UNIVERSITI SAINS MALAYSIA

First Semester Examination
2009/2010 Academic Session

November 2009

MSG 366 – Multivariate Analysis
[Analisis Multivariat]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FOURTY TWO pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT PULUH DUA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

*[**Arahan:** Jawab **semua sepuluh** [10] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. The matrix $\mathbf{A}$ below represents the performance of five students on three questions, where $a_{ij} = 1$ indicates that question i is answered correctly by student j.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) Calculate $\mathbf{A}'\mathbf{A}$ and $\mathbf{A}\mathbf{A}'$.
(b) What do the diagonal elements represent for each matrix in (a)?
(c) What do the off-diagonal elements represent for each matrix in (a)?

[8 marks]

2. Let $\mathbf{x} = (x_1, \ldots, x_n)'$ be a vector containing the number of units purchased of each of a variety of products at a market. Let $\mathbf{y} = (y_1, \ldots, y_n)'$ be a vector of unit prices, such that $y_i =$ the price/unit of item i. Let $\mathbf{T}$ be the total (net) cost of the products in $\mathbf{x}$.

(a) Formulate a matrix expression for the total cost of the products in $\mathbf{x}$.
(b) Suppose that each product is subject to a particular rate of tax, that is being given by a vector, $\mathbf{t} = (t_1, \ldots, t_n)'$. Formulate an expression in terms of matrices and vectors for the total cost of $\mathbf{x}$ including taxes.
[Hint: cost after tax = net cost $\times (1+t)$.]

[8 marks]

3. Let $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4$ be independent $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ random vectors.

(a) Find the mean vector and covariance matrices for each of the two linear combinations of random vectors

$$\mathbf{V}_1 = \tfrac{1}{4}\mathbf{X}_1 - \tfrac{1}{4}\mathbf{X}_2 + \tfrac{1}{4}\mathbf{X}_3 - \tfrac{1}{4}\mathbf{X}_4 \quad \text{and} \quad \mathbf{V}_2 = \tfrac{1}{4}\mathbf{X}_1 + \tfrac{1}{4}\mathbf{X}_2 - \tfrac{1}{4}\mathbf{X}_3 - \tfrac{1}{4}\mathbf{X}_4.$$

(b) State the distribution for each of the random vectors $\mathbf{V}_1$ and $\mathbf{V}_2$.
(c) Write out the the joint density of the random vectors $\mathbf{V}_1$ and $\mathbf{V}_2$.
(d) Are $\mathbf{V}_1$ and $\mathbf{V}_2$ independent?

[24 marks]

1. *Matriks* $\mathbf{A}$ *di bawah mewakili prestasi lima orang pelajar ke atas tiga soalan, yang mana* $a_{ij} = 1$ *menandakan bahawa soalan* i *dijawab dengan betul oleh pelajar* j.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) *Hitung* $\mathbf{A}'\mathbf{A}$ *dan* $\mathbf{A}\mathbf{A}'$.
(b) *Apakah yang diwakili oleh unsur pepenjuru bagi setiap matrik dalam (a)?*
(c) *Apakah yang diwakili oleh unsur luar pepenjuru bagi setiap matrik dalam (a)?*

[8 markah]

2. *Biar* $\mathbf{x} = (x_1, \ldots, x_n)'$ *sebagai suatu vektor yang mengandungi bilangan unit dibeli bagi setiap jenis beberapa produk di pasar. Biar* $\mathbf{y} = (y_1, \ldots, y_n)'$ *sebagai suatu vektor harga seunit, supaya* $y_i =$ *harga/unit barangan* i. *Biar* $\mathbf{T}$ *sebagai jumlah (bersih) kos produk dalam* $\mathbf{x}$.

(a) *Rumuskan suatu ungkapan matriks bagi jumlah kos produk dalam* $\mathbf{x}$.
(b) *Katakan bahawa setiap produk adalah tertakluk kepada suatu kadar cukai, yang diberi oleh suatu vektor,* $\mathbf{t} = (t_1, \ldots, t_n)'$. *Rumuskan suatu ungkapan dalam sebutan matriks dan vektor bagi jumlah kos* $\mathbf{x}$ *termasuk cukai.*
[Petunjuk: kos selepas cukai = kos bersih $\times (1+t)$.*]*

[8 markah]

3. *Biar* $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4$ *sebagai vektor rawak* $N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ *tak bersandar.*

