UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSG 367 – Time Series Analysis
[Analisis Siri Masa]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of THIRTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Discuss the following with example:

   (i) Why diagnostic checking procedure is important in the process of building a time series model?
   (ii) What is meant by over-fitting and its use in the diagnostic checking process?
   (iii) How model selection criteria such as AIC and BIC can be used in choosing the best among competing models?

[45 marks]

(c) Consider a process defined as: $Y_t = 20 - 10t + X_t$ where X_t is a process defined as below:

$$X_t = X_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1} \text{ with } X_1 = \varepsilon_1 \text{ and that } Var(X_1) = \sigma_\varepsilon^2.$$

Show that: (i) $E(X_t) = 0$ (ii) Y_t is not stationary.

Find the mean, variance and autocovariance function of $Z_t = Y_t - Y_{t-1}$. Is $\{Z_t\}$ a stationary process? Briefly explain your reason.

[35 marks]

(d) Rewrite each of the models below using the backward operator *B* and state the form of ARIMA(*p*,*d*,*q*) or SARIMA(*p*,*d*,*q*)(*P*,*D*,*Q*). [*p, d, q, P, D,* and *Q* are positive finite numbers].

(i) $Y_t = Y_{t-1} + \phi_2 Y_{t-2} - \phi_1 Y_{t-3} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-2})$

(ii) $Y_t = \varepsilon_t - (1-\theta_1)\varepsilon_{t-1} - (1-\theta_1)\varepsilon_{t-2} - \cdots$

(iii) $Y_t = (1+\phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \varepsilon_t + \theta_1\varepsilon_{t-1} - \Theta_{12}\varepsilon_{t-12} - \theta_1\Theta_{12}\varepsilon_{t-13}$

(iv) $Y_t = \varepsilon_t + 0.7\varepsilon_{t-2} + 0.49\varepsilon_{t-4} + 0.343\varepsilon_{t-6} + \ldots + 0.118\varepsilon_{t-12} + \cdots$

[20 marks]

1. (a) *Bincangkan yang berikut dengan contoh:*

   (i) *Mengapa prosedur pemeriksaan dianostik adalah penting dalam proses pembentukkan suatu model siri masa?*

   (ii) *Apakah yang dimaksudkan dengan terlebih-suai dan kegunaannya dalam proses pemeriksaan dianostik?*

   (iii) *Bagaimanakah criteria pemilihan model seperti AIC dan BIC boleh digunakan dalam memilih yang terbaik dikalangan model yang berkemungkinan?*

[45 markah]

(b) *Pertimbangkan suatu proses yang dinyatakan sebagai:* $Y_t = 20 - 10t + X_t$ *yang mana* X_t *adalah suatu proses yang diberikan seperti di bawah:*

$$X_t = X_{t-1} + \varepsilon_t - \theta\varepsilon_{t-1} \text{ dengan } X_1 = \varepsilon_1 \text{ dan juga } Var(X_1) = \sigma_\varepsilon^2$$

Tunjukkan bahawa: (i) $E(X_t) = 0$ (ii) Y_t *adalah tidak pegun.*

Cari min, varians dan fungsi autokovarians bagi $Z_t = Y_t - Y_{t-1}$. *Adakah* $\{Z_t\}$ *merupakan suatu proses yang pegun? Terangkan secara ringkas alasan kamu?*

[35 markah]

(c) *Tulis semula setiap model di bawah menggunakan pengoperasi anjak kebelakang B dan nyatakan bentuk ARKPB(p,d,q) atau bermusim ARKPBR(p,d,q)(P,D,Q). [p, d, q, P, D dan Q adalah nombor-nombor positif terhingga]*

(i) $Y_t = Y_{t-1} + \phi_2 Y_{t-2} - \phi_1 Y_{t-3} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-2})$

(ii) $Y_t = \varepsilon_t - (1-\theta_1)\varepsilon_{t-1} - (1-\theta_1)\varepsilon_{t-2} - \cdots$

(iii) $Y_t = (1+\phi_1)Y_{t-1} - \phi_1 Y_{t-2} + \varepsilon_t + \theta_1\varepsilon_{t-1} - \theta_{12}\varepsilon_{t-12} - \theta_1\theta_{12}\varepsilon_{t-13}$

(iv) $Y_t = \varepsilon_t + 0.7\varepsilon_{t-2} + 0.49\varepsilon_{t-4} + 0.343\varepsilon_{t-6} + \ldots + 0.118\varepsilon_{t-12} + \ldots$

[20 markah]

2. (a) Given two processes of autoregressive of order one, AR(1):

A: $Y_t = \lambda + \phi_1 Y_{t-1} + \varepsilon_t$
B: $Y_t - Y_{t-1} = \lambda + \phi_1 (Y_{t-1} - Y_{t-2}) + \varepsilon_t$ with $Y_1 = \varepsilon_1$

Show that process A is stationary with constant mean, $E_A(Y_t) = \mu$ such that $\lambda = \mu(1-\phi_1)$.

Show that process B is not only non-stationary but having a deterministic mean that increases over time.

