UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSG 284 – Introduction to Geometric Modelling
[Pengenalan kepada Pemodelan Geometri]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Let $\boldsymbol{C}_0(1,\ 0)$, $\boldsymbol{C}_1(1,\ 1)$, $\boldsymbol{C}_2(2,\ 0)$ and $\boldsymbol{C}_3(2,\ 1)$ be the control points of a cubic Bézier curve $\boldsymbol{P}(t)$, $t \in [0,\ 1]$. If $\boldsymbol{P}(t)$ is degree raised by one to

$$\boldsymbol{P}(t) = \boldsymbol{B}_0(1-t)^4 + 4\boldsymbol{B}_1(1-t)^3 t + 6\boldsymbol{B}_2(1-t)^2 t^2 + 4\boldsymbol{B}_3(1-t)t^3 + \boldsymbol{B}_4 t^4,$$

calculate the points $\boldsymbol{B}_i \in \mathbb{R}^2$, $i = 0,\ 1,\ \ldots,\ 4$.

(b) Let $\boldsymbol{C}_0(1,\ 1)$, $\boldsymbol{C}_1(1,\ 3)$, $\boldsymbol{C}_2(3,\ 3)$, $\boldsymbol{C}_3(3,\ 1)$ be the control points of a cubic Bézier curve $\boldsymbol{P}(t)$, $t \in [0,\ 1]$.

(i) Use the de Casteljau algorithm to show geometrically the positions of the points $\boldsymbol{P}(0.3)$ and $\boldsymbol{P}(0.5)$ in two separate figures.

(ii) Write down the sets of control points defining the curve segments obtained by subdividing $\boldsymbol{P}(t)$ at $t = 0.5$.

(c) A cubic Bézier curve $\boldsymbol{P}(t)$, $t \in [0,\ 1]$, has control points $\boldsymbol{C}_0(3,\ 1)$, $\boldsymbol{C}_1(4,\ 2)$, $\boldsymbol{C}_2(5,\ 4)$ and $\boldsymbol{C}_3(6,\ 2)$. Determine the parameter values that give the maximum and minimum values of y-component of $\boldsymbol{P}(t)$.

(d) Given four ordered points $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$, and let

$$\boldsymbol{P}_1(t) = (1-t)\boldsymbol{A} + t\boldsymbol{B}, \quad \boldsymbol{P}_2(t) = (1-t)\boldsymbol{B} + t\boldsymbol{C} \quad \text{and} \quad \boldsymbol{P}_3(t) = (1-t)\boldsymbol{C} + t\boldsymbol{D}$$

where $t \in [0,\ 1]$. If a cubic Bézier curve $\boldsymbol{P}(t)$ is defined by

$$\boldsymbol{P}(t) = \boldsymbol{A}(1-t)^3 + 3\boldsymbol{B}(1-t)^2 t + 3\boldsymbol{C}(1-t)t^2 + \boldsymbol{D}t^3,$$

show that $\boldsymbol{P}(t) = \alpha_1 \boldsymbol{P}_1(t) + \alpha_2 \boldsymbol{P}_2(t) + \alpha_3 \boldsymbol{P}_3(t)$, where $\alpha_i \geq 0$, for $i = 1,\ 2,\ 3$, and $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

[100 marks]

1. (a) *Katakan* $\boldsymbol{C}_0(1,\ 0)$, $\boldsymbol{C}_1(1,\ 1)$, $\boldsymbol{C}_2(2,\ 0)$ *dan* $\boldsymbol{C}_3(2,\ 1)$ *ialah titik-titik kawalan bagi suatu lengkung Bézier kubik* $\boldsymbol{P}(t)$, $t \in [0,\ 1]$. *Jika* $\boldsymbol{P}(t)$ *ditingkatkan darjahnya dengan satu kepada*

$$\boldsymbol{P}(t) = \boldsymbol{B}_0(1-t)^4 + 4\boldsymbol{B}_1(1-t)^3 t + 6\boldsymbol{B}_2(1-t)^2 t^2 + 4\boldsymbol{B}_3(1-t)t^3 + \boldsymbol{B}_4 t^4,$$

nilaikan titik-titik $\boldsymbol{B}_i \in \mathbb{R}^2$, $i = 0,\ 1,\ \ldots,\ 4$.