(a) *Cari vektor min dan matriks kovarians bagi setiap satu daripada dua kombinasi linear vektor rawak*

$$\mathbf{V}_1 = \tfrac{1}{4}\mathbf{X}_1 - \tfrac{1}{4}\mathbf{X}_2 + \tfrac{1}{4}\mathbf{X}_3 - \tfrac{1}{4}\mathbf{X}_4 \quad \textit{dan} \quad \mathbf{V}_2 = \tfrac{1}{4}\mathbf{X}_1 + \tfrac{1}{4}\mathbf{X}_2 - \tfrac{1}{4}\mathbf{X}_3 - \tfrac{1}{4}\mathbf{X}_4.$$

(b) *Nyatakan taburan bagi setiap vector rawak* $\mathbf{V}_1$ *dan* $\mathbf{V}_2$.
(c) *Tuliskan ketumpatan tercantum bagi vector rawak* $\mathbf{V}_1$ *dan* $\mathbf{V}_2$.
(d) *Adakah* $\mathbf{V}_1$ *and* $\mathbf{V}_2$ *tak bersandar?*

[24 markah]

4. Let $\mathbf{X}_1, \ldots, \mathbf{X}_{25}$ be a random sample of size $n = 25$ from an $N_8(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ population. Specify the distribution and variance covariance matrix of each of the following.

(a) $\bar{\mathbf{X}}$

(b) $\sqrt{n}(\bar{\mathbf{X}} - \boldsymbol{\mu})$

(c) $(n-1)\mathbf{S}$

(d) $\mathbf{B}[(n-1)\mathbf{S}]\mathbf{B}'$ where $\mathbf{B} = \begin{bmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 1 \end{bmatrix}$.

[20 marks]

5. Write a paragraph on assessing the assumption of normality.

[10 marks]

6. (a) The body of chromaticity specification had specified for a certain level of color temperature that are $x = 0.3804$ and $y = 0.3768$. In 12 color-matching trials, one subject had mean chromaticity values $\bar{x} = 0.3745$ and $\bar{y} = 0.3719$, and sample covariance matrix

$$\mathbf{S} = 10^{-5}\begin{bmatrix} 1.843 & 1.799 \\ 1.799 & 1.836 \end{bmatrix}.$$

Test the null hypothesis that the observations came from the bivariate normal with mean vector of the specified values.

(b) In a similar experiment of (a), two independent observations of another color density had these mean vectors and covariance matrices for $n_1 = n_2 = 12$ observations:

$$\bar{\mathbf{x}}_1 = [0.3781, \quad 0.3755] \qquad \bar{\mathbf{x}}_2 = [0.3772, \quad 0.3750]$$

$$\mathbf{S}_1 = 10^{-6}\begin{bmatrix} 2.64 & 3.18 \\ 3.18 & 11.42 \end{bmatrix} \qquad \mathbf{S}_2 = 10^{-6}\begin{bmatrix} 6.48 & 0.45 \\ 0.45 & 8.26 \end{bmatrix}$$

Obtain the 95% simultaneous confidence intervals for the population differences. Are the population mean vectors of the two observers the same? State the assumptions concerning the data.

(c) It is found that the population covariances in the experiment in (b) are not the same. The researcher then decided to repeat the experiment but with increased sample size, that is, $n = 40$ from each observer. Assuming that the results are the same as in (b), approximate the 95% simultaneous confidence intervals for the population differences. Justify your assumptions in this case.

[30 marks]

4. *Biar* $\mathbf{X}_1,\ldots,\mathbf{X}_{25}$ *sebagai suatu sampel rawak bersaiz* $n=25$ *dari suatu populasi* $N_8(\boldsymbol{\mu},\boldsymbol{\Sigma})$. *Tentukan taburan dan matriks varians kovarians bagi setiap yang berikut.*
   (a) $\bar{\mathbf{X}}$
   (b) $\sqrt{n}(\bar{\mathbf{X}}-\boldsymbol{\mu})$
   (c) $(n-1)\mathbf{S}$
   (d) $\mathbf{B}[(n-1)\mathbf{S}]\mathbf{B}'$ *yang mana* $\mathbf{B}=\begin{bmatrix}1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 & 0\\ 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 1\end{bmatrix}$.

[20 markah]

5. *Tuliskan suatu perenggan mengenai penilaian terhadap andaian kenormalan.*

[10 markah]

6. (a) *Suatu badan penentuan chromaticity telah menentukan bahawa nilai chromaticity terhadap suatu suhu warna pada suatu aras tertentu ialah* $x=0.3804$ *dan* $y=0.3768$. *Dalam 12 cubaan padan-warna, satu subjek mempunyai nilai min 'chromaticity'* $\bar{x}=0.3745$ *dan* $\bar{y}=0.3719$, *dan matriks kovarians sampel*

$$\mathbf{S}=10^{-5}\begin{bmatrix}1.843 & 1.799\\ 1.799 & 1.836\end{bmatrix}.$$

Uji hipotesis nul bahawa cerapan datang dari normal bivariat dengan vektor min nilai yang telah ditentukan.