[30 marks]

(b) Given an ARMA(2,1) process:

$$(1-\phi_1 B - \phi_2 B^2) Y_t = (1-\theta_1 B)\varepsilon_t$$

Show that:

$$\gamma_0 = \phi_1\gamma_1 + \phi_2\gamma_2 + [1-\theta_1(\phi_1-\theta_1)]\sigma_\varepsilon^2$$

$$\gamma_1 = \phi_1\gamma_2 + \phi_2\gamma_3 - \theta_1\sigma_\varepsilon^2$$

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} \quad \text{for } k \geq 2$$

A series of 225 observations was collected and an ARMA(2,1) model has been fitted with the following estimates: $\hat{\phi}_1 = 0.55$, $\hat{\phi}_2 = -0.25$ and $\hat{\theta}_1 = 0.20$.

Calculate the values of autocorrelation, acf for lag *k* = 1, 2, 3, 4, 5, and partial autocorrelation, pacf for lag *k* = 1 and 2. What can you say about the calculated values of acf and pacf and its underlying process.

[Given the values of acf at lag 6 through to 10 are 0.561, -0.485, 0.440, -0.295 and 0.350 respectively, and pacf at lag 3 through to 8 are -0.030, -0.051, -0.081, 0.140, 0.127, 0.062 respectively].

[30 marks]

(c) A newly employed trainee at Company ZZ has been given a time series of length 300. She has been asked to fit a suitable time series model to the data. Appendix A shows the procedures and steps that she has conducted in her analysis.
Explain with reason each of the output in Appendix A.

[40 marks]

2. (a) *Diberi dua proses autoregressive peringkat pertama, AR(1):*

A: $Y_t = \lambda = \phi_1 Y_{t-1} + \varepsilon_t$
B: $Y_t - Y_{t-1} = \lambda + \phi_1 (Y_{t-1} - Y_{t-2}) + \varepsilon_t$ *dengan* $Y_1 = \varepsilon_1$

Tunjukkan bahawa proses A adalah pegun dengan min konstan, $E(Y_t) = \mu$ *seperti mana* $\lambda = \mu(1 - \phi_1)$.

Tunjukkan bahawa proses B bukan sahaja tidak pegun malah mempunyai min tertentu yang mana ianya meningkat mengikut masa.

[30 markah]

(b) *Diberi suatu proses ARPB(2,1):*

$$(1 - \phi_1 B - \phi_2 B^2) Y_t = (1 - \theta_1 B)\varepsilon_t$$

Tunjukkan bahawa:

$$\gamma_0 = \phi_1\gamma_1 + \phi_2\gamma_2 + [1 - \theta_1(\phi_1 - \theta_1)]\sigma_\varepsilon^2$$

$$\gamma_1 = \phi_1\gamma_2 + \phi_2\gamma_3 - \theta_1\sigma_\varepsilon^2$$

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} \quad \text{untuk} \; k \geq 2$$

Suatu siri denagn 225 cerapan telah dikumpul dan suatu model ARPB(2,1) telah disuai dengan anggaran-anggaran berikut: $\hat{\phi}_1 = 0.55$, $\hat{\phi}_2 = -0.25$ *dan* $\hat{\theta}_1 = 0.20$.

Hitung nilai autokorelasi, fak untuk susulan k = 1, 2, 3, 4,5 dan 6, dannilai autokorelai separa, faks untuk susulan k = 1 dan 2. Apa yang boleh kamu katakana tentang nilai terhitung bagi fak dan faks dan juga proses yang diwakilkan.

[Diberi bahawa nilai-nilai fak pada susulan 6 hingga 10 masing-masing adalah ??, ??, ??, ??, ?? dan nilai faks pada susulan 3 hingga 8 masing-masing adalah ??, ??, ??, ??, ?? dan ??]

[30 markah]

(c) *Seorang pelatih yang baru diambil bekerja di Syarikat ZZ telah diberi suatu siri masa dengan panjang 300. Beliau telah disuruh untuk menyuaikan suatu model siri masa terhadap data tersebut. Lampiran A menunjukkan prosedur dan juga langkah yang telah dijalankan oleh pelatih tersebut.*
Terangkan dengan alasan bagi setiap output di Lampiran A.

[40 markah]

3. (a) Consider a seasonal AR(2) process, SAR(2) as given by:

$$\left(1-\phi_{12}B^{12}-\phi_{24}B^{24}\right)Y_t=\varepsilon_t$$

Using method of moments, show that the estimates of ϕ_{12} and ϕ_{24} are given by:

$$\hat{\phi}_{12}=\frac{\hat{\rho}_{12}\left(1-\hat{\rho}_{24}\right)}{1-\rho_{12}^2} \quad \text{and} \quad \hat{\phi}_{24}=\frac{\hat{\rho}_{24}-\rho_{12}^2}{1-\rho_{12}^2}$$

[25 marks]

(b) A non-stationary seasonal time series $\{S_t\}$ has 250 observations and is believed to follows an invertible SARIMA $(0,0,1)(1,1,0)_{12}$ model given by:

$$S_t=\left(1+\phi_{12}\right)S_{t-12}-S_{t-24}+\varepsilon_t-\theta_1\varepsilon_{t-1}$$

(i) Show that $Y_t=S_t-S_{t-12}$ has the variance and autocorrelation function given by:

$$\gamma_0=\frac{1+\theta_1^2}{1-\phi_{12}^2}\sigma_\varepsilon^2 \qquad \rho_{12k}=\phi_{12}^k \text{ for } k=1,2,\ldots$$

$$\rho_{12k-1}=\rho_{12k+1}=-\frac{\theta_1}{1+\theta_1^2}\phi_{12}^k \text{ for } k=0,1,2,\ldots$$

(ii) Table 1 and Table 2 in the Appendix B show the sample acf and sample pacf of $\{S_t\}$ and $\{\nabla_{12}S_t\}$. The mean and standard deviation for the original and differenced series are also given.

