UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MSG 228 – Introduction to Modelling
[Pengenalan Pemodelan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all five** [5] questions.

*[**Arahan:** Jawab **semua lima** [5] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) (i) What is meant by a characteristic length?

   (ii) Does a cloud have characteristic length?

   (iii) What is the appropriate characteristic length of a cylinder?

   (b) (i) What does it mean to say that two quantities are proportional?

   (ii) Two rectangles are geometrically similar to each other. Using just the length of one rectangle, find the area of the second rectangle by using the appropriate form factor.

   (iii) The Fibonacci sequence can be found in proportionality of flower, fruits and others. It is defined as

   $$F\ n+1 = F\ n + F\ n-1; \quad F\ 0 = 1 = F\ 1$$

   Show that $\lim_{x \to \infty} \dfrac{F\ n+1}{F\ n}$ is approximately 1.62.

   Discuss the significance of this number.

   (c) By constructing a simple emigration-extinction diagram, discusses the effect of the size of an off-shore island on the number of species on the island.

   (d) An experiment on a certain type of cells leads to a conclusion that each cell divides into two every 20 minutes.

   (i) Choose a unit of time, and find the corresponding probability of cell division.

   (ii) Write down a discrete time model which balances the amount of cells at time t and at time $t + \Delta t$.

   (iii) Solve the model

   (iv) What is your conclusion?

[100 marks]

2. (i) Suppose that in a certain part of the country 90% of the children of college educated parents get a college education, but 10% do not. Also, among parents, who are not college educated, 40% of their children get a college education, but the other 60% do not. Assuming that the current distribution of parents in that region includes 40% who are college educated and 60% who are not, what will be the distribution be three generation from now?

1. (a) (i) *Apakah yang dimaksudkan dengan panjang cirian?*

(ii) *Adakah awan mempunyai panjang cirian?*

(iii) *Apakah panjang cirian yang sesuai untuk satu silinder?*

(b) (i) *Apakah yang diertikan apabila dua kuantiti dikatakan berkadar?*

(ii) *Dua segi empat tepat sama secara geometri. Mengguna panjang satu segi empat tepat, dapatkan luas segi empat tepat kedua dengan menggunakan faktor pembentuk yang sesuai.*

(iii) *Jujukan Fibonacci terdapat pada perkadaran dalam bunga-bunga, buah-buahan dan lain-lain. Ianya tertakrif seperti:*

$$F(n+1) = F(n) + F(n-1); \quad F(0) = 1 = F(1)$$

Tunjukkan bahawa $\lim_{x \to \infty} \frac{F(n+1)}{F(n)}$ *adalah 1.62.*

Bincangkan keertian nombor ini.

(c) *Dengan membina satu gambarajah ringkas perpindahan kepupusan, bincangkan kesan saiz satu pulau berhampiran tanah besar ke atas bilangan spesis atas pulau.*

(d) *Satu ujikaji ke atas satu jenis sel mendapati bahawa satu sel terbahagi kepada dua setiap 20 minit.*

(i) *Pilih satu unit masa dan dapatkan kebarangkalian sepadan untuk pembahagian sel.*

(ii) *Tuliskan satu model masa diskret yang mengimbangi amaun sel pada masa t dan pada masa* $t + \Delta t$.

(iii) *Selesaikan model*

(iv) *Apakah kesimpulan anda?*

[100 marks]

2. (i) *Andaikan pada satu bahagian tertentu negara, 90% anak kepada ibu-bapa yang berpelajaran tinggi akan menikmati pelajaran tinggi, tetapi 10% tidak. Juga, antara ibubapa yang tiada pelajaran tinggi, 40% daripada anak mereka akan pergi ke institusi pelajaran tinggi, manakala 60% yang lain tidak. Andaikan taburan semasa ibu bapa pada tahun tersebut melibatkan 40% berpelajaran tinggi dan 60% tidak, apakah taburan tiga generasi dari sekarang?*

(ii) Assume that in problem (i), the transitions between states are not consistent from one generation to the next. Instead, for the n^{th} generation of parents, the fraction of their children getting a college education is $0.8+0.1\ -1^{\ n}$ for college educated parents and $0.4+0.1\ -1^{\ n}$ for non college educated parents. Construct a non-autonomous model for this process. Do not solve this model.