(b) *Dalam eksperimen yang serupa di (a), dua cerapan tak bersandar bagi ketumpatan warna lain mempunyai vektor min dan matriks kovarians untuk* $n_1=n_2=12$ *cerapan:*

$$\bar{\mathbf{x}}_1=[0.3781,\quad 0.3755] \qquad \bar{\mathbf{x}}_2=[0.3772,\quad 0.3750]$$

$$\mathbf{S}_1=10^{-6}\begin{bmatrix}2.64 & 3.18\\ 3.18 & 11.42\end{bmatrix} \qquad \mathbf{S}_2=10^{-6}\begin{bmatrix}6.48 & 0.45\\ 0.45 & 8.26\end{bmatrix}$$

Dapatkan suatu selang keyakinan serentak 95% bagi beza populasi. Adakah vektor min populasi bagi dua pemerhati sama? Nyatakan andaian mengenai data yang diambil kira.

(c) *Didapati bahawa the kovarians populasi dalam eksperimen di (b) adalah tidak sama. Penyelidik kemudian mengambil keputusan untuk mengulang eksperimen tetapi dengan saiz sampel yang meningkat, iaitu,* $n=40$ *dari setiap pemerhati. Dengan andaian bahawa keputusan adalah sama seperti di (b), anggarkan selang keyakinan serentak 95% bagi beza populasi. Tentusahkan andaian anda dalam kes ini.*

[30 markah]

7. The energy-attenuating qualities of 27 types of football helmets were investigated. Each helmet was struck with forces of known foot-pounds in the front, back, right and left areas until the helmet contacted, or "bottomed", against the head from inside it. The bottoming force in foot-pounds and the resulting acceleration of the helmet in gravitational (G) force are coded. Parts of the data are shown below and the output are shown in **OUTPUT A**.

	Front		Back		Right		Left	
Helmet	**Bottoming Foot-pounds**	G **force**	**Bottoming Foot-pounds**	G **force**	**Bottoming Foot-pounds**	G **force**	**Bottoming Foot-pounds**	G **force**
1	112	400	128	450	128	420	128	420
2	96	425	112	410	96	420	96	410
3	96	410	112	425	64	410	64	410
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
26	160	390	160	390	160	390	160	390
27	192	395	176	380	208	395	192	390

(a) Obtain the 95% simultaneous confidence interval on the mean difference of bottoming foot-pounds in the four areas. Give your conclusion on the equal mean bottoming foot-pounds in the four areas.

(b) Test the hypothesis that the bottoming acceleration mean vectors are equal for the front and back areas.

[30 marks]

8. Suppose that three variables X_1, X_2 and X_3 of an inventory measuring certain maternal attitudes were administered to mothers participating in a study of child development. As part of the investigation each mother had been assigned to one of four socioeconomic status classes. The data of X's and the Minitab output are provided by **OUTPUT B**. Interpret the results and give your conclusions.

[30 marks]

7. *Kualiti tenaga-ringan bagi 27 jenis topi bolasepak telah dikaji. Setiap topi dihentak dengan kuasa yang dikenali sebagai foot-pounds di bahagian hadapan, belakang, kanan dan kiri sehingga topi bersentuh, atau "bottomed", terhadap kepala dari bahagian dalam. Kuasa sentuhan dalam foot-pounds dan pecutan terhasil bagi topi dalam kuasa graviti* (G) *dicatatkan. Sebahagian daripada data ditunjukkan dibawah dan output ditunjukkan di* ***OUTPUT A***.

	Hadapan		Belakang		Kanan		Kiri	
Topi	**Sentuhan Foot-pounds**	**Kuasa G**	**Sentuhan Foot-pounds**	**Kuasa G**	**Sentuhan Foot-pounds**	**Kuasa G**	**Sentuhan Foot-pounds**	**Kuasa G**
1	112	400	128	450	128	420	128	420
2	96	425	112	410	96	420	96	410
3	96	410	112	425	64	410	64	410
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
26	160	390	160	390	160	390	160	390
27	192	395	176	380	208	395	192	390

(a) *Dapatkan selang keyakinan serentak 95% ke atas beza min bagi sentuhan foot-pounds di empat bahagian. Nyatakan kesimpulan anda ke atas kesamaan min sentuhan foot-pounds di empat bahagian.*

(b) *Uji hipotesis bahawa vektor min kelajuan sentuhan adalah sama bagi bahagian depan dan belakang.*