Discuss the appropriateness of the SARIMA $(0,0,1)(1,1,0)_{12}$ model for the series based on the given sample acf and sample pacf. Calculate the estimate for ϕ_{12}, θ_1 and σ_ε^2.

[50 marks]

(c) Consider the following SARIMA model:

$$Y_t-Y_{t-12}=\varepsilon_t-\theta_1\varepsilon_{t-1}$$

Show that the forecast error variance is given by:

$$Var\left[e_n(m)\right]=\sigma_\varepsilon^2\left[1+k\left(1-\theta_1\right)^2\right]$$

for $m=12k+r+1$, $k=0, 1, \ldots$, and $0\le r<12$.

What can you say about the model and its forecast error variance?

[25 marks]

3. (a) *Pertimbangkan suatu proses bermusim AR(2), SAR(2) yang diwakili oleh:*

$$(1-\phi_{12}B^{12}-\phi_{24}B^{24})Y_t=\varepsilon_t$$

Menggunakan kaedah momen, tunjukkan bahawa anggaran bagi ϕ_{12} *dan* ϕ_{24} *adalah diberikan oleh:*

$$\hat{\phi}_{12}=\frac{\hat{\rho}_{12}(1-\hat{\rho}_{24})}{1-\rho_{12}^2} \quad \text{and} \quad \hat{\phi}_{24}=\frac{\hat{\rho}_{24}-\rho_{12}^2}{1-\rho_{12}^2}$$

[25 markah]

(b) *Suatu siri masa bermusim tak pegun* $\{S_t\}$ *mempunyai 250 cerapan dan dipercayai mengikuti model bolehsongsang bermusim ARKPB* $(0,0,1)\times(1,1,0)_{12}$ *yang diberikan oleh:*

$$S_t=(1+\phi_{12})S_{t-12}-S_{t-24}+\varepsilon_t-\theta_1\varepsilon_{t-1}$$

(i) *Tunjukkan bahawa* $Y_t=S_t-S_{t-12}$ *mempunyai varians dan fungsi autokorelasi yang diberikan oleh:*

$$\gamma_0=\frac{1+\theta_1^2}{1-\phi_{12}^2}\sigma_\varepsilon^2 \qquad \rho_{12k}=\phi_{12}^k \text{ untuk } k=1,2,\ldots$$

$$\rho_{12k-1}=\rho_{12k+1}=-\frac{\theta_1}{1+\theta_1^2}\phi_{12}^k \text{ untuk } k=0,1,2,\ldots$$

(ii) *Jadual1 dan Jadual 4 di Lampiran B menunjukkan sampel fak dan sampel faks bagi* $\{S_t\}$ *dan* $\{\nabla_{12}S_t\}$. *Min serta sisihan piawai bagi siri asal dan siri yang telah dibezakan juga diberikan.*

Bincang kesesuaian bagi model bermusim ARKPB $(0,0,1)\times(1,1,0)_{12}$ *untuk siri tersebut berdasarkan sampel fak dan sampel faks yang diberi. Hitung anggaran bagi* ϕ_{12}, θ_1 *dan* σ_ε^2.

[50 markah]

(c) *Pertimbangkan model bermusim ARKPB yang berikut:*

$$Y_t-Y_{t-12}=\varepsilon_t-\theta_1\varepsilon_{t-1}$$

Tunjukkan bahawa varians bagi ralat telahan adalah diberikan oleh:

$$Var[e_n(m)]=\sigma_\varepsilon^2\left[1+k(1-\theta_1)^2\right]$$

untuk $m=12k+r+1$, *k = 0, 1, …, dan* $0\le r<12$.

Apakah yang boleh anda katakana mengenai model serta varians bagi ralat telahan yang sepadan?

[25 markah]

4. Consider an ARMA(1,2) model for a series with non-zero mean:

$$(1-\phi_1 B)(Y_t-\mu) = (1-\theta_1 B-\theta_2 B^2)\varepsilon_t$$

(a) Consider a special case with $d = \theta_2 = 0$.

(i) Show that the MA coefficient is given by: $\psi_k = (\phi_1-\theta_1)\phi_1^{k-1}$.

Show that the *m*-step ahead forecasts made at time *t* = *n* is given by:

$$\hat{Y}_N(m) = \mu(1-\phi_1)+\phi_1\hat{Y}_N(m-1) \quad \text{for } m \geq 2$$

and that it can be rewritten as:

$$\hat{Y}_N(m) = \mu(1-\phi_1^m)+\phi_1^m Y_N-\phi_1^{m-1}\theta_1\varepsilon_N \quad \text{for } m \geq 1$$

(ii) Show that the corresponding variance of forecast error is given by:

$$Var[e_N(m)] = \sigma_\varepsilon^2\left(1+(\phi_1-\theta_1)^2\left[\frac{1-\phi_1^{2(n-1)}}{1-\phi_1^2}\right]\right)$$

(iii) Finally, show that as $m \to \infty$:

$$\hat{Y}_N(m) \to \mu \quad \text{and} \quad Var[e_N(m)] \to \frac{(1+\theta_1^2-2\phi_1\theta_1)}{1-\phi_1^2}\sigma_\varepsilon^2$$

[40 marks]

