
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSG 252 – Linear and Integer Programming
[Pengaturcaraan Linear dan Integer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

[Arahan: Jawab **semua sembilan** [9] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Consider the following LP:

$$\begin{aligned} \text{Maximize } z &= 2x_1 + 4x_2 + 4x_3 - 3x_4 \\ \text{Subject to } & x_1 + x_2 + x_3 = 4 \\ & x_1 + 4x_2 + x_4 = 8 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

- (a) Write the dual problem.
 (b) Verify that $\mathbf{B} = (\mathbf{P}_2, \mathbf{P}_3)$ is the optimal basis by computing $\bar{c}_j$ for all nonbasic variables.
 (c) Find the associated optimal dual solution.

[10 marks]

2. Consider the following LP problem.

$$\begin{aligned} \text{Maximize } z &= 20x_1 + 6x_2 + 8x_3 \\ \text{Subject to } & 8x_1 + 2x_2 + 3x_3 \leq 200 \\ & 4x_1 + 3x_2 \leq 100 \\ & 2x_1 + x_3 \leq 50 \\ & x_3 \leq 20 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Let x_4, x_5, x_6 and x_7 denote the slack variables for the first through fourth constraints, respectively. Suppose after some number of iterations of the simplex method, a portion of the current simplex tableau is as follows:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Solution
z				$\frac{9}{4}$	$\frac{1}{2}$	0	0	500
x_1				$\frac{3}{16}$	$-\frac{1}{8}$	0	0	25
x_2				$-\frac{1}{4}$	$\frac{1}{2}$	0	0	0
x_6				$-\frac{3}{8}$	$\frac{1}{4}$	1	0	0
x_7				0	0	0	1	20

From the tableau,

- (a) indicate which of the missing numbers would be generated by the revised simplex in order to perform the next iteration.
 (b) find the set of basic variables and their values in the next iteration.

[10 marks]

1. Pertimbangkan masalah PL berikut:

$$\begin{aligned} \text{Maksimumkan } z &= 2x_1 + 4x_2 + 4x_3 - 3x_4 \\ \text{Terhadap } x_1 + x_2 + x_3 &= 4 \\ x_1 + 4x_2 + x_4 &= 8 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

- (a) Tuliskan masalah dualnya.
- (b) Sahkan bahawa $\mathbf{B} = (\mathbf{P}_2, \mathbf{P}_3)$ ialah asas optimum dengan menghitung $\bar{c}_j$ bagi semua pembolehubah tak asas.
- (c) Dapatkan penyelesaian optimum bagi dualnya.

[10 markah]

2. Pertimbangkan masalah PL berikut:

$$\begin{aligned} \text{Maksimumkan } z &= 20x_1 + 6x_2 + 8x_3 \\ \text{Subject to } 8x_1 + 2x_2 + 3x_3 &\leq 200 \\ 4x_1 + 3x_2 &\leq 100 \\ 2x_1 + x_3 &\leq 50 \\ x_3 &\leq 20 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Biar x_4, x_5, x_6 dan x_7 masing-masing mewakili pembolehubah -pembolehubah lalai bagi kekangan pertama hingga keempat. Katakan selepas beberapa lelaran simpleks, sebahagian daripada tablo semasa adalah seperti berikut:

Asas	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Penyelesaian
z				$\frac{9}{4}$	$\frac{1}{2}$	0	0	500
x_1				$\frac{3}{16}$	$-\frac{1}{8}$	0	0	25
x_2				$-\frac{1}{4}$	$\frac{1}{2}$	0	0	0
x_6				$-\frac{3}{8}$	$\frac{1}{4}$	1	0	0
x_7				0	0	0	1	20

Daripada tablo tersebut

- (a) tunjukkan nombor-nombor tertinggal yang akan dijana oleh kaedah simpleks tertilik semula untuk melakukan lelaran seterusnya.
- (b) tentukan set pembolehubah-pembolehubah asas dan nilai mereka dalam lelaran berikutnya.

[10 markah]

3. Write the dual of

$$\text{Maximize } z = \mathbf{cx} \mid \mathbf{Ax} = \mathbf{b}, \mathbf{x} \text{ unrestricted}$$

[5 marks]

4. Given this problem and its final simplex tableau:

$$\begin{aligned} \text{Maximize } z &= 9x_1 + 6x_2 \\ \text{Subject to } 12x_1 + 5x_2 &\leq 600 \\ x_2 &\leq 72 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Basic	x_1	x_2	s_1	s_2	solution
z	0	0	$\frac{9}{12}$	$\frac{2}{12}$	612
x_1	1	0	$\frac{1}{12}$	$-\frac{5}{12}$	20
x_2	0	1	0	1	72

The slack variables of the first and second constraints are represented by s_1 and s_2 respectively.

- Identify the dual prices and interpret their values.
- Determine the range of feasibility for right-hand side value of the first constraint and interpret your answer.
- Determine the range of optimality for the coefficient of x_2 in the objective function.
- If the right-hand side of the first constraint is changed to 450, what impact would that have on the values of the basic variables and z ?