[30 markah]

8. *Andaikan bahawa tiga pemboleh ubah rawak* X_1, X_2 *and* X_3 *bagi suatu inventori yang mengukur sikap keibuan tertentu di ambil kira ke atas ibu-ibu yang mengambil bahagian dalam suatu kajian pembangunan kanak-kanak. Sebagai sebahagian dari penyelidikan, setiap ibu dikelaskan dalam satu dari empat kelas status socioekonomik. Data bagi* X *dan output MINITAB diberi oleh* ***OUTPUT B***. *Tafsirkan keputusan terhasil dan nyatakan kesimpulan anda.*

[30 markah]

9. A study involving samples of a certain wolf skulls from 2 regions was done to investigate geographical and sexual differences in the skull dimensions. These variables (in millimeters) were measured on each skull:

$X_1 =$ palatal length
$X_2 =$ postpalatal length
$X_3 =$ zygomatic width
$X_4 =$ palatal width outside the first upper molars
$X_5 =$ palatal width inside the second upper premolars
$X_6 =$ width between the postglenoid foramina
$X_7 =$ interorbital width
$X_8 =$ least width of the braincase
$X_9 =$ crown length of the first upper molar

The observations for the four regional and gender samples and the output from the discriminant analysis and MANOVA are as in **OUTPUT C**.

(a) Interpret the results and give your conclusion.

(b) Can we perform a cluster analysis on the data? Explain.

[20 marks]

10. The observations for the four regional and gender samples in the above question 9 are referred. The SPSS factor analysis program was used to fit the factor models. The output is as in **OUTPUT D**. Interpret the results.

[20 marks]

9. *Suatu kajian yang melibatkan sampel tengkorak sejenis serigala dari 2 kawasan dijalankan untuk mengkaji perbezaan geografi dan jantina dalam dimensi tengkorak. Pemboleh ubah ini (dalam millimeters) diukur bagi setiap tengkorak:*

$X_1 =$ *panjang palatal*
$X_2 =$ *panjang postpalatal*
$X_3 =$ *lebar zygomatic*
$X_4 =$ *lebar palatal di luar molar atas pertama*
$X_5 =$ *lebar palatal di dalam pramolar atas kedua*
$X_6 =$ *lebar antara postglenoid foramina*
$X_7 =$ *lebar interorbital*
$X_8 =$ *lebar terpendek bagi ruang otak*
$X_9 =$ *panjang crown bagi molar atas pertama*

Cerapan bagi empat kawasan dan sampel jantina dan output dari analisis pembezalayan dan MANOVA adalah seperti dalam ***OUTPUT C****.*

(a) *Tafsirkan keputusan dan beri kesimpulan anda.*

(b) *Bolehkah kita laksanakan suatu analisis kelompok ke atas data ini? Terangkan.*

[20 markah]

10. *Cerapan bagi empat kawasan dan sampel jantina dalam soalan 9 di atas dirujuk. Program analisis faktor SPSS digunakan untuk menyuaikan model faktor. Data dan output adalah seperti dalam* ***OUTPUT D****. Tafsirkan keputusan.*

[20 markah]

APPENDIX / LAMPIRAN

FORMULAE

The notations are as given in the lectures.

1. Suppose $\mathbf{X}$ has $E(\mathbf{X}) = \boldsymbol{\mu}$ and $\text{Cov}(\mathbf{X}) = \boldsymbol{\Sigma}$. Thus $\mathbf{c}'\mathbf{X}$ has mean, $\mathbf{c}'\boldsymbol{\mu}$, and variance, $\mathbf{c}'\boldsymbol{\Sigma}\mathbf{c}$.

2. Bivariate normal p.d.f:

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{\sigma_{11}\sigma_{22}(1-\rho_{12}^2)}} \times \exp\left\{-\frac{1}{2(1-\rho_{12}^2)}\left[\left(\frac{x_1-\mu_1}{\sqrt{\sigma_{11}}}\right)^2 + \left(\frac{x_2-\mu_2}{\sqrt{\sigma_{22}}}\right) - 2\rho_{12}\left(\frac{x_1-\mu_1}{\sqrt{\sigma_{11}}}\right)\left(\frac{x_2-\mu_2}{\sqrt{\sigma_{22}}}\right)\right]\right\}$$

3. Multivariate normal p.d.f:

$$f(\boldsymbol{x}) = \frac{1}{(2\pi)^{p/2}|\boldsymbol{\Sigma}|^{1/2}} \mathbf{e}^{-(1/2)(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$