(b) Show that the one-step and two-step ahead forecasts made at *t* = *n* are respectively given by:

$$\hat{Y}_n(1) = \mu(1-\phi_1)+\phi_1 Y_n-\theta_1\varepsilon_n-\theta_2\varepsilon_{n-1},\ \hat{Y}_n(2) = \mu(1-\phi_1)+\phi_1\hat{Y}_n(1)-\theta_2\varepsilon_n$$

and also show that the *m*-step-ahead forecast is given by:

$$\hat{Y}_n(m) = \mu(1-\phi_1)+\phi_1\hat{Y}_n(m-1) \quad \text{for } m \geq 3$$

4. *Pertimbangkan suatu model ARPB ARMA(1,2)bagi suatu siri dengan min bukan kosong:*

$$(1-\phi_1 B)(Y_t-\mu)=(1-\theta_1 B-\theta_2 B^2)\varepsilon_t$$

(a) *Pertimbangkan kes khas dengan* $d=\theta_2=0$.

(i) *Tunjukkan bahawa koefisien PB adalah diberikan oleh:*

$\varphi_k=(\phi_1-\theta_1)\phi_1^{k-1}$.

Tunjukkan bahawan telahan m-langkah kehadapan yang dibuat pada waktu t = n adalah diberikan oleh:

$$\hat{Y}_N(m)=\mu(1-\phi_1)+\phi_1\hat{Y}_N(m-1) \quad \textit{untuk} \quad m\geq 2$$

Dan bahawa ia boleh ditulis semula sebagai:

$$\hat{Y}_N(m)=\mu(1-\phi_1^m)+\phi_1^m Y_N-\phi_1^{m-1}\theta_1\varepsilon_N \quad \textit{untuk} \quad m\geq 1$$

(ii) *Tunjukkan bahawa varians bagi ralat telahan yang sepadan diberikan oleh:*

$$Var[e_N(m)]=\sigma_\varepsilon^2\left(1+(\phi_1-\theta_1)^2\left[\frac{1-\phi_1^{2(m-1)}}{1-\phi_1^2}\right]\right)$$

(iii) *Akhir sekali, tunjukkan bahawa apabila* $m\to\infty$:

$$\hat{Y}_N(m)\to\mu \quad \textit{dan} \quad Var[e_N(m)]\to\left(\frac{1+\theta_1^2-2\phi_1\theta_1}{1-\phi_1^2}\right)\sigma_\varepsilon^2$$

[40 markah]

(b) *Tunjukkan bahawa telahan satu-langkah dan dua-langkah kehadapan yang dibuat pada t = n masing-masing diberikan oleh:*

$$\hat{Y}_n(1)=\mu(1-\phi_1)+\phi_1 Y_n-\theta_1\varepsilon_n-\theta_2\varepsilon_{n-1}, \quad \hat{Y}_n(2)=\mu(1-\phi_1)+\phi_1\hat{Y}_n(1)-\theta_2\varepsilon_n$$

Dan juga tunjukkan bahawa telahan m-langkah kehadapan adalah diberikan oleh:

$$\hat{Y}_n(m)=\mu(1-\phi_1)+\phi_1\hat{Y}_n(m-1) \quad \textit{untuk} \quad m\geq 3$$

(i) Consider n = 250. If estimated values for the coefficients are $\hat{\phi}_1 = -0.7$, $\hat{\theta}_1 = -0.3$, $\hat{\theta}_2 = 0.73$, $\hat{\Theta}_1 = 0.315$, $\hat{\mu} = 200$, $s_{\varepsilon}^2 = 12$ with $Y_{250} = 216$, $\varepsilon_{250} = 4$ and $\varepsilon_{249} = 12$, obtain multi-step (dynamic) forecasts for $\hat{Y}_{250}(m)$ for m = 1, 2, …, 6. Construct a 95% forecast interval for Y_{251}, Y_{252}, Y_{253} and Y_{254}. Comment on the six forecast values obtained above.

(ii) At time t = 251 the observed value is found to be 188. Calculate the updated forecast of $Y_{252}, \ldots, Y_{256}$. Compare these new forecasts with those calculated in (i) above and discuss.

(iii) At time t = 252 and t = 253 the observed value is noted as 197 and 194 respectively. Together with the information in (ii) above, obtain the 1-step-ahead forecast of $\hat{Y}_{251}(1)$, $\hat{Y}_{252}(1)$ and $\hat{Y}_{253}(1)$ and its corresponding 95% confidence interval.
Compare and what can you say about the multi-step-ahead forecast and 1-step-ahead forecast for Y_{251}, Y_{252}, Y_{253} and Y_{254}.

[60 marks]

(i) *Pertimbangkan n = 250. Sekiranya nilai teranggar bagi koefisien-koefisien adalah* $\hat{\phi}_1 = -0.7$, $\hat{\theta}_1 = -0.3$, $\hat{\theta}_2 = 0.73$, $\hat{\theta}_1 = 0.315$, $\hat{\mu} = 200$, $s_{\varepsilon}^2 = 12$ *dengan* $Y_{250} = 216$, $\varepsilon_{250} = 4$ *dan* $\varepsilon_{249} = 12$, *dapatkan nilai telahan banyak-langkah (dinamik) bagi* $\hat{Y}_{250}(m)$ *untuk m = 1, 2, ..., 6. Bina selang telahan 95% bagi* Y_{251}, Y_{252}, Y_{253} *dan* Y_{254} *. Komen terhadap enam nilai telahan yang diperoleh di atas.*

