UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MAT 516 – Curve and Surface Methods for CAGD
[Kaedah Lengkung dan Permukaan untuk RGBK]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. The n th degree Bezier curve is defined by

$$P(t) = \sum_{i=0}^{n} B_{n,i}(t) P_i \quad , 0 \le t \le 1$$

where $B_{n,i}(t) = \binom{n}{i}(1-t)^{n-i} t^i$ and $P_i, i = 0,1,\ldots,n$ are the control points.

(a) A cubic Bezier curve can be represented in a matrix form TMP

where $T = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix}$, $P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$

Write down the matrix M.

(b) Let $Q(t) = P(t+1)$

(i) Use (a) find a matrix C so that $Q(t) = TCMP$?

(ii) Write the control points Q_0, Q_1, Q_2 and Q_3 of $Q(t)$ in terms of P_0, P_1, P_2 and P_3.

(iii) Construct the control polygon of $Q(t)$ and $P(t)$ on the same figure, and show that $P(t)$ and $Q(t)$ satisfy the C^2 continuity at the common point.

(c) If $P^*(t)$ is obtained by translating $P(t)$ with a vector $\underline{V}$, find the control points of $P^*(t)$.

[25 marks]

2. A quadratic rational Bezier curve is defined by

$$r(t) = \frac{\sum_{i=0}^{2} B_{2,i}(t) w_i P_i}{\sum_{i=0}^{2} B_{2,i}(t) w_i} \quad , 0 \le t \le 1$$

with control points P_0, P_1, P_2 and the corresponding weights w_0, w_1, w_2.

1. *Lengkung Bezier dengan darjah n ditakrifkan sebagai*

$$P(t) = \sum_{i=0}^{n} B_{n,i}(t)\, P_i \quad ,0 \le t \le 1$$

dengan $B_{n,i}(t) = \binom{n}{i}(1-t)^{n-i}t^i$ *dan* $P_i, i = 0,1,...,n$ *adalah titik kawalan.*

(a) *Suatu lengung Bezier kubik dapat diwakilkan dalami suatu bentuk matriks TMP*

dengan $T = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix}, \quad P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$

Tuliskan matriks M.

(b) *Andaikan* $Q(t) = P(t+1)$

(i) *Guna (a) cari matriks C supaya* $Q(t) = TCMP$?

(ii) *Tuliskan titik kawalan* Q_0, Q_1, Q_2 *dan* Q_3 *untuk* $Q(t)$ *dalam sebutan* P_0, P_1, P_2 *dan* P_3.

(iii) *Bina poligon kawalan* $Q(t)$ *dan* $P(t)$ *pada rajah yang sama dan tunjukkan* $P(t)$ *dan* $Q(t)$ *memenuhi syarat keselanjaran* C^2 *pada titik sepunya.*

(c) *Jika* $P^*(t)$ *diperolehi dengan menganjakan* $P(t)$ *dengan suatu vektor* $\underline{V}$, *cari titik kawalan* $P^*(t)$.

[25 markah]

2. *Suatu lengkung Bezier nisbah kuadratik ditakrifkan sebagai*

$$r(t) = \frac{\sum_{i=0}^{2} B_{2,i}(t)\, w_i P_i}{\sum_{i=0}^{2} B_{2,i}(t)\, w_i} \quad ,0 \le t \le 1$$

dengan titik kawalan P_0, P_1, P_2 *dan pemberat yang sepadans* w_0, w_1, w_2.

