UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSG 265 – Design and Analysis of Experiments
[Rekabentuk dan Analisis Uji Kaji]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIXTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. The quality control department of a fabric finishing plant is studying the effect of several factors on the dyeing of cotton-synthetic cloth used to manufacture men's shirts. Three operators, three cycle times and two temperatures were selected, and three small specimens of cloth were dyed under each set of conditions. The finished cloth was compared to a standard, and a numerical score was assigned. The score data were given in the table below:

Cycle Time (A)	Temperature (B)					
	300			350		
	Operator (C)			Operator (C)		
	1	2	3	1	2	3
40	23	30	31	24	38	34
	24	28	32	23	36	36
	25	26	29	28	35	39
50	36	34	33	37	34	34
	35	38	34	39	38	36
	36	39	35	35	36	31
60	28	35	26	26	36	28
	24	35	27	29	37	26
	27	34	25	25	34	24

(i) Given that $\sum\sum\sum\sum y_{ijkl}^2 = 55299$ and $y_{....} = 1707$. Complete the following ANOVA table.

Source of variation	Sum of Squares	Degrees of freedom	Mean Square	F_0
A	426.333			
B				
C	279.000			
AB				
AC	349.667			
BC				
ABC				
Error				
Total				

(ii) State the linear statistical model for this experiment. Based on the ANOVA table in part (i), analyze the data and state the conclusion. Use $\alpha = 0.01$.

(iii) Analyze the residuals for the model obtained in part (ii). (Refer Appendix)

(iv) Obtain the point estimates of the mean score at each cycle times and temperature levels.

1. *Bahagian kawalan kualiti bagi penghasilan fabrik sedang mengkaji kesan beberapa faktor ke atas proses pewarnaan baju kapas sintetik yang digunakan dalam penghasilan kemeja lelaki. Tiga operator, tiga masa kitaran dan dua suhu dipilih, dan tiga spesimen kecil kain diwarnakan dibawah setiap keadaan. Kain yang telah siap diwarnakan dibanding dengan tahap piawai yang ditetapkan dan skornya didapatkan. Skor tersebut diberikan dalam jadual di bawah:*

Masa Kitaran (A)	*Suhu (B)*					
	300			*350*		
	Operator (C)			*Operator (C)*		
	1	*2*	*3*	*1*	*2*	*3*
40	*23*	*30*	*31*	*24*	*38*	*34*
	24	*28*	*32*	*23*	*36*	*36*
	25	*26*	*29*	*28*	*35*	*39*
50	*36*	*34*	*33*	*37*	*34*	*34*
	35	*38*	*34*	*39*	*38*	*36*
	36	*39*	*35*	*35*	*36*	*31*
60	*28*	*35*	*26*	*26*	*36*	*28*
	24	*35*	*27*	*29*	*37*	*26*
	27	*34*	*25*	*25*	*34*	*24*

(i) Diberikan bahawa $\sum\sum\sum\sum y_{ijkl}^2 = 55299$ *and* $y_{....} = 1707$ *. Lengkapkan jadual ANOVA berikut:*

Punca Ubahan	*Hasil Tambah Kuasa Dua*	*Darjah Kebebasan*	*Min Kuasa Dua*	F_0
A	*426.333*			
B				
C	*279.000*			
AB				
AC	*349.667*			
BC				
ABC				
Ralat				
Jumlah				

(ii) Nyatakan model linear statistik bagi uji kaji ini. Berdasarkan jadual ANOVA di bahagian (i), jalankan analisis data dan nyatakan kesimpulan anda. Guna $\alpha = 0.01$.

(iii) Jalankan analisis bagi reja bagi model yang diperoleh dalam bahagian (i). (Rujuk lampiran)

(iv) Dapatkan anggaran titik bagi min skor pada setiap masa kitaran dan aras suhu.

(v) Use Tukey's test to determine which levels of cycle time are significantly different.

(vi) Assume that the replicates are used as blocking variables. Present the ANOVA table for the full model of the three-factor factorial design.

[100 marks]

2. (a) The yield of corn planting is being studied. The type of seed and type of fertilizer are the two factors that will be considered in this study. Three levels of seeds type and five levels of fertilizers type are randomly selected and two replicates of the factorial experiment is conducted. The sums of squares of the data are shown in the following table.

