarkan jumlah pekerjaan berserta batas ralat yang bersesuaian.

[100 markah]

Appendix

Sample	Sampel variance
$\sum_{i=1}^{n} \frac{y_i}{n}$	$\frac{s^2}{n}\left(\frac{N-n}{N}\right), s^2 = \frac{\sum_{i=1}^{n} y_i^2 - n\bar{y}^2}{n-1}$
$N\bar{y}$	$N^2 \frac{s^2}{n}\left(\frac{N-n}{N}\right)$
$\frac{a}{n}$	$\frac{\hat{p}\ 1-\hat{p}}{n-1}\left(\frac{N-n}{N}\right)$
$\frac{\sum_{i=1}^{n} N_i \bar{y}_i}{N}$	$\sum_{i=1}^{n} \frac{N_i^2}{N^2}\left(\frac{N_i - n_i}{N_i}\right)\frac{s_i^2}{n_i}$
$\sum_{i=1}^{n} \frac{N_i \hat{p}_i}{N}$	$\sum_{i=1}^{n} \frac{N_i^2}{N^2}\left(\frac{N_i - n_i}{N_i}\right)\frac{\hat{p}_i\ 1-\hat{p}_i}{n_i - 1}$
$\frac{\bar{y}}{\bar{x}}$	$\left(\frac{N-n}{nN}\right)\left(\frac{1}{\mu_x^2}\right)\left(\frac{\sum_{i=1}^{n} y_i - r x_i\ ^2}{n-1}\right)$
$\bar{y} + b\ \mu_x - \bar{x}$, $b = \frac{\sum_{i=1}^{n} y_i - \bar{y}\ \ x_i - \bar{x}}{\sum_{i=1}^{n} x_i - \bar{x}\ ^2}$	$\left(\frac{N-n}{Nn}\right)\left(\frac{1}{n-2}\right)\left(\sum_{i=1}^{n} y_i - \bar{y}\ ^2 - b^2 \sum_{i=1}^{n} x_i - \bar{x}\ ^2\right)$
$\mu_x + \bar{d}$	$\left(\frac{N-n}{Nn}\right)\left(\frac{\sum_{i=1}^{n} d_i - \bar{d}\ ^2}{n-1}\right)$
$\left(\frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} m_i}\right)$	$\left(\frac{N-n}{Nn\bar{M}^2}\right)\left(\frac{\sum_{i=1}^{n} y_i - \bar{y} m_i\ ^2}{n-1}\right)$

Sampel	Sample Variance
$\frac{\sum_{i=1}^{n} a_i}{\sum_{i=1}^{n} m_i}$	$\left(\frac{N-n}{Nn-\bar{M}^2}\right)\frac{\sum_{i=1}^{n} a_i-\hat{p}m_i^{\,2}}{n-1}$
$M\bar{y}$	$M^2\left(\frac{N-n}{Nn\bar{M}^2}\right)\frac{\sum_{i=1}^{n} y_i-\bar{y}m_i^{\,2}}{n-1}$
$\hat{\mu}=\frac{1}{n\bar{M}}\sum_{i=1}^{n} M_i\bar{y}_i$	$\left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)S_b^2+\frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{S_i^2}{m_i}\right)$ with $S_b^2=\frac{\left(\sum_{i=1}^{n} M_i\bar{y}_i-\bar{M}\hat{\mu}\right)^2}{n-1}$ $S_i^2=\frac{\sum_{j=1}^{m_i} y_{ij}-\bar{y}_i^{\,2}}{m_i-1}$
$\hat{\mu}_r=\frac{\sum_{i=1}^{n} M_i\bar{y}_i}{\sum_{i=1}^{n} M_i}$	$\left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)S_r^2+\frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{S_i^2}{m_i}\right)$ with $S_r^2=\frac{\sum_{i=1}^{n_i} M_i\bar{y}_i-\hat{\mu}_r M_i^{\,2}}{n-1}$
$\hat{p}=\frac{\sum_{i=1}^{n} M_i\hat{p}_i}{\sum_{i=1}^{n} M_i}$	$\left(\frac{N-n}{N}\right)\left(\frac{1}{n\bar{M}^2}\right)S_r^2+\frac{1}{nN\bar{M}^2}\sum_{i=1}^{n} M_i^2\left(\frac{M_i-m_i}{M_i}\right)\left(\frac{\hat{p}_i\hat{q}_i}{m_i-1}\right)$ with $S_r^2=\frac{\sum_{i=1}^{n_i} M_i\hat{p}_i-\hat{p}M_i^{\,2}}{n-1}$

Sample Size
$n=\frac{N\sigma^2}{N-1\ D+\sigma^2}\ ;\ D=\frac{B^2}{4}\ ;\ D=\frac{B^2}{4N^2}$
$n=\frac{\sum_{i=1}^{L}\frac{N_i^2\sigma_i^2}{w_i}}{N^2D+\sum_{i=1}^{L}N_i\sigma_i^2}\ ;\ w_i=\frac{n_i}{n}$
$n=\frac{\left(\sum_{k=1}^{L}N_k\sigma_k/\sqrt{C_k}\right)\left(\sum_{i=1}^{L}N_i\sigma_i^2\sqrt{C_i}\right)}{N^2D+\sum_{i=1}^{L}N_i\sigma_i^2}\ ;n_i=\frac{n\,N_i\sigma_i/\sqrt{C_i}}{\sum_{i=1}^{L}N_i\sigma_i/\sqrt{C_i}}$
$n=\frac{\left(\sum_{i=1}^{L}N_i\sigma_i\right)^2}{N^2D+\sum_{i=1}^{L}N_i\sigma_i^2}\ ;\ n_i=n\left(\frac{N_i\sigma_i}{\sum_{i=1}^{L}N_i\sigma_i}\right)$
$n=\frac{\sum_{i=1}^{L}N_i\sigma_i^2}{ND+\frac{1}{N}\sum_{i=1}^{L}N_i\sigma_i^2}\ ;\ n_i=n\left(\frac{N_i}{\sum_{i=1}^{L}N_i}\right)$
$n=\frac{\sum_{i=1}^{L}N^2{}_i p_i q_i/a_i}{N^2D+\sum_{i=1}^{L}N_i p_i q_i}\ ;\ n_i=n\left(\frac{N_i\sqrt{p_i q_i/c_i}}{\sum_{i=1}^{L}N_i\sqrt{p_i q_i/c_i}}\right)$
$n=\frac{N\sigma^2}{ND+\sigma^2}\ ;\ D=\frac{B^2\mu x^2}{4}\ ;\ D=\frac{B^2}{4}\ ;\ D=\frac{B^2}{4N^2}$
$n=\frac{N\sigma_r{}^2}{ND+\sigma_r{}^2}\ ;\ D=\frac{B^2\ \bar{M}^{\ 2}}{4}\ ;\ D=\frac{B^2}{4N^2}$
$n=\frac{c-c_0\ \sum_{i=1}^{L}N_i\sigma_i/\sqrt{c_i}}{\sum_{i=1}^{L}N_i\sigma_i\sqrt{c_i}}$