4. If $\mathbf{X} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then
   (a) $\mathbf{aX} \sim N(\mathbf{a}'\boldsymbol{\mu}, \mathbf{a}'\boldsymbol{\Sigma}\mathbf{a})$
   (b) $\mathbf{AX} \sim N_q(\mathbf{A}\boldsymbol{\mu}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}')$
   (c) $\mathbf{X}+\mathbf{d} \sim N_p(\boldsymbol{\mu}+\mathbf{d}, \boldsymbol{\Sigma})$, $\mathbf{d}$ is a vector of constant
   (d) $(\mathbf{X}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{X}-\boldsymbol{\mu}) \sim \chi_p^2$

5. Let $\mathbf{X}_j \sim N_p(\boldsymbol{\mu}_j, \boldsymbol{\Sigma}),\ j=1,\ldots,n$ be mutually independent. Then $\mathbf{V}_1 = \sum_{j=1}^{n} c_j \mathbf{X}_j \sim N_p\left(\sum_{j=1}^{n} c_i \boldsymbol{\mu}_j, \left(\sum_{j=1}^{n} c_j^2\right)\boldsymbol{\Sigma}\right)$. Moreover, $\mathbf{V}_1$ and $\mathbf{V}_2 = \sum_{j=1}^{n} b_j \mathbf{X}_j$ are jointly multivariate normal with covariance matrix

$$\begin{bmatrix} \left(\sum_{j=1}^{n} c_j^2\right)\boldsymbol{\Sigma} & (\mathbf{b}'\mathbf{c})\boldsymbol{\Sigma} \\ (\mathbf{b}'\mathbf{c})\boldsymbol{\Sigma} & \left(\sum_{j=1}^{n} b_j^2\right)\boldsymbol{\Sigma} \end{bmatrix}.$$

6. If $\mathbf{A}_1 \sim W_{m_1}(\mathbf{A}_1|\boldsymbol{\Sigma})$ independently of $\mathbf{A}_2$, which $\mathbf{A}_2 \sim W_{m_2}(\mathbf{A}_2|\boldsymbol{\Sigma})$, then $\mathbf{A}_1 + \mathbf{A}_2 \sim W_{m_1+m_2}(\mathbf{A}_1 + \mathbf{A}_2|\boldsymbol{\Sigma})$. Also, if $\mathbf{A} \sim W_m(\mathbf{A}|\boldsymbol{\Sigma})$, then

$$\mathbf{CAC}' \sim W_m(\mathbf{CAC}'|\mathbf{C\Sigma C}').$$

7. One-sampel :

(a) $T^2 = n(\bar{\mathbf{X}} - \boldsymbol{\mu})' \mathbf{S}^{-1} (\bar{\mathbf{X}} - \boldsymbol{\mu})$

$$\bar{\mathbf{X}} = \frac{1}{n}\sum_{j=1}^{n} \mathbf{X}_j, \quad \mathbf{S} = \frac{1}{n-1}\sum_{j=1}^{n} (\mathbf{X}_j - \bar{\mathbf{X}})(\mathbf{X}_j - \bar{\mathbf{X}})'$$

$$T^2 \sim \frac{(n-1)p}{(n-p)} F_{p,n-p}$$

(b) $100(1-\alpha)\%$ simultaneous confidence intervals for $\mathbf{a}'\boldsymbol{\mu}$:

$$\mathbf{a}'\bar{\mathbf{X}} \pm \sqrt{\frac{p(n-1)}{n(n-p)} F_{p.n-p}(\alpha)(\mathbf{a}'\mathbf{S}\mathbf{a})}$$

(c) $100(1-\alpha)\%$ Bonferroni confidence interval for $\mu_i,\ i=1,2,\ldots,p$:

$$\bar{x}_i \pm t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{s_{ii}}{n}}$$

(d) $100(1-\alpha)\%$ large sample confidence interval for $\mu_i : i = 1, 2, \ldots p$

$$\bar{x}_i \pm \sqrt{\chi_p^2(\alpha)}\sqrt{\frac{s_{ii}}{n}}$$

8. Paired comparisons

(a) $T^2 = n(\bar{\mathbf{D}}-\boldsymbol{\delta})'\,\boldsymbol{\delta}_d^{-1}(\bar{\mathbf{D}}-\boldsymbol{\delta})$

$$\bar{\mathbf{D}} = \frac{1}{n}\sum_{j=1}^{n}\mathbf{D}_j \qquad \mathbf{S}_d = \frac{1}{n-1}\sum_{j=1}^{n}(\mathbf{D}_j-\bar{\mathbf{D}})(\mathbf{D}_j-\bar{\mathbf{D}})'$$

$$T^2 \sim \left[\frac{(n-1)p}{n-p}\right]F_{p,n-p}$$

(b) $100(1-\alpha)\%$ simultaneous confidence interval for δ_i:

$$\bar{d}_i \pm \sqrt{\frac{(n-1)p}{n-p}F_{p,n-p}(\alpha)}\sqrt{\frac{s_{d_i}^2}{n}}$$