Source of variation	Sum of squares
Type of seed (A)	512.867
Type of fertilizer (B)	449.467
Interaction AB	143.133
Error	136.000
Total	1241.467

(i) Analyze the data fom this experiment. State the conclusion. Use $\alpha = 0.05$.

(ii) Estimate the variance components.

(iii) Reanalyze the yield of corn planting experiment, assuming that type of fertilizers are fixed factor. Use the restricted model to estimate the appropriate parameters and model components.

[60 marks]

(b) State a general model and give the analysis of variance table for the three-stage nested design. Assume that factor A and B fixed and factor C random.

[40 marks]

(v) *Gunakan ujian Tukey's bagi menentukan tahap masa kitaran yang berbeza secara bererti.*

(vi) *Andaikan bahawa replika digunakan sebagai pemboleh ubah blok. Berikan jadual ANOVA bagi model penuh untuk reka bentuk faktorial tiga faktor.*

[100 markah]

2. (a) *Hasil bagi penanaman jagung dikaji. Jenis benih dan baja yang digunakan merupakan dua faktor yang dipertimbangkan dalam kajian ini. Tiga aras bagi jenis benih dan lima aras bagi jenis baja dipilih secara rawak dan uji kaji faktorial dengan dua replika dilakukan. Hasil tambah kuasa dua bagi data yang digunakan ditunjukkan dalam jadual berikut:*

Punca Ubahan	*Hasil Tambah Kuasa Dua*
Jenis benih (A)	*512.867*
Jenis baja (B)	*449.467*
Saling tindak AB	*143.133*
Ralat	*136.000*
Jumlah	*1241.467*

(i) *Jalankan analisis data bagi uji kaji ini. Nyatakan kesimpulan anda. Guna* $\alpha = 0.05$.

(ii) *Dapatkan anggaran titik bagi komponen varians.*

(iii) *Analisis semula uji kaji hasil penanaman jagung di atas dengan mengandaikan jenis baja adalah faktor tetap. Guna model terhad bagi menganggarkan parameter yang bersesuaian dan komponen varians.*

[60 markah]

(b) *Nyatakan model am dan berikan analisis varians bagi reka bentuk tersarang tiga peringkat. Andaikan faktor A dan B adalah tetap dan faktor C adalah rawak.*

[40 markah]

3. An engineer is interested in the effects of cutting speed (A), tool geometry (B) and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 2^3 factorial design are run. The results follow:

			Treatment	Replicate		
A	B	C	Combination	I	II	III
-	-	-	(1)	22	31	25
+	-	-	a	32	43	29
-	+	-	b	35	34	50
+	+	-	ab	55	47	46
-	-	+	c	44	45	38
+	-	+	ac	40	37	36
-	+	+	bc	60	50	54
+	+	+	abc	39	41	47

$SS_T = 2095.33$ $SS_A = 0.67$ $SS_B = 770.67$ $SS_C = 280.17$

$SS_{AB} = 16.67$ $SS_{AC} = 468.17$ $SS_{BC} = 48.17$ $SS_{ABC} = 28.17$

(i) Estimate the factor effects. Which effects appear to be large?

(ii) Use the analysis of variance to confirm your conclusions for part (i).

(iii) Analyze the residuals plots. Are there any obvious problems? (Refer Appendix)

(iv) State a regression model for predicting machine tool life (in hours) based on the results of this experiment.

(v) Suppose that the data above we had confounded *ABC* in replicate I, *AB* in replicate II and *BC* in replicate III. Construct the analysis of variance table.

[100 marks]

4. Heat treating is often used to carbonize metal parts. The thickness of the carbonized layer is a critical output variable from this process, and it is usually measured by performing a carbon analysis on the gear pitch. Six factors were studied in a 2^{6-2}_{IV} design: *A* = furnace temperature, *B* = cycle time, *C* = carbon concentration, *D* = duration of the carbonizing cycle, *E* = carbon concentration of the diffuse cycle and *F* = duration of the diffuse cycle. The experiment is shown in the table below:

3. *Seorang jurutera berminat untuk mengkaji kesan kelajuan pemotong (A), peralatan geometri (B) dan sudut pemotongan (C) ke atas jangka hayat (dalam jam) bagi satu peralatan mesin. Dua aras bagi setiap faktor dipilih, dan tiga replika bagi reka bentuk faktorial* 2^3 *dijalankan. Hasil yang diperoleh adalah seperti berikut:*