(b) *Katakan* $\boldsymbol{C}_0(1,\ 1)$, $\boldsymbol{C}_1(1,\ 3)$, $\boldsymbol{C}_2(3,\ 3)$, $\boldsymbol{C}_3(3,\ 1)$ *ialah titik-titik kawalan bagi suatu lengkung Bézier kubik* $\boldsymbol{P}(t)$, $t \in [0,\ 1]$.

(i) *Gunakan algoritma de Casteljau untuk menunjukkan kedudukan titik-titik* $\boldsymbol{P}(0.3)$ *dan* $\boldsymbol{P}(0.5)$ *di dua gambarajah yang berasingan.*

(ii) *Tuliskan set titik kawalan bagi segmen-segmen lengkung yang diperolehi dengan sub-bahagi* $\boldsymbol{P}(t)$ *pada* $t = 0.5$.

(c) *Satu lengkung Bézier kubik* $\boldsymbol{P}(t)$, $t \in [0,\ 1]$, *mempunyai titik-titik kawalan* $\boldsymbol{C}_0(3,\ 1)$, $\boldsymbol{C}_1(4,\ 2)$, $\boldsymbol{C}_2(5,\ 4)$ *dan* $\boldsymbol{C}_3(6,\ 2)$. *Tentukan nilai-nilai parameter yang memberi nilai maksimum dan nilai minimum untuk komponen* y *bagi* $\boldsymbol{P}(t)$.

(d) *Diberi empat titik teratur* $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ *dan* $\boldsymbol{D}$, *katakan*

$$\boldsymbol{P}_1(t) = (1-t)\boldsymbol{A} + t\boldsymbol{B}, \quad \boldsymbol{P}_2(t) = (1-t)\boldsymbol{B} + t\boldsymbol{C} \quad \text{dan} \quad \boldsymbol{P}_3(t) = (1-t)\boldsymbol{C} + t\boldsymbol{D}$$

di mana $t \in [0,\ 1]$. *Jika suatu lengkung Bézier kubik* $\boldsymbol{P}(t)$ *ditakrif sebagai*

$$\boldsymbol{P}(t) = \boldsymbol{A}(1-t)^3 + 3\boldsymbol{B}(1-t)^2 t + 3\boldsymbol{C}(1-t)t^2 + \boldsymbol{D}t^3,$$

tunjukkan bahawa $\boldsymbol{P}(t) = \alpha_1 \boldsymbol{P}_1(t) + \alpha_2 \boldsymbol{P}_2(t) + \alpha_3 \boldsymbol{P}_3(t)$, *di mana* $\alpha_i \geq 0$, *untuk* $i = 1,\ 2,\ 3$, *dan* $\alpha_1 + \alpha_2 + \alpha_3 = 1$.

[100 markah]

2. Let $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ be a knot vector where n and k are positive integers. The normalized B-spline basis functions of order k are defined recursively by

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

and

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad k > 1.$$

(a) Suppose $\boldsymbol{u} = (0, 0, 0, 1, 2, 2, 2)$. Obtain the B-spline basis functions of order 3 defined on this knot vector.

(b) Suppose $u_0 < u_1 < \ldots < u_{n+k}$. Prove that $N_i^k(u) > 0$ for $u \in (u_i, u_{i+k})$.

(c) Let $\boldsymbol{u} = (0, 1, 2, 3, 4, 5)$. A B-spline curve of order 3 is defined by

$$\boldsymbol{P}(u) = \boldsymbol{D}_0 N_0^3(u) + \boldsymbol{D}_1 N_1^3(u) + \boldsymbol{D}_2 N_2^3(u), \qquad u \in [2, 3],$$

where $\boldsymbol{D}_0, \boldsymbol{D}_1, \boldsymbol{D}_2 \in \mathbb{R}^2$ are the de Boor points.

(i) Show that the matrix form of $\boldsymbol{P}(u)$ can be expressed locally as

$$\frac{1}{2}\begin{bmatrix} t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{D}_0 \\ \boldsymbol{D}_1 \\ \boldsymbol{D}_2 \end{bmatrix},$$

where $t \in [0, 1]$.

(ii) Prove that $\boldsymbol{P}(u)$ lies within the convex hull of the de Boor polygon $\boldsymbol{D}_0 \boldsymbol{D}_1 \boldsymbol{D}_2$.