$\bar{d}_i = i^{th}$ element of $\bar{\mathbf{d}}$

$s_{d_i}^2 = i^{th}$ diagonal element of $\mathbf{S}_d$

9. Repeated Measure Design

(a) Let $\mathbf{C}$ be a contrast matrix

$T^2 = n(\mathbf{C}\bar{\mathbf{x}})'\,[\mathbf{CSC}']^{-1}\mathbf{C}\bar{\mathbf{x}}$

$$T^2 \sim \frac{(n-1)(q-1)}{n-q+1}F_{q-1,\,n-q+1}(\alpha)$$

(b) $100(1-\alpha)\%$ simultaneous confidence intevals for $\mathbf{c}'\boldsymbol{\mu}$:

$$\mathbf{c}'\bar{\mathbf{x}} \pm \sqrt{\frac{(n-1)(q-1)}{n-q+1}F_{q-1,n-q+1}(\alpha)}\sqrt{\frac{\mathbf{c}'\mathbf{S}\mathbf{c}}{n}}$$

10. Two independent samples:

(a) $$T^2 = \left[\bar{\mathbf{X}}_1 - \bar{\mathbf{X}}_2 - (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)\right]' \left[\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\mathbf{S}_p\right]^{-1} \left[\bar{\mathbf{X}}_1 - \bar{\mathbf{X}}_2 - (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)\right]$$

$$T^2 \sim \frac{(n_1+n_2-2)p}{n_1+n_2-p-1} F_{p,n_1+n_2-p-1}$$

$$\mathbf{S}_p = \frac{n_i-1}{n_i+n_2-2}\mathbf{S}_1 + \frac{n_2-1}{n_1+n_2-2}\mathbf{S}_2$$

$$\mathbf{S}_i = \frac{\sum_{j=1}^{n_i}(\mathbf{x}_{ij}-\bar{\mathbf{x}}_i)(\mathbf{x}_{ij}-\bar{\mathbf{x}}_i)'}{n_i-1}$$

(b) $100(1-\alpha)\%$ simultaneous confidence intervals for $\mathbf{a}'(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)$:

$$\mathbf{a}'(\bar{\mathbf{X}}_1 - \bar{\mathbf{X}}_2) \pm c\sqrt{\mathbf{a}'\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\mathbf{S}_p\,\mathbf{a}}$$

where $c^2 = \frac{(n_1+n_2-2)p}{n_1+n_2-p-1} F_{p,n_1+n_2-p-1}(\alpha)$

(c) For large $n_1 - p$, and $n_2 - p$, $100(1-\alpha)\%$ simultaneous confidence interval for $\mathbf{a}'(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_2)$:

$$\mathbf{a}'(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2) \pm c\sqrt{\mathbf{a}'\left(\frac{1}{n_1}\mathbf{S}_1+\frac{1}{n_2}\mathbf{S}_2\right)\mathbf{a}}$$

where $c^2 = \chi_p^2(\alpha)$

11. One-way MANOVA:

(a) $$\mathbf{B} = \sum_{\ell=1}^{g} n_\ell (\bar{\mathbf{x}}_\ell - \bar{\mathbf{x}})(\bar{\mathbf{x}}_\ell - \bar{\mathbf{x}})'$$

$$\mathbf{W} = \sum_{\ell=1}^{g}\sum_{j=1}^{n_\ell} (\mathbf{x}_{\ell j} - \bar{\mathbf{x}}_\ell)(\mathbf{x}_{\ell j} - \bar{\mathbf{x}}_\ell)'$$

$$\Lambda^* = \frac{|\mathbf{W}|}{|\mathbf{B}+\mathbf{W}|}$$

(b) Bartlett: If $\sum n_\ell = n$ is large,

$$-\left(n-1-\frac{p+g}{2}\right)\ln\Lambda^* = -\left(n-1-\frac{p+g}{2}\right)\ln\left(\frac{|\mathbf{W}|}{|\mathbf{B}+\mathbf{W}|}\right) \sim \chi^2_{p(g-1)}$$

(c) $100(1-\alpha)\%$ simultaneous confidence intervals for $\tau_{ki}-\tau_{\ell i}$:

$$\bar{x}_{ki}-\bar{x}_{\ell i} \pm t_{n-g}\left(\frac{\alpha}{pg(g-1)}\right)\sqrt{\frac{w_{ii}}{n-g}\left(\frac{1}{n_k}+\frac{1}{n_\ell}\right)}, \quad n=\sum_{\ell=1}^{g} n_\ell$$

$i=1,2,\ldots,p, \quad \ell<k=1,2,\ldots,g$

$w_{ii}=i^{th}$ diagonal element of $\mathbf{W}$.