(a) Show that the curve reduces to a quadratic Bezier curve when $w_0 = w_1 = w_2 = w \neq 0.$

(b) If $Q = r\left(\frac{1}{2}\right)$, write Q in terms of w_i and P_i, $i = 0,1,2.$
What will happen to Q if $w_1 \to \infty.$

(c) If $t(u) = \frac{\lambda u}{(1-u)+\lambda u}$; $0 \le u \le 1$ is substituted into the equation of rational Bezier quadratic, show that it becomes

$$r(u) = \frac{(1-u)^2 w_0 P_0 + 2u(1-u)\lambda w_1 P_1 + u^2 \lambda^2 w_2 P_2}{(1-u)^2 w_0 + 2u(1-u)\lambda w_1 + u^2 \lambda^2 w_2}.$$

If $\lambda^2 w_2 = w_0$, show that $r(u)$ reduces to

$$r(u) = \frac{\left((1-u)^2 P_0 + 2u(1-u)\frac{w_1}{\sqrt{w_0 w_2}} P_1 + u^2 P^2\right)}{(1-u)^2 + 2u(1-u)\frac{w_1}{\sqrt{w_0 w_2}} + u^2}.$$

(d) Let $k = \frac{w_1^2}{w_0 w_2}$ be the shape parameter of the conic, states the range or value of k so that $r(u)$ becomes parabola, ellips and hyperbola.

[25 marks]

3. A B-spline curve of degree p is defined by $r(u) = \sum_{i=0}^{n} N_{i,p}(u) P_i,\ t_p \le t < t_{m-p}$
$P_i, i = 0,1,2,...,n$ are the control points with knot vector $U = \{u_0, u_1, ..., u_m\}$. $N_{i,p}(u)$ is the normalised B-spline basis function of degree p defined by

$$N_{i,0}(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

And $N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{p+1+i} - u}{u_{p+1+i} - u_{i+1}} N_{i+1,p-1}$ where $0 \le i \le n$

(a) Write down the relationship between m, n and $p.$

(b) Explain the condition on $u_0, u_1, ..., u_m$ so that B-Spline curve of degree p interpolates the first point $P_0.$

(a) *Tunjukkan bahawa lengkung terturun kebentuk Bezier kuadaratik apabila* $w_0 = w_1 = w_2 = w \neq 0.$

(b) *Jika* $Q = r\left(\frac{1}{2}\right)$, *tuliskan Q dalam sebutan* w_i *dan* P_i, $\quad i = 0,1,2.$
Apa akan terjadi kepada Q jika $w_1 \to \infty$.

(c) *Jika* $t(u) = \dfrac{\lambda u}{(1-u)+\lambda u}; \quad 0 \le u \le 1$ *digantikan kedalam persamaan Bezier nisbah kuadratik tunjukkan ia akan menjadi*

$$r(u) = \frac{(1-u)^2 w_0 P_0 + 2u(1-u)\lambda w_1 P_1 + u^2 \lambda^2 w_2 P_2}{(1-u)^2 w_0 + 2u(1-u)\lambda w_1 + u^2 \lambda^2 w_2}.$$

Jika $\lambda^2 w_2 = w_0$, *tunjukkan bahawa* $r(u)$ *terturun kepada*

$$r(u) = \frac{\left((1-u)^2 P_0 + 2u(1-u)\dfrac{w_1}{\sqrt{w_0 w_2}} P_1 + u^2 P^2\right)}{(1-u)^2 + 2u(1-u)\dfrac{w_1}{\sqrt{w_0 w_2}} + u^2}.$$

(d) *Biar* $k = \dfrac{w_1^2}{w_0 w_2}$ *sebagai parameter bentuk suatu keratan kon, nyatakan selang atau nilai k supaya* $r(u)$ *menjadi suatu parabola, elips dan hiperbola.*

[25markah]

3. *Suatu lengkung splin-B dengan darjah p ditakrifkan sebagai*

$$r(u) = \sum_{i=0}^{n} N_{i,p}(u) P_i, \; t_p \le t < t_{m-p}$$

$P_i, i = 0,1,2,...,n$ *adalah titik kawalan dengan vektor knot* $U = \{u_0, u_1, ..., u_m\}$. $N_{i,p}(u)$ *adalah fungsi asas splin-B ternormal dengan darjah p ditakrifkan sebagai*

$$N_{i,0}(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0 & \text{ditempat lain} \end{cases}$$

dan $N_{i,p}(u) = \dfrac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \dfrac{u_{p+1+i} - u}{u_{p+1+i} - u_{i+1}} N_{i+1,p-1}$ *dengan* $0 \le i \le n$

(a) *Tuliskan hubungan antara* m, n *dan* p.