[100 marks]

2. *Katakan* $\boldsymbol{u} = (u_0,\ u_1,\ \ldots,\ u_{n+k})$ *ialah suatu vektor knot di mana* n *dan* k *adalah nombor integer positif. Fungsi asas splin-B ternormal berperingkat* k *ditakrif secara rekursi sebagai*

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & di\ tempat\ lain \end{cases}$$

dan

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad k > 1.$$

(a) *Andaikan* $\boldsymbol{u} = (0,\ 0,\ 0,\ 1,\ 2,\ 2,\ 2)$. *Dapatkan fungsi-fungsi asas splin-B berperingkat* 3 *yang ditakrif pada vektor knot ini.*

(b) *Andaikan* $u_0 < u_1 < \ldots < u_{n+k}$. *Buktikan bahawa* $N_i^k(u) > 0$ *untuk* $u \in (u_i,\ u_{i+k})$.

(c) *Katakan* $\boldsymbol{u} = (0,\ 1,\ 2,\ 3,\ 4,\ 5)$. *Suatu lengkung splin-B berperingkat* 3 *ditakrif sebagai*

$$\boldsymbol{P}(u) = \boldsymbol{D}_0 N_0^3(u) + \boldsymbol{D}_1 N_1^3(u) + \boldsymbol{D}_2 N_2^3(u), \qquad u \in [2, 3],$$

di mana $\boldsymbol{D}_0,\ \boldsymbol{D}_1,\ \boldsymbol{D}_2 \in \mathbb{R}^2$ *ialah titik-titik de Boor.*

(i) *Tunjukkan bahawa perwakilan matriks bagi* $\boldsymbol{P}(u)$ *boleh diungkapkan secara setempat sebagai*

$$\frac{1}{2}\begin{bmatrix} t^2 & t & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{D}_0 \\ \boldsymbol{D}_1 \\ \boldsymbol{D}_2 \end{bmatrix},$$

di mana $t \in [0, 1]$.

(ii) *Buktikan bahawa* $\boldsymbol{P}(u)$ *terletak di dalam hul cembung bagi poligon de Boor* $\boldsymbol{D}_0 \boldsymbol{D}_1 \boldsymbol{D}_2$.

[100 markah]

3. (a) A biquadratic Bézier surface is defined by

$$\boldsymbol{S}(u,v) = \sum_{i=0}^{2}\sum_{j=0}^{2} \boldsymbol{C}_{i,j}\, B_j^2(v)\, B_i^2(u), \qquad 0 \le u, v \le 1,$$

where

$$\boldsymbol{C}_{0,0} = (1,\ 1,\ 1), \quad \boldsymbol{C}_{0,1} = (2,\ 1,\ 2), \quad \boldsymbol{C}_{0,2} = (3,\ 1,\ 1),$$
$$\boldsymbol{C}_{1,0} = (1,\ 2,\ 3), \quad \boldsymbol{C}_{1,1} = (a,\ b,\ c), \quad \boldsymbol{C}_{1,2} = (3,\ 2,\ 3),$$
$$\boldsymbol{C}_{2,0} = (1,\ 3,\ 1), \quad \boldsymbol{C}_{2,1} = (2,\ 3,\ 2), \quad \boldsymbol{C}_{2,2} = (3,\ 3,\ 1),$$

and a, b, $c \in \square$.

(i) Evaluate a, b and c such that $\boldsymbol{S}(0.5,\ 0.5) = (2,\ 2,\ 2)$.

(ii) Determine the unit normal vector to surface $\boldsymbol{S}$ at $(u,\ v) = (0,\ 0)$.

(iii) Determine the principal normal vector to boundary curve $\boldsymbol{S}(0,\ v)$ at $v = 0$.