(iii) Suppose that when a certain skyscraper is disturbed from rest, its top sways back and forth in such a way that its maximum deflection from the vertical axis of the building to either side is always 90% of its previous maximum deflection on the other side of that axis, which always occur 8 seconds earlier. After a strong wind gust, if the top of the building is 2 feet from that vertical axis: (i) how many times will the top of the building cross the vertical axis before the amplitude of the oscillation falls below 3 inches? (ii) Approximate how long will it take for this to happen?

[100 marks]

3. (a) With the aid of diagrams, explain the law of supply and the law of demand.

(b) The increase or decrease of the price of a certain commodity can be expressed as

$$P_{n+1} = P_n + \text{ (Demand Force)}$$

where P_n represent the price at any given time step *n*. Using the laws of supply and demand, find an appropriate expression for the Demand Force and hence derive the linear price model

$$P_{n+1} = aP_n + b, \quad a \leq 1, \quad b \text{ any value.}$$

(c) (i) Find the equilibrium point of the price model

$$P_{n+1} = 39 - 1.25\,P_n$$

(ii) Determines its stability

(iii) Discuss the significance of your answers above

(iv) Determine whether the price oscillates around the equilibrium point

(v) Discuss significance of your answer to (iv).

(d) The following gives the coordinates of three points on supply and demand curves of a cobweb diagram. [(Price, Quantity)]

$4.4, 1.4$, $\quad 4.4, 0.8$, $\quad 6.8, 0.8$,

Does the price reach on equilibrium or does it spiral out without bound?

[100 marks]

(ii) *Katakan pada masalah (i), transisi antara keadaan-keadaan tidak tetap dari satu generasi ke seterusnya. Untuk ibu bapa genarasi ke-n, kadar anak ibu bapa yang berpelajaran tinggi mendapat pelajaran tinggi adalah* $0.8+0.1(-1)^n$ *sementara* $0.4+0.1(-1)^n$ *untuk anak kepada ibu bapa tanpa pelajaran tinggi. Bina satu model tak berautonomi untuk proses ini. Jangan selesaikan model ini.*

(iii) *Andaikan bahawa apabila satu bangunan pencakar langit di sasar daripada keadaan pegun bahagian atas akan bergerak depan ke belakang dengan sedemikian pesongan maksimum daripada paksi mencacang bangunan adalah 90% pesongan maksimum sebelumnya (pasangan bertentangan). Ini berlaku 8 saat sebelumnya. Selepas satu tiupan angin yang kuat, bahagian atas berada 2 kaki daripada paksi. (i) Berapakali bahagian atas akan merintangi paksi sebelum ayunan menjadi kurang daripada 3 inci? (ii) Berapa lamakah untuk ini berlaku?*

[100 markah]

3. (a) *Dengan menggunakan gambarajah, terangkan hukum pembekal dan hukum permintaan.*

(b) *Peningkatan atau penurunan harga suatu komoditi boleh di ungkapkan seperti*

$$P_{n+1} = P_n + \text{(Daya Permintaan)}$$

dengan P_n *mewakili harga pada sebarang langkah masa n. Mengguna hukum-hukum pembekalan dan permintaan, dapatkan satu ungkapan sesuai untuk Daya Permintaan dan dengan ini dapatkan model linear harga*

