UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MAA 102 – Calculus for Science Students II
[Kalkulus untuk Pelajar Sains II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>FIVE</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>LIMA</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u>:   Answer **<u>all ten</u>** [10] questions.

*[**<u>Arahan:</u>**   Jawab **<u>semua sepuluh</u>** [10] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1.   Determine whether the following sequence converges or diverges:

(a) $\left\{ \dfrac{n+ -1^{\,n}}{n} \right\}$ (b) $\left\{ \dfrac{n^{\frac{3}{4}} \sin n^2}{n+4} \right\}$

[10 marks]

2.   Test the convergence of the series:

(a) $\displaystyle\sum_{n=1}^{\infty} -1^{\,n+1} \left(\dfrac{5}{4} \right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} a_n$, where $a_1 = 4$ and $a_{n+1} = \dfrac{3a_n}{2n+1}$

[8 marks]

3.   Find the Taylor polynomial of degree 3 for $f\ x\ = \sqrt{x}$ about $x=4$.
      Hence, approximate $\sqrt{6}$.

[7 marks]

4.   Find the center, radius and interval of convergence of the power series
      $\displaystyle\sum_{n=1}^{\infty} \dfrac{-1^{\,n}\, x^n}{9n^2 -1}.$

[15 marks]

5.   Determine whether $\displaystyle\int_{1}^{\infty} \dfrac{x}{x^3 +1}\, dx$ is convergent or divergent.

[5 marks]

6.   (a)   Find and sketch the domain of the $f\ x,y\ = \sqrt{4-x^2 - y^2} + \ln\ 1-x^2\ .$

      (b)   Let $z=f\ x,y$ where f is differentiable, $x=r\cos\theta$ and $y=r\sin\theta.$
            If $f_x\ 0,2\ =-1$ and $f_y\ 0,2\ =3$, find $\dfrac{\partial z}{\partial r}$ and $\dfrac{\partial z}{\partial \theta}$ when $r=2$ and $\theta=\dfrac{\pi}{2}$.

[11 marks]

1.   *Tentukan sama ada jujukan berikut menumpu atau mencapah*:

     (a) $\left\{ \dfrac{n + \left(-1\right)^n}{n} \right\}$ (b) $\left\{ \dfrac{n^{3/4}\sin n^2}{n+4} \right\}$

                                                            [10 markah]

2.   *Uji penumpuan bagi siri*:

     (a) $\displaystyle\sum_{n=1}^{\infty} \left(-1\right)^{n+1}\left(\dfrac{5}{4}\right)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} a_n$, $a_1 = 4\ dan\ a_{n+1} = \dfrac{3a_n}{2n+1}$

                                                            [8 markah]

3.   *Dapatkan polinomial Taylor berdarjah 3 bagi $f\left(x\right) = \sqrt{x}$ sekitar $x = 4$.*
       Seterusnya, anggarkan $\sqrt{6}$.

                                                            [7 markah]

4.   *Dapatkan pusat, jejari dan selang penumpuan bagi siri kuasa $\displaystyle\sum_{n=1}^{\infty} \dfrac{\left(-1\right)^n x^n}{9n^2 - 1}$.*

                                                            [15 markah]

5.   *Tentukan sama ada $\displaystyle\int_{1}^{\infty} \dfrac{x}{x^3 + 1}\,dx$ menumpu atau mencapah.*

                                                            [5 markah]

6.   (a)   *Dapatkan dan lakarkan domain bagi $f\left(x,y\right) = \sqrt{4 - x^2 - y^2} + \ln\left(1 - x^2\right)$.*

       (b)   *Biar $z = f\left(x,y\right)$ yang mana f terbezakan, $x = r\cos\theta$ dan $y = r\sin\theta$. Jika*

              $f_x\left(0,2\right) = -1\ dan\ f_y\left(0,2\right) = 3$, dapatkan $\dfrac{\partial z}{\partial r}$ dan $\dfrac{\partial z}{\partial \theta}$ apabila

              $r = 2\ dan\ \theta = \dfrac{\pi}{2}$.

                                                            [11 markah]

7.   Let $f\ x,y,z\ =z\ x-y^{\ 5}+xy^2z^3$. Find the direction and magnitude of the largest directional derivative of f at $\ 2,1,-1\ $.
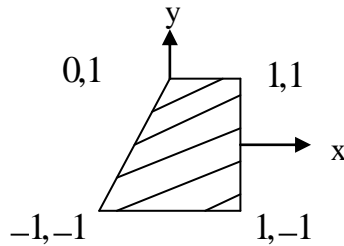
[7 marks]

8.   Show that the production function $Q\ x,y\ =Ax^\alpha y^\beta$, with $\alpha+\beta=1$ and A is a constant, is maximized with respect to the fixed cost $\ px+qy=k\ $ when

$$x=\frac{\alpha k}{p}\quad\text{and}\quad y=\frac{\beta k}{q}.$$

[8 marks]

9.   (a)   Set up (but do not evaluate) an integral

(i) $\iint_{D} xy^2 dA$, where D is the shaded region.

(ii) $\int\limits_{0}^{2}\int\limits_{0}^{\sqrt{1-x^2}}\sqrt{4-x^2-y^2}\ dydx$ in terms of polar form.

(b)   Compute the area of the region R, that bounded by the $x-$axis and the parabola $y=9-x^2$.

[14 marks]

10.   (a)   Find the integrating factor for the equation $x\dfrac{dy}{dx}-3y=x^3$. Hence, find the particular solution when $\ y\ 1\ =1$.

(b)   A hard boiled egg at $98°C$ is put in a sink of $18°C$ water. After 5 minutes, the egg`s temperature is $38°C$. When will its temperature be $20°C$?

[15 marks]

7. *Biar* $f\ (x,y,z)\ =z\ (x-y)^5+xy^2z^3.$ *Dapatkan arah dan magnitud terbitan berarah yang terbesar bagi f pada* $(2,1,-1)$.
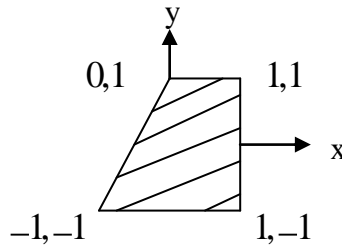
   [7 markah]

8. *Tunjukkan bahawa fungsi pengeluaran* $Q\ (x,y)\ =Ax^\alpha y^\beta$, *dengan* $\alpha+\beta=1$ *dan A merupakan pemalar, adalah maksimum terhadap kos tetap* $px+qy=k$ *apabila*
   $$x=\frac{\alpha k}{p}\ dan\ y=\frac{\beta k}{q}.$$

   [8 markah]

9. (a) *Tentukan (tanpa menilai) suatu kamiran*

   (i) $\iint_D xy^2\,dA,\ \ D$ *merupakan rantau berlorek.*

   (ii) $\int_0^2 \int_0^{\sqrt{1-x^2}} \sqrt{4-x^2-y^2}\ dy\,dx$ *dalam bentuk polar.*

   (b) *Hitung luas rantau R, yang dibatasi oleh paksi - x dan parabola* $y=9-x^2$.

   [14 markah]

10. (a) *Dapatkan faktor pengamir bagi persamaan* $x\dfrac{dy}{dx}-3y=x^3$. *Seterusnya, dapatkan penyelesaian khusus apabila* $y(1)=1$.

    (b) *Sebiji telur rebus pada suhu* $98°C$ *dimasukkan ke dalam sink yang suhu airnya* $18°C$. *Selepas 5 minit, suhu telur adalah* $38°C$. *Bilakah suhu telur menjadi* $20°C$?

    [15 markah]

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