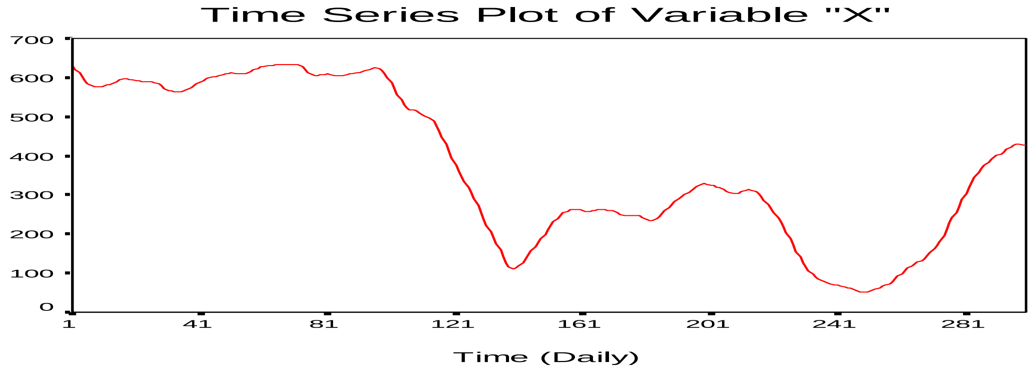
(ii) *Pada waktu t = 251 nilai dicerap dijumpai sebagai 188. Hitung nilai telahan kemaskini bagi* $Y_{252}, \ldots, Y_{256}$*. Bandingkan nilai-nilai telahan ini dengan nilai-nilai yang diperoleh dalam (i) di atas dan bincangkan,.*

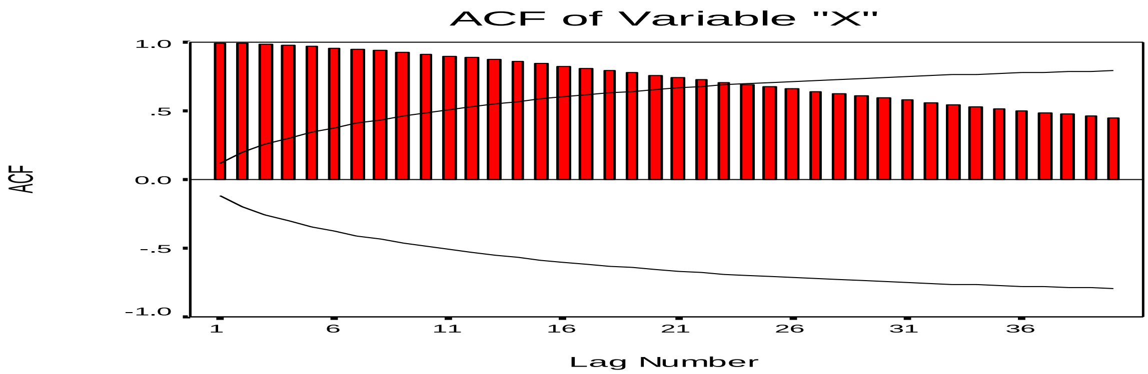
(iii) *Pada waktu t = 252 dan t = 253 nilai yang dicerap masing-masing dicatatkan sebagai 197 dan 194. Bersama maklumat yang terdapat dalam (ii) di atas, dapatkan nilai telahan 1-langkah kehadapan bagi*
$\hat{Y}_{251}(1)$, $\hat{Y}_{252}(1)$ *dan* $\hat{Y}_{253}(1)$ *dan selang keyakinan 95% yang sepadan.*
Bandingkan dan apakah yang boleh dikatakan mengenai telahan banyak-langkah kehadapan dan telahan 1-langkah kehadapan bagi Y_{251}, Y_{252}, Y_{253} *dan* Y_{254}.

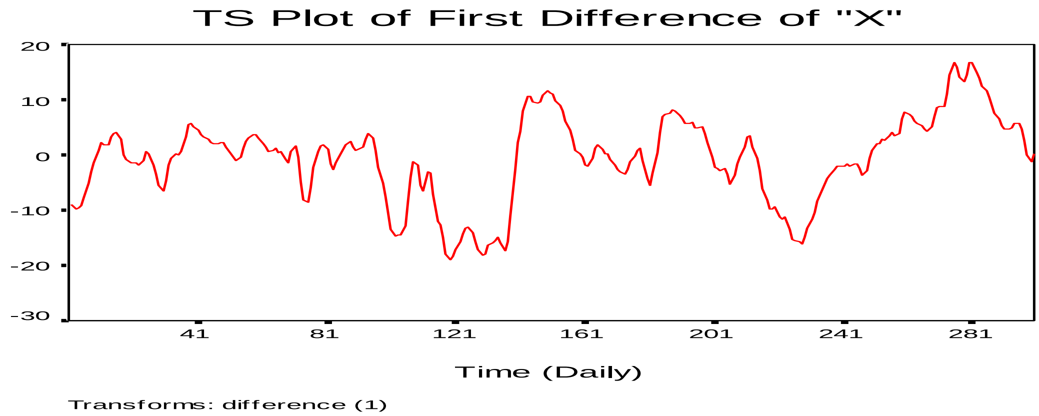
[60 markah]

