UNIVERSITI SAINS MALAYSIA

First Semester Examination
2009/2010 Academic Session

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MST 564 – Statistical Reliability
[Kebolehpercayaan Statistik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of THIRTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) There are four related probability functions in describing lifetime distributions. Each of these functions can be used to compute reliabilities, but they offer four different perspectives. Specifying any one of these functions will uniquely and completely characterize the failure or survival process.

   (i) Name the four functions.
   (ii) Describe each of the functions.
   (iii) Give an example to illustrate the functions.

   (b) The bathtub curve characterizes human mortality and engineering failures. Summarize the distinguishing features of the bathtub curve. Draw a picture to illustrate the curve.

   (c) A simplified form of the bathtub curve is based upon linear and constant hazard rates:

$$\lambda(t)=\begin{cases} c_0-c_1t+\lambda & 0\le t\le \dfrac{c_0}{c_1} \\ \lambda & \dfrac{c_0}{c_1}<t\le t_0 \\ c_2(t-t_0)+\lambda & t_0<t \end{cases}$$

   Find $R(t)$.

[100 marks]

2. (a) For a three-parameter Weibull distribution with $\beta=1.54$ hr, $\alpha=8500$, and $t_0=50$ hr, find the following:

   (i) $R(150\ \text{hr})$

   (ii) *MTTF*

   (iii) t_{median}

   (iv) The design life for a reliability of 0.98

1. (a) *Terdapat empat fungsi kebarangkalian yang berkaitan dalam menghuraikan taburan-taburan masahayat. Setiap fungsi ini dapat mengira kebolehpercayaan, tetapi mereka memberi perspektif yang berbeza. Memperincikana sebarang fungsi ini akan mencirikan proses gagal atau hidup secara unik dan lengkap.*

    (i) *Namakan empat fungsi ini.*
    (ii) *Huraikan setiap fungsi ini.*
    (iii) *Berikan satu contoh untuk mengilustrasi fungsi ini.*

(b) *Lengkungan 'bathtub' mencirikan mortaliti manusia dan kegagalan kejuruteraan. Perihalkan ciri-ciri penting bagi lengkungan 'bathtub'. Lukis suatu gambar untuk mengilustrasi lengkungan itu.*

(c) *Satu bentuk yang mudah bagi lengkungan 'bathtub' adalah berdasarkan pada kadar-kadar bahaya linear dan rata:*

$$\lambda(t)=\begin{cases} c_0 - c_1 t + \lambda & 0 \le t \le \dfrac{c_0}{c_1} \\ \lambda & \dfrac{c_0}{c_1} < t \le t_0 \\ c_2(t-t_0)+\lambda & t_0 < t \end{cases}$$

Cari $R(t)$.

[100 markah]

2. (a) *Bagi suatu taburan Weibull tiga-parameter dengan* $\beta = 1.54$ *jam,* $\alpha = 8500,$ *dan* $t_0 = 50$ *jam, cari yang berikut:*

    (i) $R(150 \text{ jam})$

    (ii) *MTTF*

    (iii) t_{median}

    (iv) *Hayat rekabentuk bagi suatu kebolehpercayaan 0.98*

(b) The failure rate on a new brake drum design is estimated to be

$$\lambda(t) = 1.2\times 10^{-6}\exp(10^{-4}t)$$

per set, where t is in kilometers of normal driving. Forty vehicles are each test-driven for 15,000 km.

(i) How many failures are expected, assuming that the vehicles with failed drives are removed from the test?

(ii) What is the probability that more than two vehicles will fail?

(c) Sketch the survival function for an exponential random variable with mean time to failure of 10 hours. Determine the probability that a component having this survival function will operate at least 20 hours before failure.

[100 marks]

3. (a) The survival times (in days after transplant) for the original $n = 69$ members of the Stanford Heart Transplant Program (see Crowley and Hu, 1977) were as follows:

Table 2(c). Survival Time After Heart Transplant (Days)

15	3	624	46	127	64	1350	280
23	10	1024	39	730	136	1775	1
836	60	1536	1549	54	47	51	1367
1264	44	994	51	1106	897	253	147
51	875	322	838	65	815	551	66
228	65	660	25	589	592	63	12
499	305	29	456	439	48	297	389
50	339	68	26	30	237	161	14
167	110	13	1	1			

(Source: Crowley, J. and Hu, M. (1977). The Covariance Analysis of Heart Transplant Data. *Journal of the American Statistical Association*, **72**, 27-36)

Obtain the 'best fit' parameters for the fitted distribution. What are your conclusions about the survival distribution of these patients?

