
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

MAA 101 – Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all ten** [10] questions.

Arahan : Jawab **semua sepuluh** [10] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Find the interval of x that satisfies $\left| \frac{x}{x-4} \right| \leq 3$.
- (b) Given that $f(x) = x^2 + 3$ and $g(x) = 2x + 3$, find all values of x such that $f(g(x)) = g(f(x))$.
[9 marks]
2. (a) Given that $f(x) = \frac{2x-6}{3x+3}$
- (i) State the domain and range of f .
- (ii) Is f one to one function? If yes, find the inverse of f .
- (b) Let f be a function for which $f(2) = -3$ and $f'(x) = \sqrt{x^2 + 5}$. If $g(x) = x^2 f\left(\frac{x}{x-1}\right)$, find $g'(2)$.
[10 marks]
3. Evaluate the following limit. The L'Hospital rule can be applied whenever applicable.
- (a) $\lim_{x \rightarrow 0} \frac{x^2 - 3x + 2}{x^2 - 1}$.
- (b) $\lim_{x \rightarrow +\infty} \frac{2x^2 + 3x}{e^x}$.
- (c) $\lim_{x \rightarrow +\infty} [x - \ln(e^x + e^{-x})]$. Hint: $\ln e^x = x$.
[11 marks]
4. Given $f(x) = \begin{cases} x^2 - mx - 6, & x < 2 \\ x^2 + 2, & x \geq 2 \end{cases}$
- (a) Find m so that $f(x)$ is continuous for all x .
- (b) Hence, with the value m obtained in (a), does $f(x)$ differentiable at $x = 2$? Explain your answer.
[10 marks]

1. (a) Cari selang bagi x yang memenuhi $\left| \frac{x}{x-4} \right| \leq 3$.
- (b) Diberi bahawa $f(x) = x^2 + 3$ and $g(x) = 2x + 3$, cari semua nilai x supaya $f(g(x)) = g(f(x))$.

[9 markah]

2. (a) Diberi bahawa $f(x) = \frac{2x-6}{3x+3}$
- (i) Nyatakan domain and julat f .
- (ii) Adakah f fungsi satu ke satu? Jika ya, cari songsangan f .
- (b) Diberi bahawa f suatu fungsi yang mana $f'(x) = -3$ dan $f(x) = \sqrt{x^2 + 5}$. If $g(x) = x^2 f\left(\frac{x}{x-1}\right)$, find $g'(2)$.

[10 markah]

3. Nilaikan had berikut. Hukum L'Hospital boleh digunakan di tempat yang sesuai.

(a) $\lim_{x \rightarrow 0} \frac{x^2 - 3x + 2}{x^2 - 1}$.

(b) $\lim_{x \rightarrow +\infty} \frac{2x^2 + 3x}{e^x}$.

(c) $\lim_{x \rightarrow +\infty} [x - \ln(e^x + e^{-x})]$. Hint: $\ln e^x = x$.

[11 markah]

4. Diberi $f(x) = \begin{cases} x^2 - mx - 6, & x < 2 \\ x^2 + 2, & x \geq 2 \end{cases}$

- (a) Cari m supaya f selanjut untuk semua nilai x .
- (b) Seterusnya, dengan nilai m yang diperolehi di bahagian (a), adakah $f(x)$ terbezakan pada $x = 2$? Jelaskan jawapan anda.

[10 markah]

5. Find the derivative of the following functions.

(a) $f(x) = \frac{x - \sin x}{x^3}$.

(b) $e^y + \ln(x + y^2) = 2x^3$.

(c) $y = \frac{e^{3x^2}}{\sqrt[3]{(x^3 + 1)^5} (4x - 7)^{-2}}$. Use logarithmic differentiation.

[10 marks]

6. (a) Let $h(x) = (f(x))^3$, where f is a differentiable function. If $f(0) = -\frac{1}{2}$ and $f'(0) = \frac{8}{3}$, find the equation of the tangent line to the graph of h at $x = 0$.