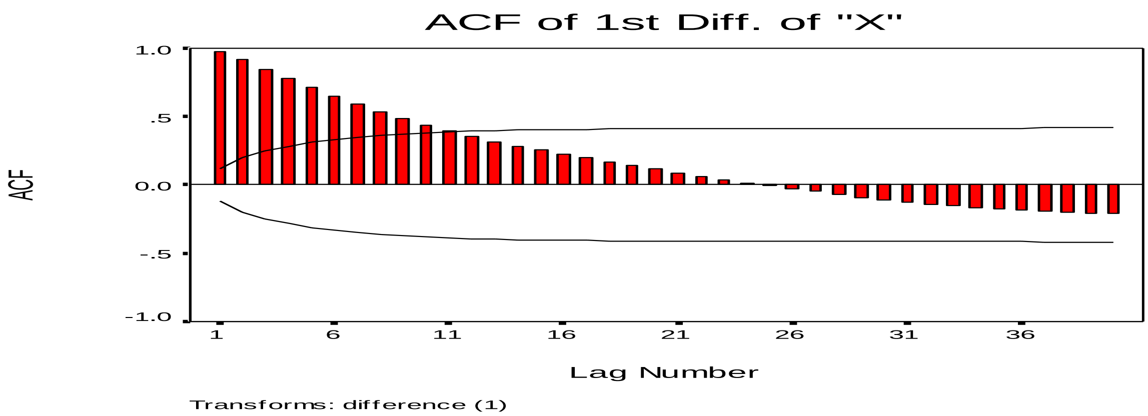
APPENDIX A

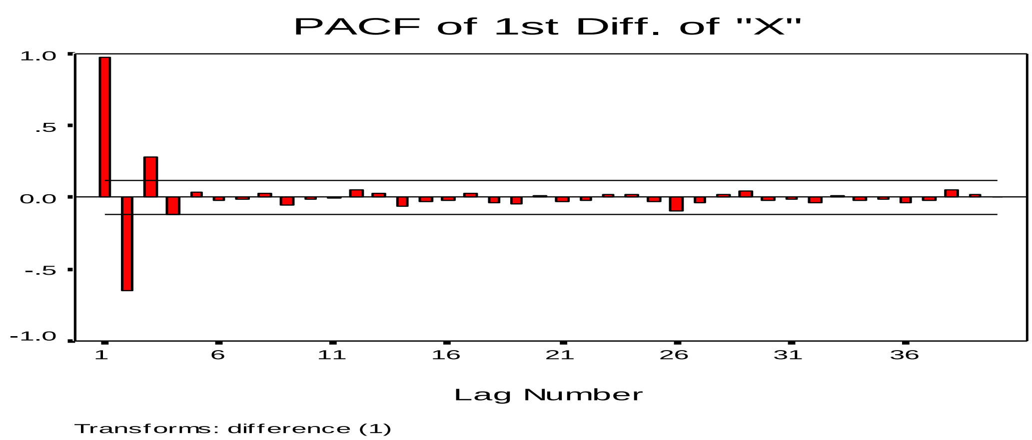
STEP 1






TS Plot of First Difference of "X"
20
10
0
-10
-20
-30
41
81
121
161
201
241
281
Time (Daily)
Transforms: difference (1)


ACF of 1st Diff. of "X"
ACF
1.0
.5
0.0
-.5
-1.0
1
6
11
16
21
26
31
36
Lag Number
Transforms: difference (1)


PACF of 1st Diff. of "X"
1.0
.5
0.0
-.5
-1.0
1
6
11
16
21
26
31
36
Lag Number
Transforms: difference (1)

STEP 2

Dependent Variable: 1^{st} Diff. of X
Method: Least Squares
Included observations: 298 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.961442	0.015541	61.86407	0.0000
MA(1)	0.814071	0.033924	23.99722	0.0000

R-squared	0.977806	Mean dependent var	-0.649042
Adjusted R-squared	0.977731	S.D. dependent var	7.629079
S.E. of regression	1.138462	Akaike info criterion	3.103922
Log likelihood	-460.4844	Schwarz criterion	3.128735

Residuals Analysis of ARMA(1,1)

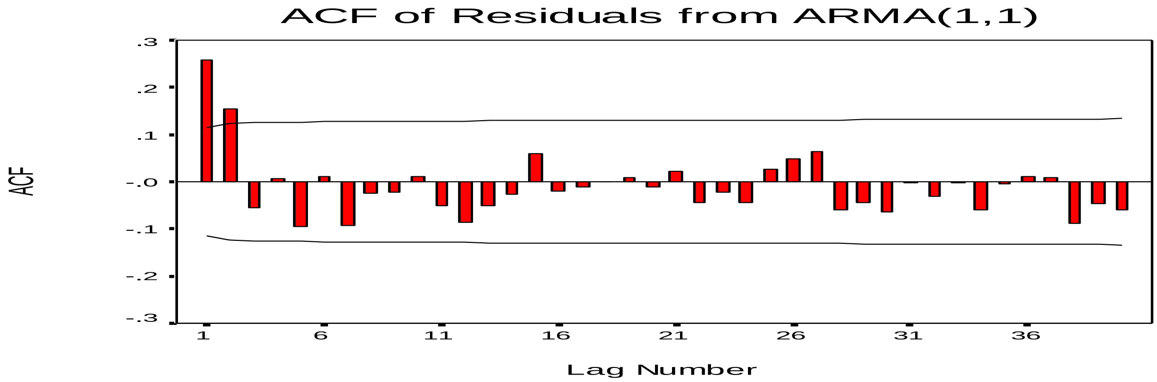
	Residuals				Residuals-squared			
Lag	ACF	PACF	Q-Stat	Prob	ACF	PACF	Q-Stat	Prob
1	0.288	0.288	25.04		0.220	0.220	14.60	
2	0.196	0.124	36.70		0.255	0.217	34.23	
3	-0.070	-0.172	38.20	0.000	0.160	0.075	41.97	0.000
4	0.037	0.084	38.63	0.000	0.130	0.040	47.09	0.000
5	-0.077	-0.073	40.46	0.000	0.105	0.030	50.42	0.000
6	0.026	0.036	40.67	0.000	0.002	-0.072	50.42	0.000
9	-0.013	0.030	42.76	0.000	0.020	0.028	50.58	0.000
12	-0.078	-0.075	45.31	0.000	0.010	-0.005	50.80	0.000
18	-0.018	0.018	48.49	0.000	-0.043	-0.028	52.99	0.000
24	-0.086	-0.078	51.93	0.000	0.213	0.131	87.38	0.000
36	0.036	0.005	56.81	0.008	-0.019	-0.027	97.36	0.000