$$P_{n+1} = aP_n + b, \quad a \le 1, \quad b \text{ sebarang nilai.}$$

(c) (i) *Dapatkan titik keseimbangan bagi model harga*

$$P_{n+1} = 39 - 1.25\,P_n$$

(ii) *Tentukan kestabilan titik ini*

(iii) *Bincangkan keertian jawapan anda di atas*

(iv) *Tentukan jika harga berayun sekeliling titik keseimbangan.*

(v) *Bincangkan keertian jawapan anda untuk (iv).*

(d) *Berikut adalah koordinat tiga titik atas lengkungan-lengkungan pembekalan dan permintaan untuk satu rajah sarang labah-labah. [(Harga, Kuantiti)]*

$$(4.4, 1.4), \quad (4.4, 0.8), \quad (6.8, 0.8),$$

Adakah harga akan menuju kepada satu keseimbangan atau pun pilin keluar tanpa batasan?

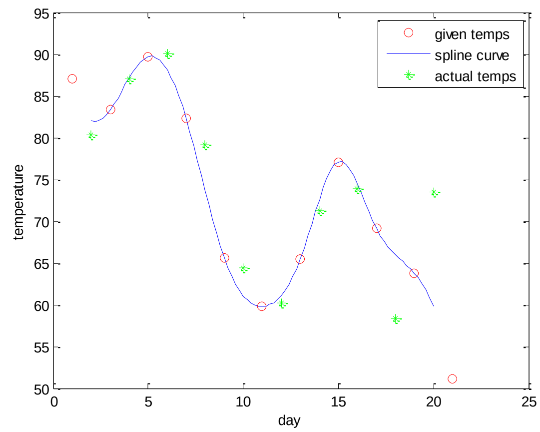
[100 markah]

4. (a) Given $n+1$ data points, we can find a polynomial of degree n interpolating these points.

   (i) Briefly explain the problems that crop up when we try to interpolate points with a high degree polynomial?

   (ii) How does the use of piecewise polynomials avoid these problems?

(b) Temperatures in the temperate zone can change drastically from day to day. A monthly temperature was obtained. Temperature from odd days were used to general a cubic spline interpolation, while temperature from the even days were used to test the 'goodness' of the interpolation. The results is as follows



Explain why the interpolation is not as good as expected.

(c) Part of the Refrigerant Pressure Temperature table is given below

Temperature (^oF)	1	2	3	4
Pressure (p sig)	16.4	17.1	17.9	18.7

Using standard linear least square technique, find a linear model that fits these values.

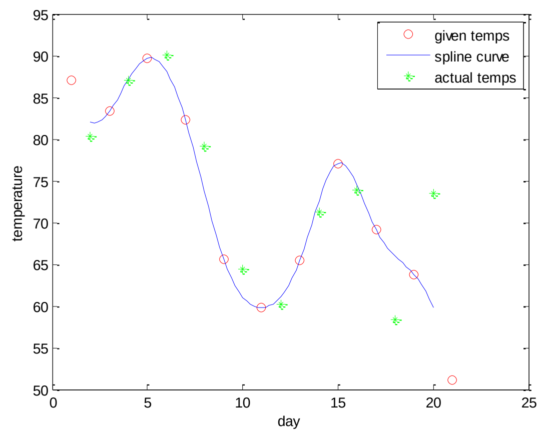
[100 marks]

4. (a) *Di beri* $n+1$ *titik, kita boleh dapatkan satu polynomial berdarjah n menginterpolasikan titik-titik ini.*

   (i) *Secara ringkas, terangkan masalah-masalah yang timbul jika kita interpolasikan titik-titik dengan polynomial berdarjah tinggi?*

   (ii) *Bagaimana penggunaan polynomial tertakrif cebis-demi-cebis mengatasi masalah ini?*

(b) *Suhu pada zon sederhana boleh bertukar secara keterlaluan dari hari ke hari. Satu suhu bulanan telah diperolehi. Suhu dari hari-hari ganjil digunakan untuk menjana satu interpolasi splin kubik, manakala suhu-suhu hari hari genap digunakan menguji 'kebaikan' interpolasi. Hasilnya tertera dalam gambarajah berikut.*



Terangkan mengapa interpolasi tidak seelok yang dijanka.