(b) *Terangkan syarat pada* $u_0, u_1, ..., u_m$ *supaya lengkung splin-B dengan darjah p menginterpolasikan titik pertama* P_0.

(c) If $U = \{-1,0,1,2,3,4,5,6,7,8\}$ is the given knot vector.

(i) Find the maximum number of control points and the interval of u to generate a B-spline curve of degree 3.

(ii) Evaluate $N_{2,3}(2.5)$.

[25 marks]

4. A Bezier patch of degree $m \times n$ is defined by

$$\sum_{i=0}^{m}\sum_{j=0}^{n} B_{m,i}(u)\,B_{n,j}(v)\,P_{ij}$$

where $0 \leq u, v \leq 1$ and P_{ij} are the Bezier control points. $B_{m,i}(u)$ and $B_{n,i}(v)$ are Bernstein basis functions.

(a) A Bezier patch of degree 2×2 can be written as $UMWM^TV^T$ where $U = \left[1\,u\,u^2\right]$, $V = \left[1\,v\,v^2\right]$, M, W and M^T are 3×3 matrices. Write down the matrix M and W.

(b) When the degree in the u direction is increased to $m+1$, that is

$$\sum_{i=0}^{m+1}\sum_{j=0}^{n} B_{m+1,i}(u)\,B_{n,j}(v)\,\hat{P}_{ij} = \sum_{i=0}^{m}\sum_{j=0}^{n} B_{m,i}(u)\,B_{n,j}(w)\,P_{ij}.$$

Show $\hat{P}_{ij} = \dfrac{i}{m+1}P_{i-1,j} + \left(1 - \dfrac{i}{m+1}\right)P_{ij}, \quad i = 0,1,2,\ldots,m+1,\ j = 0,1,\ldots,n.$

[25 marks]

(c) *Jika diberikan* $U = \;-1,0,1,2,3,4,5,6,7,8$ *suatu vektor knot.*

(i) *Cari bilangan titik kawalan yang maksimum dan selang nilai u untuk menjana suatu lengkung splin-B berdarjah 3.*

(ii) *Nilaikan* $N_{2,3}\;\;2.5$.

[25 markah]

4. *Suatu tampalan Bezier dengan darjah* $m \times n$ *ditakrifkan sebagai*

$$\sum_{i=0}^{m}\sum_{j=0}^{n} B_{m,i}\;\;u\;\;B_{n,j}\;\;v\;\;P_{ij}$$

dengan $0 \le u, v \le 1$ *dan* P_{ij} *adalah titik kawalan.* $B_{m,i}\;\;u$ *dan* $B_{n,i}\;\;v$ *adalah fungsi asas Bernstein.*

(a) *Suatu tampalan Bezier dengan darjah* 2×2 *boleh ditulis sebagai* $UMWM^TV^T$ *dengan* $U = \begin{bmatrix} 1 & u & u^2 \end{bmatrix}$, $V = \begin{bmatrix} 1 & v & v^2 \end{bmatrix}$, M, W *dan* M^T *adalah matriks* 3×3. *Tuliskan matriks M dan W.*

(b) *Apabila darjah pada arah u ditingkatkan kepada* $m+1$, *iaitu*

$$\sum_{i=0}^{m+1}\sum_{j=0}^{n} B_{m+1,i}\;\;u\;\;B_{n,j}\;\;v\;\;\hat{P}_{ij} = \sum_{i=0}^{m}\sum_{j=0}^{n} B_{m,i}\;\;u\;\;B_{n,j}\;\;w\;\;P_{ij}.$$

Tunjuk $\hat{P}_{ij} = \dfrac{i}{m+1}P_{i-1,j} + \left(1 - \dfrac{i}{m+1}\right)P_{ij}, \quad i = 0,1,2,\ldots,m+1,\; j = 0,1,\ldots,n.$

[25 markah]

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