(b) *Kadar kegagalan bagi rekabentuk 'brake drum' yang baru dianggar*

$$\lambda(t) = 1.2 \times 10^{-6} \exp(10^{-4} t)$$

bagi satu set, di mana t adalah dalam kilometer bagi pemanduan biasa. Empat puluh buah kenderaan dipandu uji untuk 15,000 km.

(i) *Berapa banyak kegagalan dijangka, diberi andaian bahawa kenderaan yang gagal dikeluarkan dari ujian?*

(ii) *Apakah kebarangkalian bahawa lebih daripada dua kenderaan akan gagal?*

(c) *Lakar fungsi survival bagi pemboleh ubah rawak eksponen dengan min masa kegagalan (MTTF) 10 jam. Dapatkan kebarangkalian bahawa sebuah komponen yang mempunyai fungsi survival ini akan berfungsi sekurang-kurangnya 20 jam sebelum kegagalan.*

[100 marks]

3\. *(a)* *Masa-masa survival (dalam hari selepas pemindahan) bagi $n = 69$ pesakit yang asal untuk Program Transplant Jantung Stanford (lihat Crowley and Hu, 1977) adalah seperti yang berikut:*

Jadual 2(c). *Masa Survival Selepas Transplant Jantung (Hari)*

15	3	624	46	127	64	1350	280
23	10	1024	39	730	136	1775	1
836	60	1536	1549	54	47	51	1367
1264	44	994	51	1106	897	253	147
51	875	322	838	65	815	551	66
228	65	660	25	589	592	63	12
499	305	29	456	439	48	297	389
50	339	68	26	30	237	161	14
167	110	13	1	1			

(Sumber: Crowley, J. and Hu, M. (1977). The Covariance Analysis of Heart Transplant Data. Journal of the American Statistical Association, ***72****, 27-36)*

Dapatkan parameter yang paling sesuai bagi taburan yang disuai. Apakah kesimpulan anda tentang taburan survival bagi pesakit-pesakit ini?

(b) The data given below are survival times T_i (in months) after an operation:

$$8,\ 1+,\ 7,\ 7,\ 4+,\ 5,\ 7+,\ 5,\ 6+,\ 3$$

(i) Calculate by hand the Kaplan-Meier estimate of survival $\hat{S}(t)$.

(ii) Sketch the Kaplan-Meier curve and mark the censored data.

(iii) According to the Kaplan-Meier estimate, how large is the probability that a patient survives at least half a year?

(c) When do you use non-parametric or parametric methods to estimate the survival and hazard functions? What are the strengths and weaknesses of these two methods?

[100 marks]

4. (a) What are the reasons for modeling survival data? Describe the Proportional Hazards Model which is a basic model for survival data.

(b) Are there differences between the Accelerated Failure Time (AFT) Model and the usual Linear Regression Model? If there are differences, what are they?

(c) Discuss two AFT models and their implications for hazard functions.

(d) Surgical treatment of ovarian cancer may be followed by a course of chemotherapy. Consider a clinical trial which involved 26 women with minimal residual disease and who had experienced surgical excision of all tumour masses greater than 2 cm in diameter. Following surgery, the patients were further classified according to whether the residual disease was completely or partially excised. The age of the patient and their performance status were also recorded at the start of the trial. The response variable was the survival time in days following randomization to one or other of the two chemotherapy treatments: cyclophosphamide alone (single) and cyclophosphamide combined with adriamycin (combined). The variables in the data set are therefore as follows:

TIME:	Survival time in days
CENS:	Censoring indicator (0=censored, 1=uncensored)
TREAT:	Treatment (1=single, 2=combined)
AGE:	Age of patient in years
RDISEASE:	Extent of residual disease (1=incomplete, 2=complete)
PERF:	Performance status (1=good, 2=poor)

The data are given in Table 4(d).