- A new variable with coefficients $\begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ is added to the original problem.

What impact would that have on the current solution?

- A new constraint $x_1 + x_2 \leq 80$ is added to the problem. What impact would that have on the current solution?

[15 marks]

3. Tuliskan dual bagi

$$\text{Maksimumkan } z = \mathbf{cx} \mid \mathbf{Ax} = \mathbf{b}, \mathbf{x} \text{ tak tersekat}$$

[5 markah]

4. Diberikan masalah berikut dan tablo simpleks terakhirnya:

$$\begin{aligned} \text{Maksimumkan } z &= 9x_1 + 6x_2 \\ \text{Subject to } 12x_1 + 5x_2 &\leq 600 \\ x_2 &\leq 72 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Asas	x_1	x_2	s_1	s_2	Penyelesaian
z	0	0	$\frac{9}{12}$	$\frac{27}{12}$	612
x_1	1	0	$\frac{1}{12}$	$-\frac{5}{12}$	20
x_2	0	1	0	1	72

Pembolehubah lalai bagi kekangan pertama dan kedua masing-masing diwakili oleh s_1 dan s_2 .

- Camkan harga-harga dual dan terjemahkan nilai-nilainya.
- Tentukan julat ketersauran nilai sebelah kanan bagi kekangan pertama dan terjemahkan jawapan anda.
- Tentukan julat keoptimuman bagi pekali x_2 di dalam fungsi matlamat.
- Jika nilai sebelah kanan kekangan pertama diubah kepada 450, apakah kesannya terhadap nilai-nilai pembolehubah asas dan z ?
- Suatu pembolehubah baru dengan pekali-pekali $\begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ ditambahkan kepada masalah asal. Apakah kesannya ke atas penyelesaian semasa?
- Suatu kekangan baru $x_1 + x_2 \leq 80$ ditambahkan kepada masalah tersebut. Apakah kesannya ke atas penyelesaian semasa?

[15 markah]

5. Consider the following integer programming and the optimal simplex tableau of its LP relaxation problem.

$$\begin{aligned}
 &\text{Minimize } z = 3x_1 + 2x_2 + 3x_3 \\
 &\text{Subject to } \quad x_1 + 4x_2 + x_3 \geq 7 \\
 &\quad \quad \quad 2x_1 + x_2 + x_4 \geq 10 \\
 &\quad \quad \quad x_1, x_2, x_3, x_4 \geq 0 \text{ and integer}
 \end{aligned}$$

Basic	x_1	x_2	x_3	x_4	s_1	s_2	Solution
z	$-\frac{5}{2}$	0	$-\frac{5}{2}$	0	$-\frac{1}{2}$	0	$\frac{7}{2}$
x_2	$\frac{1}{4}$	1	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	$\frac{7}{4}$
x_4	$\frac{7}{4}$	0	$-\frac{1}{4}$	1	$\frac{1}{4}$	-1	$\frac{33}{4}$

The surplus variables of the first and second constraints are represented by s_1 and s_2 respectively.

Use the cutting plane algorithm to find the integer solution.

[10 marks]

6. Solve the following 0-1 problem.

$$\begin{aligned}
 &\text{Maximize } z = 30x_1 + 90x_2 + 40x_3 + 15x_4 \\
 &\text{Subject to } \quad 35x_1 + 10x_2 + 25x_3 + 90x_4 \leq 120 \\
 &\quad \quad \quad 4x_1 + 2x_2 + 7x_3 + 3x_4 \leq 12 \\
 &\quad \quad \quad x_1 + x_2 \leq 1 \\
 &\quad \quad \quad x_1, x_2, x_3, x_4 = 0, 1
 \end{aligned}$$

[5 marks]

7. Consider the following pure integer programming problem.

$$\begin{aligned}
 &\text{Maximize } z = 120x_1 + 50x_2 \\
 &\text{Subject to } \quad 14x_1 + 5x_2 \leq 70 \\
 &\quad \quad \quad 2x_1 + 3x_2 \leq 18 \\
 &\quad \quad \quad x_1, x_2 \geq 0 \text{ and integer}
 \end{aligned}$$

- Graph the problem and identify the LP relaxation solution on the graph.
- Solve using the branch and bound method and show the subproblems on your graph.

[10 marks]

5. Pertimbangkan masalah pengaturcaraan integer berikut dan tablo simpleks optimum bagi masalah PL tak ketat .

$$\text{Minimumkan } z = 3x_1 + 2x_2 + 3x_3$$

$$\text{Terhadap } x_1 + 4x_2 + x_3 \geq 7$$

$$2x_1 + x_2 + x_4 \geq 10$$

$$x_1, x_2, x_3, x_4 \geq 0 \text{ dan integer}$$

Asas	x_1	x_2	x_3	x_4	s_1	s_2	Penyelesaian
z	$-\frac{5}{2}$	0	$-\frac{5}{2}$	0	$-\frac{1}{2}$	0	$\frac{7}{2}$
x_2	$\frac{1}{4}$	1	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	$\frac{7}{4}$
x_4	$\frac{7}{4}$	0	$-\frac{1}{4}$	1	$\frac{1}{4}$	-1	$\frac{33}{4}$

Pembolehkan- pembolehkan lebihan bagi kekangan pertama dan kedua masing-masing diwakili oleh s_1 dan s_2 .