			Gabungan	Replika		
A	B	C	Rawatan	I	II	III
-	-	-	(1)	22	31	25
+	-	-	a	32	43	29
-	+	-	b	35	34	50
+	+	-	ab	55	47	46
-	-	+	c	44	45	38
+	-	+	ac	40	37	36
-	+	+	bc	60	50	54
+	+	+	abc	39	41	47

$SS_T = 2095.33$ $SS_A = 0.67$ $SS_B = 770.67$ $SS_C = 280.17$

$SS_{AB} = 16.67$ $SS_{AC} = 468.17$ $SS_{BC} = 48.17$ $SS_{ABC} = 28.17$

(i) Anggarkan kesan faktor. Yang manakah memberikan kesan yang besar?

(ii) Gunakan analisis varians bagi mengesahkan dapatan anda dalam bahagian (i).

(iii) Jalankan analisis reja. Adakah terdapat sebarang tanda ketidakcukupan model? (Rujuk lampiran)

(iv) Nyatakan model regresi bagi meramal jangka hayat peralatan mesin (jam) berdasarkan keputusan uji kaji yang didapati.

(v) Pertimbangkan data di atas dengan pembauran ABC dilakukan dalam replika I, AB dalam replika II dan BC dalam replika III. Dapatkan jadual analisis varians.

[100 markah]

4. *Rawatan pemanasan biasanya digunakan dalam pengkarbonan bahagian logam. Ketebalan lapisan karbon merupakan pemboleh ubah sambutan dalam proses ini dan biasanya diukur dengan menjalankan analisis karbon ke atas pemasangan gear.* *Enam faktor dikaji dalam reka bentuk* 2_{IV}^{6-2} *: A = suhu tungku, B = masa kitaran, C = kepekatan karbon, D = masa bagi kitaran karbon, E = kepekatan karbon bagi penyebaran kitaran dan F = tempoh penyebaran kitaran. Eksperimen yang dijalankan ditunjukkan dalam jadual di bawah:*

Standard Order	Run Order	*A*	*B*	*C*	*D*	*E*	*F*	Yield
1	5	-	-	-	-	-	-	74
2	7	+	-	-	-	+	-	190
3	8	-	+	-	-	+	+	133
4	2	+	+	-	-	-	+	127
5	10	-	-	+	-	+	+	115
6	12	+	-	+	-	-	+	101
7	16	-	+	+	-	-	-	54
8	1	+	+	+	-	+	-	144
9	6	-	-	-	+	-	+	119
10	9	+	-	-	+	+	+	188
11	14	-	+	-	+	+	-	135
12	13	+	+	-	+	-	-	170
13	11	-	-	+	+	+	-	126
14	3	+	-	+	+	-	-	175
15	15	-	+	+	+	-	+	126
16	4	+	+	+	+	+	+	193

$A = 50.5 \quad B = -1 \quad C = -13 \quad D = 37 \quad E = 34.5 \quad F = 4.5$
$AB = -4 \quad AC = -2.5 \quad AD = 4 \quad AE = 1 \quad BD = 4.5$
$CD = 14.5 \quad DE = -22 \quad ABD = 0.5 \quad ABF = 6$

(i) What type of design did the experimenter used?

(ii) State the complete defining relation and the aliases for this design. Choose $I = ABCE$ and $I = BCDF$ as the design generators.

(iii) Based on the estimates of the factor effects and its normal probability plot, which factors can be identified as important?