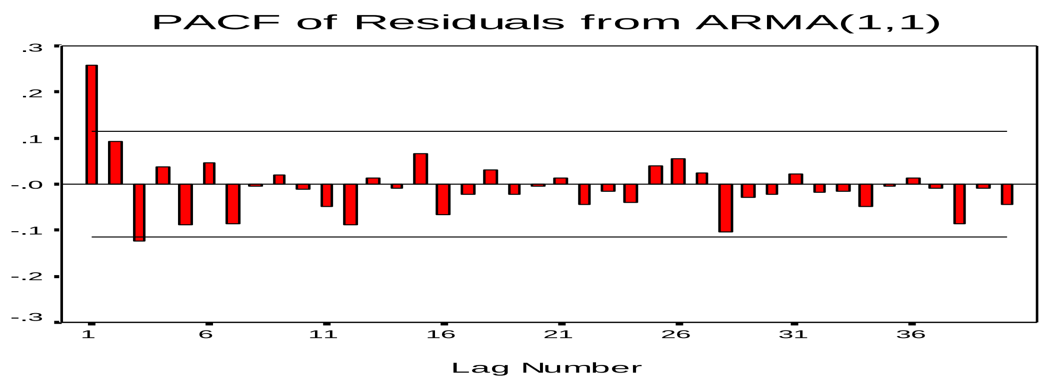
ARCH-LM Test: Lag 1

Obs*R-squared	14.40693	Probability	0.000147

ARCH-LM Test: Lag 12

Obs*R-squared	30.76511	Probability	0.002139





STEP 3a

Dependent Variable: 1st Diff of X
Method: Least Squares
Included observations: 297 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	1.399340	0.061336	22.81446	0.0000
AR(2)	-0.447636	0.061078	-7.328954	0.0000
MA(1)	0.653330	0.052015	12.56051	0.0000
Schwarz criterion	2.991391	Akaike info criterion		2.954081

STEP 3b

Dependent Variable: 1st Diff of X
Method: Least Squares
Included observations: 298 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.939440	0.020006	46.95776	0.0000
MA(1)	1.103652	0.053847	20.49604	0.0000
MA(2)	0.375905	0.053631	7.009100	0.0000
Schwarz criterion	2.997140	Akaike info criterion		2.959921

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STEP 3c

Residuals Analysis of ARMA(2,1)

	Residuals				Residuals-squared			
Lag	ACF	PACF	Q-Stat	Prob	ACF	PACF	Q-Stat	Prob
3	-0.039	-0.038	0.68		0.224	0.142	59.70	
4	0.020	0.021	0.81	0.369	0.272	0.173	82.15	0.000
5	-0.003	-0.005	0.81	0.668	0.103	-0.064	85.34	0.000
6	0.056	0.055	1.75	0.626	0.013	-0.089	85.39	0.000
9	0.045	0.048	2.82	0.831	-0.037	-0.036	86.57	0.000
12	-0.047	-0.048	3.66	0.932	-0.023	-0.016	87.16	0.000
24	-0.092	-0.099	11.37	0.955	0.094	0.004	115.58	0.000

ARCH-LM Test: Lag 1

Obs*R-squared	30.85022	Probability	0.000000

STEP 4a

Dependent Variable: 1st Diff of X
Method: ML - ARCH (Marquardt) - Normal distribution
Included observations: 297 after adjustments

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	1.398897	0.072270	19.35645	0.0000
AR(2)	-0.445571	0.070304	-6.337780	0.0000
MA(1)	0.673700	0.053530	12.58550	0.0000
Variance Equation				
C	0.205166	0.088291	2.323746	0.0201
RESID(-1)^2	0.367633	0.114142	3.220835	0.0013
GARCH(-1)	0.462123	0.135647	3.406815	0.0007
Schwarz criterion	2.860635	Akaike info criterion		2.786014
Inverted AR Roots	.91	.49		
Inverted MA Roots	-.67			

Residuals Analysis of ARMA(2,1)-GARCH(1,1)