(b) *Data yang diberi di bawah adalah masa-masa survival* T_i *(dalam bulan) selepas suatu pembedahan:*

$$8,\ 1+,\ 7,\ 7,\ 4+,\ 5,\ 7+,\ 5,\ 6+,\ 3$$

(i) *Hitung secara manual anggaran Kaplan-Meier bagi survival* $\hat{S}(t)$.

(ii) *Lakar lengkungan Kaplan-Meier dan tandakan data tertapis.*

(iii) *Mengikut anggaran Kaplan-Meier, berapa besar kebarangkalian bahawa seorang pesakit hidup sekurang-kurangnya setengah tahun?*

(c) *Bilakah anda menggunakan kaedah-kaedah tak berparameter atau berparameter untuk menganggar fungsi-fungsi survival dan hazard? Apakah kebaikan dan kekurangan kedua-dua kaedah ini?*

[100 markah]

4. (a) *Apakah sebab-sebab bagi pemodelan data survival? Huraikan Model Bahaya Berkadaran yang merupakan model asas bagi data survival.*

(b) *Adakah terdapat perbezaan antara Model Masa Kegagalan Terpecut (AFT) dan Model Regresi Linear yang biasa? Jika ada, apakah perbezaannya?*

(c) *Bincangkan dua model (AFT) dan implikasinya bagi fungsi hazard.*

(d) *Rawatan surgeri bagi kanser ovari boleh diikuti dengan satu kursus kemoterapi Pertimbangkan suatu cubaan klinikal yang melibatkan 26 orang wanita dengan penyakit reja minimal dan yang mengalami pemotongan surgeri bagi semua tumor yang melebihi garis pusat 2 sentimeter. Selepas pembedahan, pesakit-pesakit dikelaskan lagi mengikut sama ada penyakit reja itu dipotong secara lengkap atau tidak lengkap. Umur pesakit dan prestasinya dicatat pada awal percubaan. Pemboleh ubah sambutan ialah masa survival dalam hari mengikut proses rawakan bagi dua rawatan chemoterapi. Pemboleh ubah dalam set data adalah seperti berikut:*

TIME:	*Masa survival dalam hari*
CENS:	*Petunjuk tapis (0=tertapis, 1=tak tertapis)*
TREAT:	*Rawatan (1=tunggal, 2=kombinasi)*
AGE:	*Umur pesakit dalam tahun*
RDISEASE:	*Sejauh mana penyakit reja merebak (1=tidak lengkap, 2=lengkap)*
PERF:	*Status prestasi (1=baik, 2=lemah)*

Data diberi dalam Jadual 4(d).

Table 4(d). Survival Times of 26 Ovarian Cancer Patients.

Patient	TIME	CENS	TREAT	AGE	RDISEASE	PERF
1	156	1	1	66	2	2
2	1040	0	1	38	2	2
3	59	1	1	72	2	1
4	421	0	2	53	2	1
5	329	1	1	43	2	1
6	769	0	2	59	2	2
7	365	1	2	64	2	1
8	770	0	2	57	2	1
9	1227	0	2	59	1	2
10	268	1	1	74	2	2
11	475	1	2	59	2	2
12	1129	0	2	53	1	1
13	464	1	2	56	2	2
14	1206	0	2	44	2	1
15	638	1	1	56	1	2
16	563	1	2	55	1	2
17	1106	0	1	44	1	1
18	431	1	1	50	2	1
19	855	0	1	43	1	2
20	803	0	1	39	1	1
21	115	1	1	74	2	1
22	744	0	2	50	1	1
23	477	0	1	64	2	1
24	448	0	1	56	1	2
25	353	1	2	63	1	2
26	377	0	2	58	1	1

(i) In modelling the data in Table 4(d), fit a range of models to these data. Which model is most satisfactory? Give your justification.

(ii) Then identify which prognostic factors are associated with the survival times of the patients. What is your conclusion?

(iii) Is there a difference in the effect of the two chemotherapy treatments on the hazard of death? If there is a treatment difference, is it consistent over age?

(iv) Obtain the estimated median survival time for patients of a given age on a given treatment.