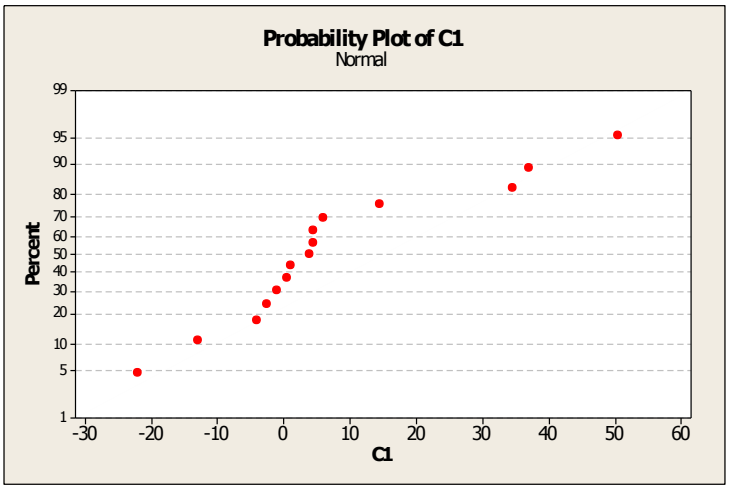
Susunan Piawai	Susunan Larian	*A*	*B*	*C*	*D*	*E*	*F*	Hasil
1	5	-	-	-	-	-	-	74
2	7	+	-	-	-	+	-	190
3	8	-	+	-	-	+	+	133
4	2	+	+	-	-	-	+	127
5	10	-	-	+	-	+	+	115
6	12	+	-	+	-	-	+	101
7	16	-	+	+	-	-	-	54
8	1	+	+	+	-	+	-	144
9	6	-	-	-	+	-	+	119
10	9	+	-	-	+	+	+	188
11	14	-	+	-	+	+	-	135
12	13	+	+	-	+	-	-	170
13	11	-	-	+	+	+	-	126
14	3	+	-	+	+	-	-	175
15	15	-	+	+	+	-	+	126
16	4	+	+	+	+	+	+	193

$A = 50.5 \quad B = -1 \quad C = -13 \quad D = 37 \quad E = 34.5 \quad F = 4.5$
$AB = -4 \quad AC = -2.5 \quad AD = 4 \quad AE = 1 \quad BD = 4.5$
$CD = 14.5 \quad DE = -22 \quad ABD = 0.5 \quad ABF = 6$

(i) Apakan jenis reka bentuk uji kaji yang digunakan?

(ii) Nyatakan hubungan pentakrif yang lengkap dan alias-alias bagi reka bentuk ini. Pilih $I = ABCE$ *dan* $I = BCDF$ *sebagai penjana reka bentuk.*

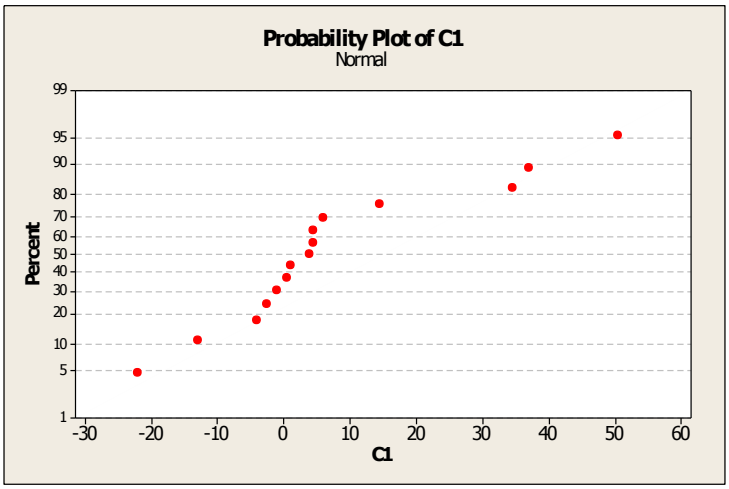
(iii) Berdasarkan nilai penganggar bagi kesan faktor dan plot kebarangkalian normal bagi kesan-kesan tersebut, faktor manakah yang boleh dikenalpasti sebagai faktor yang penting?



(iv) Perform statistical tests to confirm your result in part (iii).

(v) Fit an appropriate regression model based on the result obtained in part (iv).

[100 marks]



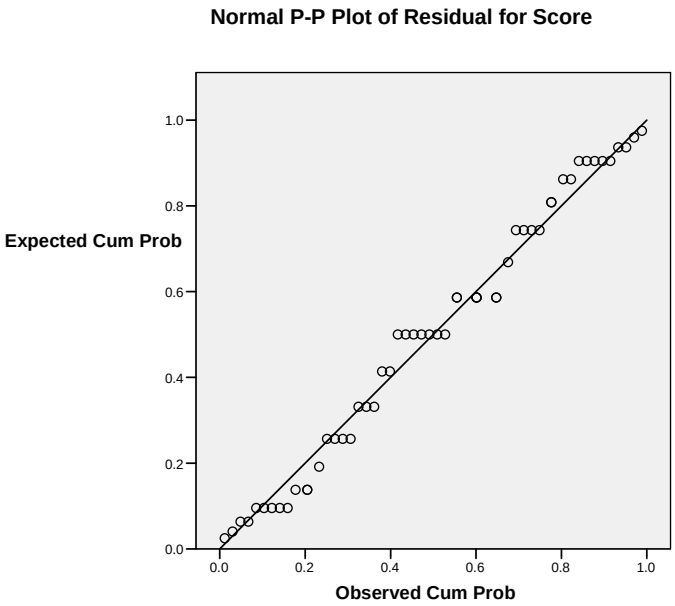
(iv) Jalankan ujian statistik bagi mengesahkan keputusan yang didapati dalam bahagian (iii).