(b) Given a bilinearly blended Coons patch $\boldsymbol{F}(u,\ v)$, $0 \le u, v \le 1$, which its four boundary curves are polynomials and has the following properties:

$$\boldsymbol{F}(0,\ 0) = (1,\ 1,\ 0), \quad \boldsymbol{F}_u(0,\ 0) = (1,\ 0,\ 1), \quad \boldsymbol{F}_v(0,\ 0) = (0,\ 1,\ -1),$$
$$\boldsymbol{F}(1,\ 0) = (7,\ 2,\ 1), \quad \boldsymbol{F}_u(1,\ 0) = (1,\ 1,\ -1), \quad \boldsymbol{F}_v(1,\ 0) = (-1,\ 1,\ 1),$$
$$\boldsymbol{F}(0,\ 1) = (1,\ 5,\ 1), \quad \boldsymbol{F}_u(0,\ 1) = (1,\ 0,\ 1), \quad \boldsymbol{F}_v(0,\ 1) = (0,\ 1,\ 1),$$
$$\boldsymbol{F}(1,\ 1) = (5,\ 5,\ 2), \quad \boldsymbol{F}_u(1,\ 1) = (1,\ 0,\ -1), \quad \boldsymbol{F}_v(1,\ 1) = (1,\ -1,\ 0).$$

$\boldsymbol{F}_u$ and $\boldsymbol{F}_v$ indicate the partial derivatives $\boldsymbol{F}$ with respect to u and v respectively. Evaluate the patch point $\boldsymbol{F}$ at $(u,\ v) = (0.5,\ 0.5)$.

[100 marks]

3. (a) *Satu permukaan Bézier bikuadratik ditakrif oleh*

$$\boldsymbol{S}(u,v)=\sum_{i=0}^{2}\sum_{j=0}^{2}\boldsymbol{C}_{i,j}\,B_j^2(v)\,B_i^2(u), \qquad 0\le u,v\le 1,$$

di mana

$\boldsymbol{C}_{0,0}=(1,\ 1,\ 1)$, $\boldsymbol{C}_{0,1}=(2,\ 1,\ 2)$, $\boldsymbol{C}_{0,2}=(3,\ 1,\ 1)$,
$\boldsymbol{C}_{1,0}=(1,\ 2,\ 3)$, $\boldsymbol{C}_{1,1}=(a,\ b,\ c)$, $\boldsymbol{C}_{1,2}=(3,\ 2,\ 3)$,
$\boldsymbol{C}_{2,0}=(1,\ 3,\ 1)$, $\boldsymbol{C}_{2,1}=(2,\ 3,\ 2)$, $\boldsymbol{C}_{2,2}=(3,\ 3,\ 1)$,

dan a, b, $c\in\mathbb{R}$.

(i) *Nilaikan* a, b *dan* c *supaya* $\boldsymbol{S}(0.5,\ 0.5)=(2,\ 2,\ 2)$.

(ii) *Tentukan vektor normal unit kepada* $\boldsymbol{S}$ *pada* $(u,\ v)=(0,\ 0)$.

(iii) *Tentukan vektor normal prinsipal kepada lengkung sempadan* $\boldsymbol{S}(0,\ v)$ *pada* $v=0$.

(b) *Diberi satu tampalan Coons teraduan bilinear* $\boldsymbol{F}(u,\ v)$, $0\le u,v\le 1$, *yang empat lengkung sempadannya adalah polinomial dan mempunyai ciri-ciri berikut:*

$\boldsymbol{F}(0,\ 0)=(1,\ 1,\ 0)$, $\boldsymbol{F}_u(0,\ 0)=(1,\ 0,\ 1)$, $\boldsymbol{F}_v(0,\ 0)=(0,\ 1,\ -1)$,
$\boldsymbol{F}(1,\ 0)=(7,\ 2,\ 1)$, $\boldsymbol{F}_u(1,\ 0)=(1,\ 1,\ -1)$, $\boldsymbol{F}_v(1,\ 0)=(-1,\ 1,\ 1)$,
$\boldsymbol{F}(0,\ 1)=(1,\ 5,\ 1)$, $\boldsymbol{F}_u(0,\ 1)=(1,\ 0,\ 1)$, $\boldsymbol{F}_v(0,\ 1)=(0,\ 1,\ 1)$,
$\boldsymbol{F}(1,\ 1)=(5,\ 5,\ 2)$, $\boldsymbol{F}_u(1,\ 1)=(1,\ 0,\ -1)$, $\boldsymbol{F}_v(1,\ 1)=(1,\ -1,\ 0)$.

$\boldsymbol{F}_u$ *dan* $\boldsymbol{F}_v$ *menandakan terbitan separa* $\boldsymbol{F}$ *terhadap* u *dan* v *masing-masing. Nilaikan titik tampalan* $\boldsymbol{F}$ *pada* $(u,\ v)=(0.5,\ 0.5)$.

[100 markah]

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