[100 marks]

Table 4(d). *Masa Survival bagi 26 Pesakit Kanser Ovari.*

Patient	*TIME*	*CENS*	*TREAT*	*AGE*	*RDISEASE*	*PERF*
1	156	1	1	66	2	2
2	1040	0	1	38	2	2
3	59	1	1	72	2	1
4	421	0	2	53	2	1
5	329	1	1	43	2	1
6	769	0	2	59	2	2
7	365	1	2	64	2	1
8	770	0	2	57	2	1
9	1227	0	2	59	1	2
10	268	1	1	74	2	2
11	475	1	2	59	2	2
12	1129	0	2	53	1	1
13	464	1	2	56	2	2
14	1206	0	2	44	2	1
15	638	1	1	56	1	2
16	563	1	2	55	1	2
17	1106	0	1	44	1	1
18	431	1	1	50	2	1
19	855	0	1	43	1	2
20	803	0	1	39	1	1
21	115	1	1	74	2	1
22	744	0	2	50	1	1
23	477	0	1	64	2	1
24	448	0	1	56	1	2
25	353	1	2	63	1	2
26	377	0	2	58	1	1

(i) *Dalam pemodelan data dalam Jadual 4(d), suaikan beberapa model kepada data ini. Model manakah yang paling memuaskan? Beri justifikasi anda.*

(ii) *Kemudian kenalpasti faktor-faktor prognostik yang berkenaan dengan masa survival bagi pesakit. Apakah kesimpulan anda?*

(iii) *Adakah terdapat perbezaan dalam kesan bagi dua rawatan chemoterapi pada hazard kematian? Jika terdapat perbezaan rawatan, adakah ia konsisten dengan umur?*

(iv) *Dapatkan aggaran median masa survival bagi pesakit diberi umur dan rawatan.*

[100 markah]

APPENDIX

Summary of Reliability Formulae

$$F(t) = \int_0^t f(t)dt$$

$$R(t) = 1 - F(t)$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$h(t) = \frac{f(t)}{R(t)}$$

$$H(t) = \int_0^t h(t)dt$$

$$R(t) = e^{-H(t)}$$

$$H(t) = -\ln R(t)$$

$$MTTF = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt$$

APPENDIX (contd)

Summary of Reliability Formulae (contd)

Lifetime following an **Exponential Distribution:**

$$f(t) = \lambda e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$R(t) = e^{-\lambda t}$$
$$h(t) = \lambda$$
$$H(t) = \lambda t$$
$$MTTF = \frac{1}{\lambda}$$

Lifetime following a **Weibull Distribution:**

$$f(t) = \beta \alpha^{-\beta} t^{\beta - 1} \exp\left[-\left(\frac{t}{\alpha} \right)^{\beta} \right]$$

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha} \right)^{\beta}}$$

$$R(t) = e^{-\left(\frac{t}{\alpha} \right)^{\beta}}$$

$$h(t) = \beta \alpha^{-\beta} t^{\beta - 1}$$

$$H(t) = \left(\frac{t}{\alpha} \right)^{\beta}$$

$$MTTF = \alpha \Gamma\left(1 + \frac{1}{\beta} \right)$$

$$\text{Design Life} = t_R = \alpha(-\ln R)^{1/\beta}$$

APPENDIX (contd)

Summary of Reliability Formulae (contd)

Lifetime following a **Normal Distribution:**

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\frac{(t-\mu)^2}{\sigma^2}\right]$$

$$F(t) = \Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$h(t) = \frac{f(t)}{1-\Phi\left(\frac{t-\mu}{\sigma}\right)}$$

Lifetime following a **Lognormal Distribution:**

$$f(t) = \frac{1}{\sqrt{2\pi}st} \exp\left[-\frac{1}{2s^2}\left(\ln\frac{t}{t_{median}}\right)^2\right]$$