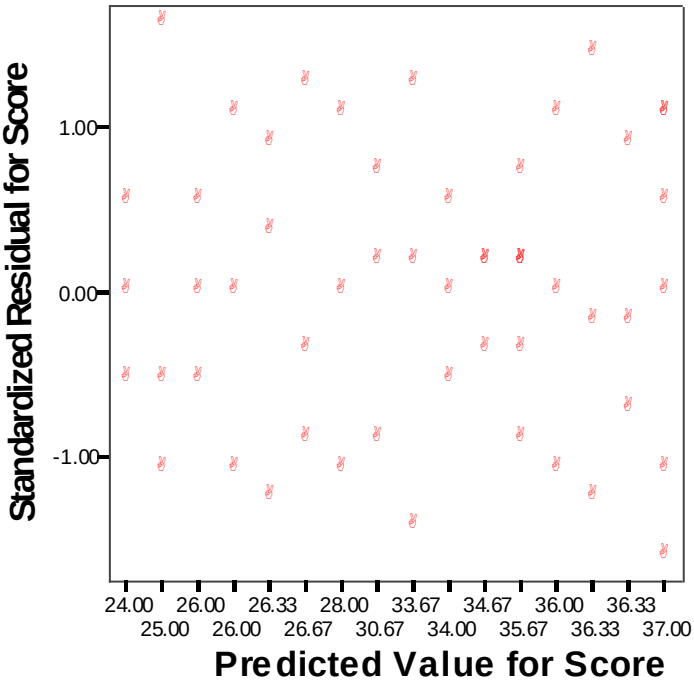
(v) Suaikan model regresi yang bersesuaian dengan keputusan yang didapati dalam bahagian (iv).

[100 markah]