Gunakan algorithm satah potongan untuk mendapatkan penyelesaian integer.

[10 markah]

6. Selesaikan masalah 0-1 berikut.

$$\text{Maksimumkan } z = 30x_1 + 90x_2 + 40x_3 + 15x_4$$

$$\text{Subject to } 35x_1 + 10x_2 + 25x_3 + 90x_4 \leq 120$$

$$4x_1 + 2x_2 + 7x_3 + 3x_4 \leq 12$$

$$x_1 + x_2 \leq 1$$

$$x_1, x_2, x_3, x_4 = 0, 1$$

[5 markah]

7. Pertimbangkan masalah pengaturcaraan integer tulen berikut:

$$\text{Maksimumkan } z = 120x_1 + 50x_2$$

$$\text{Terhadap } 14x_1 + 5x_2 \leq 70$$

$$2x_1 + 3x_2 \leq 18$$

$$x_1, x_2 \geq 0 \text{ dan integer}$$

- (a) Grafkan masalah ini dan tentukan penyelesaian bagi masalah PL tak ketat di atas graf.
- (b) Selesaikan dengan kaedah cabang dan batas dan tunjukkan submasalah-submasalah di atas graf.

[10 markah]

8. Given the following goal model:

$$\begin{aligned}
 &\text{Minimize } z = P_1 v_1 + P_2 u_2 + P_3 u_3 \\
 &\text{Subject to } \begin{aligned}
 &2x_1 + 4x_2 + u_1 - v_1 = 80 \\
 &2x_1 + 4x_2 + u_2 - v_2 = 120 \\
 &x_1 + u_3 - v_3 = 30 \\
 &x_1, x_2, u_i, v_i \geq 0 \quad i=1, 2, 3
 \end{aligned}
 \end{aligned}$$

- (a) Determine the values of the deviation variables in the objective function that will minimize the objective.
- (b) Find the values of the decision variables that result from your solution.

[10 marks]

9. A rural clinic hires its staff from nearby cities and towns on a part-time basis. The clinic attempts to have a general practitioner (GP), a nurse and an internist on duty during at least a portion of each week. The clinic has a weekly budget of RM1,200. A GP charges the clinic RM40 per hour, a nurse charges RM20 per hour and an internist charges RM150 per hour. The clinic has established the following goals in order of priority.

- (i) A nurse should be available at least 30 hours per week.
- (ii) A weekly budget of RM1,200 should not be exceeded.
- (iii) A GP or internist should be available at least 20 hours per week.
- (iv) An internist should be available at least six hours per week.

Formulate a goal programming model for determining the number of hours to hire each staff member to satisfy the various goals.

[15 marks]

8. Diberikan model gol berikut:

$$\begin{aligned} \text{Minimumkan } z &= P_1 v_1 + P_2 u_2 + P_3 u_3 \\ \text{Terhadap } 2x_1 + 4x_2 + u_1 - v_1 &= 80 \\ 2x_1 + 4x_2 + u_2 - v_2 &= 120 \\ x_1 + u_3 - v_3 &= 30 \\ x_1, x_2, u_i, v_i &\geq 0 \quad i = 1, 2, 3 \end{aligned}$$

- (a) Dapatkan nilai-nilai pembolehubah sisihan di dalam fungsi objektif yang meminimumkan objektif.
- (b) Tentukan nilai-nilai pembolehubah keputusan yang terhasil daripada penyelesaian anda.

[10 markah]

9. Sebuah klinik desa mengambil stafnya dari bandar-bandar dan pekan-pekan berdekatan untuk bekerja separuh masa. Klinik tersebut cuba mengadakan seorang doktor umum, seorang jururawat dan seorang doktor pakar untuk bertugas sekurang-kurangnya pada sebahagian daripada setiap minggu. Klinik itu mempunyai peruntukan mingguan sebanyak RM1,200. Seorang doktor umum perlu dibayar RM40 per jam, seorang jururawat perlu dibayar RM20 per jam dan seorang doktor pakar perlu dibayar RM150 per jam. Klinik tersebut telah menetapkan gol-gol berikut mengikut keutamaan.

- (i) Seorang jururawat mestilah boleh bekerja sekurang-kurangnya 30 jam per minggu.
- (ii) Peruntukan mingguan sebanyak RM1,200 mestilah tidak dilebihi.
- (iii) Seorang doktor umum atau doktor pakar mestilah boleh bekerja sekurang-kurangnya 20 jam per minggu.
- (iv) Seorang doktor pakar mestilah boleh bekerja sekurang-kurangnya enam jam per minggu.

Rumuskan suatu model pengaturcaraan gol untuk menentukan bilangan jam setiap staf perlu diambil untuk memenuhi gol-gol berkaitan.

[15 markah]