(b) Using the Mean Value Theorem, show that $\sqrt{x} < 1 + \frac{x}{4}$ for $x \geq 4$. Hint: let $f(x) = \sqrt{x}$.

[9 marks]

7. Let $f(x) = \frac{2x^2 + 3x}{e^x}$. Find

(a) all asymptotes of f .

(b) the interval on which f is increasing or decreasing.

(c) the local maximum and minimum values of f (if any).

(d) the interval of concavity and the inflection points (if exist).

Hence, sketch the graph of f .

[13 marks]

5. Cari pembezaan fungsi-fungsi berikut.

(a) $f(x) = \frac{x - \sin x}{x^3}$.

(b) $e^y + \ln(x + y^2) = 2x^3$.

(c) $y = \frac{e^{3x^2}}{\sqrt[3]{(x^3 + 1)^5 (4x - 7)^{-2}}}$. Gunakan pembezaan logaritma.

[10 markah]

6. (a) Biarkan $h(x) = (f(x))^3$, yang mana f ialah fungsi boleh beza. Jika $f(0) = -\frac{1}{2}$ dan $f'(0) = \frac{8}{3}$, cari persamaan garis tangen kepada graf h pada $x=0$.

(b) Tunjukkan bahawa dengan menggunakan Teorem Nilai Min, $\sqrt{x} < 1 + \frac{x}{4}$ untuk $x \geq 4$. Petunjuk: biarkan $f(x) = \sqrt{x}$.

[9 markah]

7. Biarkan $f(x) = \frac{2x^2 + 3x}{e^x}$. Cari

(a) semua asimptot bagi f .

(b) selang f menokok atau menyusut.

(c) nilai maximum and minimum tempatan (jika ada).

(d) selang kecekungan f dan titik lengkung balas (jika wujud).

Seterusnya, lakarkan graf untuk f .

[13 markah]

8. Evaluate the following integral.

(a) $\int_0^1 x^2 \sqrt{x^3 + 9} \, dx.$

(b) $\int \frac{x^4 - x^2 + 2}{x^2(x-1)} \, dx.$

(c) $\int_0^{\pi/2} [f(x) + f''(x)] \cos x \, dx,$ if $f(0)$ and $f'\left(\frac{\pi}{2}\right)$ are defined, $f\left(\frac{\pi}{2}\right) = 5$ and $f'(0) = -1.$

[11 marks]

9. Given f is a continuous function such that

$$\int_0^x f(t) \, dt = x \sin x + \int_0^x \frac{f(t)}{1+t^2} \, dt$$

for all x . Using the fundamental theorem of calculus, find $f(x)$.

[5 marks]

10. Let \mathbf{R} be the region bounded by the graphs of equations $y = (x-2)^2$ and $y = 4$.

(a) Sketch the region \mathbf{R} and find its area.

(b) Find the volume of the solid obtained by rotating the region \mathbf{R} about the y -axis and x -axis.

[12 marks]

8. *Nilaikan kamiran berikut.*

(a) $\int_0^1 x^2 \sqrt{x^3 + 9} \, dx.$

(b) $\int \frac{x^4 - x^2 + 2}{x^2(x-1)} \, dx.$

(c) $\int_0^{\pi/2} [f(x) + f''(x)] \cos x \, dx,$ jika $f(0)$ and $f'(\frac{\pi}{2})$ tertakrif, $f(\frac{\pi}{2}) = 5$ dan $f'(0) = -1.$

[11 markah]

9. *Diberi f suatu fungsi yang selanjur,*

$$\int_0^x f(t) \, dt = x \sin x + \int_0^x \frac{f(t)}{1+t^2} \, dt$$

untuk semua x . Cari $f(x)$ dengan menggunakan teorem asas kalkulus.

[5 markah]

10. *Biarkan R rantau yang di batasi oleh persamaan $y = (x-2)^2$ dan $y = 4$.*

(a) *Lakarkan kawasan R dan cari luas rantau tersebut.*

(b) *Cari isipadu bongkah kisanan yang terhasil apabila rantau R dikisarkan terhadap paksi-y dan paksi-x.*

[12 markah]