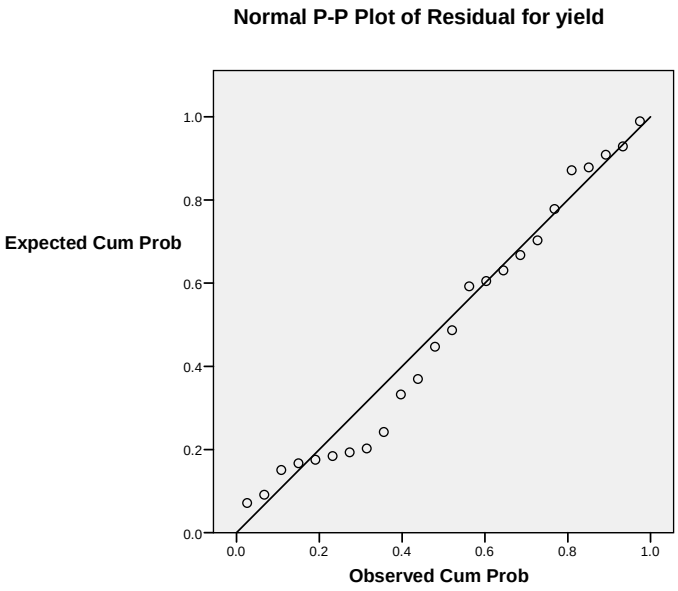
APPENDIX

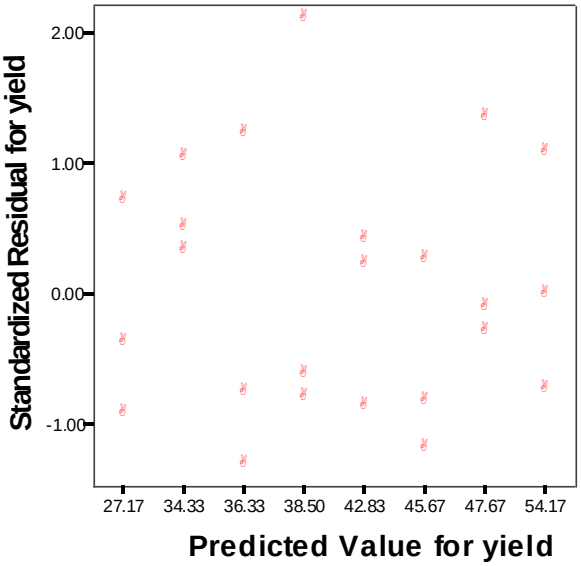
Residual plots for Question 1





Residual plots for Question 3





MSG265/4 – DESIGN AND ANALYSIS OF EXPERIMENTS

1. Two-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_B = \sum_{j=1}^{b} \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$SS_{Subtotal} = \sum_{i=1}^{a}\sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn}$$

$$SS_{AB} = SS_{Subtotal} - SS_A - SS_B$$

$$SS_E = SS_T - SS_{Subtota} \quad \text{atau} \quad SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

Expected Mean Square
Random Effects Model:

$$E[MS_A] = \sigma^2 + n\sigma_{\tau\beta}^2 + bn\sigma_\tau^2$$

$$E[MS_B] = \sigma^2 + n\sigma_{\tau\beta}^2 + an\sigma_\beta^2$$

$$E[MS_{AB}] = \sigma^2 + n\sigma_{\tau\beta}^2$$

$$E[MS_E] = \sigma^2$$

Restricted Mixed Model:
(*A*: fixed, *B*: random)

$$E[MS_A] = \sigma^2 + n\sigma_{\tau\beta}^2 + \frac{bn\sum_{i=1}^{a}\tau_i^2}{a-1}$$

$$E[MS_B] = \sigma^2 + an\sigma_\beta^2$$

$$E[MS_{AB}] = \sigma^2 + n\sigma_{\tau\beta}^2$$

$$E[MS_E] = \sigma^2$$

2. Three-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - \frac{y_{....}^2}{abcn}$$

$$SS_A = \sum_{i=1}^{a}\frac{y_{i...}^2}{bcn} - \frac{y_{....}^2}{abcn}$$

$$SS_B = \sum_{j=1}^{b}\frac{y_{.j..}^2}{acn} - \frac{y_{....}^2}{abcn}$$

$$SS_C = \sum_{k=1}^{c}\frac{y_{..k.}^2}{abn} - \frac{y_{....}^2}{abcn}$$

$$SS_{Subtotal(AB)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y_{ij..}^2}{cn} - \frac{y_{....}^2}{abcn}$$

$$SS_{AB} = SS_{Subtotal(AB)} - SS_A - SS_B$$

$$SS_{Subtotal(AC)} = \sum_{i=1}^{a}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$$SS_{AC} = SS_{Subtotal(AC)} - SS_A - SS_C$$

$$SS_{Subtotal(BC)} = \sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{.jk.}^2}{an} - \frac{y_{....}^2}{abcn}$$

$$SS_{BC} = SS_{Subtotal(BC)} - SS_B - SS_C$$

$$SS_{Subtotal(ABC)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$$SS_E = SS_T - SS_{Subtotal} \quad \text{or} \quad SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

3. Three-stage Nested Design

$$SS_T = \sum_{i}\sum_{j}\sum_{k}\sum_{l}\left(y_{ijkl} - \bar{y}_{....}\right)^2$$

$$SS_A = bcn\sum_{i}\left(\bar{y}_{i...} - \bar{y}_{....}\right)^2$$

$$SS_{B(A)} = cn\sum_{i}\sum_{j}\left(\bar{y}_{ij..} - \bar{y}_{....}\right)^2$$

$$SS_{C(B)} = n\sum_{i}\sum_{j}\sum_{k}\left(\bar{y}_{ijk.} - \bar{y}_{....}\right)^2$$

$$SS_E = SS_T - SS_A - SS_{B(A)} - SS_{C(B)}$$

Expected Mean Square (A and B Fixed, C Random)

$$E[MS_A] = \sigma^2 + n\sigma_\gamma^2 + \frac{bcn\sum_{i=1}^{a}\tau_i^2}{a-1}$$

$$E[MS_{B\ A}] = \sigma^2 + n\sigma_\gamma^2 + \frac{cn\sum\sum\beta_{j\ i}}{a\ b-1}$$

$$E[MS_{C\ B}] = \sigma^2 + n\sigma_\gamma^2$$

$$E[MS_E] = \sigma^2$$

- ooo O ooo -