$$F(t) = \Phi\left(\frac{1}{s}\ln\frac{t}{t_{median}}\right)$$

$$R(t) = 1 - \Phi\left(\frac{1}{s}\ln\frac{t}{t_{median}}\right)$$

$$MTTF = t_{median}\exp\left(\frac{s^2}{2}\right)$$

$$t_R = t_{median}\exp(sz_{1-R})$$

TABLE A.9
Gamma function

x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$	x	$\Gamma(x)$
1.01	.99433	1.51	.88659	2.01	1.00427	2.51	1.33875
1.02	.98884	1.52	.88704	2.02	1.00862	2.52	1.34830
1.03	.98355	1.53	.88757	2.03	1.01306	2.53	1.35798
1.04	.97844	1.54	.88818	2.04	1.01758	2.54	1.36779
1.05	.97350	1.55	.88887	2.05	1.02218	2.55	1.37775
1.06	.96874	1.56	.88964	2.06	1.02687	2.56	1.38784
1.07	.96415	1.57	.89049	2.07	1.03164	2.57	1.39807
1.08	.95973	1.58	.89142	2.08	1.03650	2.58	1.40844
1.09	.95546	1.59	.89243	2.09	1.04145	2.59	1.41896
1.10	.95135	1.60	.89352	2.10	1.04649	2.60	1.42962
1.11	.94740	1.61	.89468	2.11	1.05161	2.61	1.44044
1.12	.94359	1.62	.89592	2.12	1.05682	2.62	1.45140
1.13	.93993	1.63	.89724	2.13	1.06212	2.63	1.46251
1.14	.93642	1.64	.89864	2.14	1.06751	2.64	1.47377
1.15	.93304	1.65	.90012	2.15	1.07300	2.65	1.48519
1.16	.92980	1.66	.90167	2.16	1.07857	2.66	1.49677
1.17	.92670	1.67	.90330	2.17	1.08424	2.67	1.50851
1.18	.92373	1.68	.90500	2.18	1.09000	2.68	1.52040
1.19	.92089	1.69	.90678	2.19	1.09585	2.69	1.53246
1.20	.91817	1.70	.90864	2.20	1.10180	2.70	1.54469
1.21	.91558	1.71	.91057	2.21	1.10785	2.71	1.55708
1.22	.91311	1.72	.91258	2.22	1.11399	2.72	1.56964
1.23	.91075	1.73	.91467	2.23	1.12023	2.73	1.58237
1.24	.90852	1.74	.91683	2.24	1.12657	2.74	1.59528
1.25	.90640	1.75	.91906	2.25	1.13300	2.75	1.60836
1.26	.90440	1.76	.92137	2.26	1.13954	2.76	1.62162
1.27	.90250	1.77	.92376	2.27	1.14618	2.77	1.63506
1.28	.90072	1.78	.92623	2.28	1.15292	2.78	1.64868
1.29	.89904	1.79	.92877	2.29	1.15976	2.79	1.66249
1.30	.89747	1.80	.93138	2.30	1.16671	2.80	1.67649
1.31	.89600	1.81	.93408	2.31	1.17377	2.81	1.69068
1.32	.89464	1.82	.93685	2.32	1.18093	2.82	1.70506
1.33	.89338	1.83	.93969	2.33	1.18819	2.83	1.71963
1.34	.89222	1.84	.94261	2.34	1.19557	2.84	1.73441
1.35	.89115	1.85	.94561	2.35	1.20305	2.85	1.74938
1.36	.89018	1.86	.94869	2.36	1.21065	2.86	1.76456
1.37	.88931	1.87	.95184	2.37	1.21836	2.87	1.77994
1.38	.88854	1.88	.95507	2.38	1.22618	2.88	1.79553
1.39	.88785	1.89	.95838	2.39	1.23412	2.89	1.81134
1.40	.88726	1.90	.96177	2.40	1.24217	2.90	1.82736
1.41	.88676	1.91	.96523	2.41	1.25034	2.91	1.84359
1.42	.88636	1.92	.96877	2.42	1.25863	2.92	1.86005
1.43	.88604	1.93	.97240	2.43	1.26703	2.93	1.87673
1.44	.88581	1.94	.97610	2.44	1.27556	2.94	1.89363
1.45	.88566	1.95	.97988	2.45	1.28421	2.95	1.91077
1.46	.88560	1.96	.98374	2.46	1.29298	2.96	1.92814
1.47	.88563	1.97	.98769	2.47	1.30188	2.97	1.94574
1.48	.88575	1.98	.99171	2.48	1.31091	2.98	1.96358
1.49	.88595	1.99	.99581	2.49	1.32006	2.99	1.98167
1.50	.88623	2.00	1	2.50	1.32934	3.00	2

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