	Residuals				Residuals-squared			
Lag	ACF	PACF	Q-Stat	Prob	ACF	PACF	Q-Stat	Prob
3	-0.011	-0.010	0.24		0.087	0.083	3.86	
4	-0.078	-0.078	2.07	0.150	0.046	0.047	4.49	0.034
5	-0.019	-0.018	2.17	0.337	0.098	0.113	7.39	0.025
6	0.077	0.073	3.97	0.265	-0.024	-0.019	7.56	0.056
9	0.071	0.070	5.75	0.452	-0.026	-0.041	9.37	0.154
12	-0.057	-0.064	7.40	0.596	-0.037	-0.021	10.05	0.346
24	-0.007	-0.021	13.42	0.893	-0.036	-0.044	19.54	0.550

ARCH-LM Test: Lag 1

Obs*R-squared	0.255513	Probability	0.613220

STEP 4b

Dependent Variable: DIFFX
Method: ML - ARCH (Marquardt) - Normal distribution
Included observations: 298 after adjustments

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.950597	0.015328	62.01707	0.0000
MA(1)	0.804527	0.038826	20.72122	0.0000
		Variance Equation		
C	0.223037	0.101183	2.204296	0.0275
RESID(-1)^2	0.363746	0.094105	3.865304	0.0001
GARCH(-1)	0.475396	0.132173	3.596763	0.0003
Schwarz criterion	2.979936	Schwarz criterion		2.979936

Residuals Analysis of ARMA(1,1)-GARCH(1,1)

	Residuals				Residuals-squared			
Lag	AC	PAC	Q-Stat	Prob	AC	PAC	Q-Stat	Prob
2	0.134	0.071	25.92		-0.065	-0.066	1.41	
3	-0.065	-0.125	27.20	0.000	0.057	0.061	2.41	0.121
4	-0.044	-0.010	27.79	0.000	0.062	0.055	3.57	0.168
5	-0.070	-0.036	29.28	0.000	0.056	0.061	4.52	0.211
6	0.050	0.081	30.05	0.000	-0.007	-0.006	4.53	0.339
12	-0.086	-0.093	33.05	0.000	-0.024	0.003	10.58	0.391
24	0.015	0.000	39.75	0.012	0.033	0.013	18.97	0.647

STEP 5

Dependent Variable: DIFFX
Included observations: 297 after adjustments
LOG(GARCH) = C(4) + C(5)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(6)*RESID(-1)/@SQRT(GARCH(-1)) + C(7)*LOG(GARCH(-1))

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	1.384881	0.071028	19.49768	0.0000
AR(2)	-0.432861	0.067947	-6.370618	0.0000
MA(1)	0.660503	0.056161	11.76079	0.0000
		Variance Equation		
C(4)	-0.495711	0.133786	-3.705263	0.0002

C(5)	0.591069	0.158697	3.724518	0.0002
C(6)	-0.022590	0.081752	-0.276317	0.7823
C(7)	0.804151	0.091178	8.819528	0.0000
Schwarz criterion	2.869514	Mean dependent var		-0.618215

APPENDIX/LAMPIRAN B

Table 1: Series $\{S_t\}$, mean = 7.497, std deviation = 9.447

lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ACF	0.436	-0.047	0.005	-0.044	0.158	0.384	0.131	-0.074	-0.019	-0.067
PACF	0.436	-0.293	0.216	-0.227	0.457	0.007	-0.028	-0.048	0.044	-0.122
lag	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ACF	0.398	0.926	0.375	-0.085	-0.024	-0.062	0.145	0.358	0.091	-0.112
PACF	0.825	0.582	-0.576	0.066	-0.230	-0.002	-0.034	0.059	0.033	-0.016
lag	**21**	**22**	**23**	**24**	**25**	**26**	**28**	**30**	**32**	**34**
ACF	-0.051	-0.085	0.353	0.839	0.313	-0.119	-0.071	0.340	-0.137	-0.092
PACF	-0.009	0.084	-0.253	0.047	0.096	-0.027	0.088	-0.012	0.041	-0.006
lag	**35**	**36**	**37**	**38**	**40**	**42**	**44**	**46**	**47**	**48**
ACF	0.308	0.752	0.261	-0.143	-0.067	0.325	-0.152	-0.091	0.266	0.670
PACF	-0.074	0.027	0.065	-0.003	0.032	-0.021	-0.009	-0.005	-0.032	0.042

Table 2: Series $\{\nabla_{12} S_t\}$, mean = 0.758, std deviation = 1.575

lag	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
ACF	0.450	-0.029	-0.018	-0.080	-0.105	-0.087	-0.024	0.012	-0.044	-0.050
PACF	0.450	-0.291	0.178	-0.227	0.077	-0.137	0.110	-0.086	-0.016	-0.034
lag	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
ACF	0.297	0.511	0.128	-0.005	0.060	-0.072	-0.118	-0.051	0.007	-0.052
PACF	0.492	0.108	-0.195	0.219	-0.072	0.032	-0.067	0.096	-0.104	-0.065
lag	**21**	**22**	**23**	**24**	**25**	**26**	**28**	**30**	**32**	**34**
ACF	-0.160	-0.060	0.222	0.195	-0.078	-0.035	-0.121	-0.040	-0.044	-0.023
PACF	-0.084	0.097	-0.049	-0.119	-0.048	-0.032	-0.047	0.030	0.032	0.038
lag	**35**	**36**	**37**	**38**	**40**	**42**	**44**	**46**	**47**	**48**
ACF	0.122	-0.031	-0.174	-0.024	-0.098	-0.052	-0.111	0.105	0.136	-0.051
PACF	-0.048	-0.053	-0.058	0.069	0.029	-0.120	0.005	0.057	-0.020	0.066

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