(c) *Sebahagian daripada sifir Tekanan Suhu Penyejuk di beri seperti berikut:*

Suhu (^oF)	1	2	3	4
Tekanan (p sig)	16.4	17.1	17.9	18.7

Menggunakan teknik piawai ganda dua terkecil linear, dapatkan satu model linear yang mencocok nilai-nilai ini.

[100 markah]

5. (a) A basic SIR model for spread of endemics can be written as

$$\frac{dS}{dt} = -\beta S_0 I \;\; t$$

$$\frac{dI}{dt} = \beta S_0 I \;\; t \;\; - \lambda I \;\; t$$

$$\frac{dR}{dt} = \gamma I \;\; t$$

where S, I, R are respectively the susceptible, infective and removed class. Also $S_0 = S \;\; 0$

(i) Assuming that the population remains constant, find solutions in terms of $S \;\; 0 \;\; , I \;\; 0$ and $R \;\; 0$.

(ii) Show that if $\beta S_0 < \gamma$, the disease will not cause an epidemics

(iii) What happens if $\beta S_0 > \gamma$?

(b) Consider the Anderson-May model of epidemics

$$\frac{dx}{dt} = \mu - \;\; \mu + \lambda \;\; t \;\;\; x \;\; t$$

$$\frac{d\lambda}{dt} = \;\; \nu + \mu \;\; \lambda \;\; t \;\;\; R_0 x \;\; t \;\; -1$$

where

x is fraction of susceptible, λ the force of infection, μ the birth/death rate, R_0 the basic reproductive number and v the recovery rate.

(i) Show that the endemic equilibria is given by

$$x^* = \frac{1}{R_0}, \quad \lambda^* = \mu \;\; R_0 - 1$$

(ii) Define an appropriate Jacobean matrix for the model and hence show that the equilibrium x^*, λ^* is asymptotically stable.

(c) Show that a simple SIS model will give a model similar to the logistic population model.

[100 marks]

5. (a) *Model asas SIR untuk penyakit merebak boleh di tulis seperti:*

$$\frac{dS}{dt} = -\beta S_0 I\ t$$
$$\frac{dI}{dt} = \beta S_0 I\ t - \lambda I\ t$$
$$\frac{dR}{dt} = \gamma I\ t$$

dengan S, I, R masing-masing kelas mudah berjangkit, jangkit dan sembuh. *Juga* $S_0 = S\ 0$

(i) *Menganggap bahawa jumlah populasi tidak berubah, dapatkan penyelesaian dalam sebuatan-sebutan. Juga* $S\ 0$, $I\ 0$ *dan* $R\ 0$.

(ii) *Tunjukkan bahawa jika* $\beta S_0 < \gamma$, *penyakit tidak akan merebak*

(iii) *Apa terjadi jika* $\beta S_0 > \gamma$?

(b) *Pertimbangkan model Anderson-May untuk penyakit merebak*

$$\frac{dx}{dt} = \mu - \mu t\ \lambda\ t\ \ x\ t$$
$$\frac{d\lambda}{dt} = \nu + \mu\ \lambda\ t\ \ R_0 x\ t - 1$$

dengan
x *pecahan kelas mudah berjangkit,* λ *daya penyakit,* μ *kadar kelahiran/kematian ,* R_0 *nombor asas pembiakan dan v kadar sembuh*

(i) *Tunjukkan bahawa keseimbangan endemi di beri oleh*

$$x^* = \frac{1}{R_0}, \quad \lambda^* = \mu\ R_0 - 1$$

(ii) *Takrifkan suatu matriks Jacobean yang sesuai untuk model dan dengan ini tunjukkan bahawa keseimbangan* x^*, λ^* *adalah stabil secara asimptot.*

(c) *Tunjukkan bahawa model ringkas SIS memberi satu model serupa dengan model logistik populasi.*

[100 markah]

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