OUTPUT A

Row	Area	FootPound	G Force
1	Front	112	400
2	Front	96	425
3	Front	96	410
.	.	.	.
.	.	.	.
26	Front	160	390
27	Front	192	395
28	Back	128	450
29	Back	112	410
30	Back	112	425
.	.	.	.
.	.	.	.
53	Back	160	390
54	Back	176	380
55	Right	128	420
56	Right	96	420
57	Right	64	410
.	.	.	.
.	.	.	.
80	Right	160	390
81	Right	208	395
82	Left	128	420
83	Left	96	410
84	Left	64	410
.	.	.	.
.	.	.	.
107	Left	160	390
108	Left	192	390

Descriptive Statistics: FootPound, G Force by Area

Variable	Area	N	Mean	Median	TrMean	StDev
FootPoun	Back	27	146.30	160.00	147.12	24.36
	Front	27	128.15	128.00	128.16	28.25
	Left	27	116.74	112.00	115.84	31.00
	Right	27	118.52	112.00	117.12	34.16
G Force	Back	27	413.70	410.00	413.20	21.29
	Front	27	407.78	400.00	407.80	21.18
	Left	27	403.89	400.00	404.60	16.07
	Right	27	403.89	400.00	404.20	21.63

Variable	Area	SE Mean	Minimum	Maximum	Q1	Q3
FootPoun	Back	4.69	96.00	176.00	128.00	160.00
	Front	5.44	64.00	192.00	112.00	160.00
	Left	5.97	64.00	192.00	96.00	128.00
	Right	6.57	64.00	208.00	96.00	144.00
G Force	Back	4.10	380.00	460.00	400.00	425.00
	Front	4.08	365.00	450.00	395.00	425.00
	Left	3.09	360.00	430.00	395.00	420.00
	Right	4.16	350.00	450.00	390.00	420.00

...16/-

Covariances: Front _Pound, Back_Pound, Right_pound, Left_pound

	Front _P	Back_Pou	Right_po	Left_pou
Front _P	798.131			
Back_Pou	480.877	593.447		
Right_po	609.459	404.148	1166.952	
Left_pou	572.809	418.234	913.140	961.276

Matrix COVA1

798.13	480.88	609.46	572.81
480.88	593.45	404.15	418.23
609.46	404.15	1166.95	913.14
572.81	418.23	913.14	961.28

Matrix InverseCOVA1

0.00318713	-0.00178341	-0.00065433	-0.00050167
-0.00178341	0.00342910	0.00031048	-0.00072417
-0.00065433	0.00031048	0.00347408	-0.00304529
-0.00050167	-0.00072417	-0.00304529	0.00454709

OUTPUT B

General Linear Model

etween-Subjects Factor

		N
CLASS	1	8
	2	5
	3	4
	4	4

Descriptive Statistics

	CLASS	Mean	Std. Deviation	N
X1	1	18.00	1.852	8
	2	13.80	1.304	5
	3	13.00	1.414	4
	4	10.00	1.414	4
	Total	14.52	3.415	21
X2	1	20.00	1.512	8
	2	15.20	1.304	5
	3	14.00	.816	4
	4	9.00	1.414	4
	Total	15.62	4.307	21
X3	1	19.75	1.282	8
	2	14.20	1.924	5
	3	15.00	1.633	4
	4	11.00	1.155	4
	Total	15.86	3.678	21

s Test of Equality of Covariance Matr[a]

Box's M	16.407
F	.845
df1	12
df2	442.402
Sig.	.604

Tests the null hypothesis that the observed covari matrices of the dependent variables are equal ac

a. Design: Intercept+CLASS

Bartlett's Test of Sphericity[a]

Likelihood Ratio	.024
Approx. Chi-Squ	5.570
df	5
Sig.	.351

Tests the null hypothesis that the residual matrix is proportional to an identity matrix

a. Design: Intercept+CLASS

Multivariate Tests[c]

Effect		Value	F	Hypothesis df	Error df	Sig.
Intercep	Pillai's Trace	.996	132.704[a]	3.000	15.000	.000
	Wilks' Lambda	.004	132.704[a]	3.000	15.000	.000
	Hotelling's Trace	226.541	132.704[a]	3.000	15.000	.000
	Roy's Largest Ro	226.541	132.704[a]	3.000	15.000	.000
CLASS	Pillai's Trace	1.161	3.577	9.000	51.000	.002
	Wilks' Lambda	.048	10.121	9.000	36.657	.000
	Hotelling's Trace	15.642	23.752	9.000	41.000	.000
	Roy's Largest Ro	15.375	87.127[b]	3.000	17.000	.000

a. Exact statistic

b. The statistic is an upper bound on F that yields a lower bound on the sign

c. Design: Intercept+CLASS

Levene's Test of Equality of Error Variances[a]

	F	df1	df2	Sig.
X1	.581	3	17	.635
X2	1.081	3	17	.384
X3	.350	3	17	.789

Tests the null hypothesis that the error varianc dependent variable is equal across groups.

a. Design: Intercept+CLASS

Tests of Between-Subjects Effects

Source	Dependent Varia	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Moc	X1	190.438[a]	3	63.479	25.214	.000
	X2	340.152[b]	3	113.384	62.582	.000
	X3	232.271[c]	3	77.424	34.366	.000
Intercept	X1	3640.048	1	3640.048	445.814	.000
	X2	4105.745	1	4105.745	266.158	.000
	X3	4356.367	1	4356.367	933.635	.000
CLASS	X1	190.438	3	63.479	25.214	.000
	X2	340.152	3	113.384	62.582	.000
	X3	232.271	3	77.424	34.366	.000
Error	X1	42.800	17	2.518		
	X2	30.800	17	1.812		
	X3	38.300	17	2.253		
Total	X1	4663.000	21			
	X2	5494.000	21			
	X3	5551.000	21			
Corrected Tot:	X1	233.238	20			
	X2	370.952	20			
	X3	270.571	20			

a. R Squared = .816 (Adjusted R Squared = .784)

b. R Squared = .917 (Adjusted R Squared = .902)

c. R Squared = .858 (Adjusted R Squared = .833)

Between-Subjects SSCP Matrix

			X1	X2	X3
Hypothesis	Intercep	X1	640.048	865.891	982.133
		X2	865.891	105.745	229.200
		X3	982.133	229.200	356.367
	CLASS	X1	190.438	252.990	207.371
		X2	252.990	340.152	274.057
		X3	207.371	274.057	232.271
Error		X1	42.800	11.200	18.200
		X2	11.200	30.800	3.800
		X3	18.200	3.800	38.300

Based on Type III Sum of Squares

Residual SSCP Matrix

		X1	X2	X3
Sum-of-Squares and Cross-Produc	X1	42.800	11.200	18.200
	X2	11.200	30.800	3.800
	X3	18.200	3.800	38.300
Covariance	X1	2.518	.659	1.071
	X2	.659	1.812	.224
	X3	1.071	.224	2.253
Correlation	X1	1.000	.308	.450
	X2	.308	1.000	.111
	X3	.450	.111	1.000

Based on Type III Sum of Squares

Estimated Marginal Means

CLASS

				95% Confidence Interval	
Dependent Varia	CLASS	Mean	Std. Error	Lower Bound	Upper Bound
X1	1	18.000	.561	16.816	19.184
	2	13.800	.710	12.303	15.297
	3	13.000	.793	11.326	14.674
	4	10.000	.793	8.326	11.674
X2	1	20.000	.476	18.996	21.004
	2	15.200	.602	13.930	16.470
	3	14.000	.673	12.580	15.420
	4	9.000	.673	7.580	10.420
X3	1	19.750	.531	18.630	20.870
	2	14.200	.671	12.784	15.616
	3	15.000	.750	13.417	16.583
	4	11.000	.750	9.417	12.583

Post Hoc Tests
CLASS

Multiple Comparisons

Bonferroni

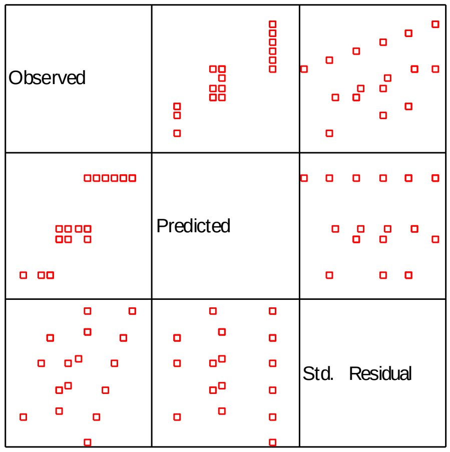
Dependent Varia	(I) CLASS	(J) CLASS	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
X1	1	2	4.20*	.905	.001	1.50	6.90
		3	5.00*	.972	.000	2.10	7.90
		4	8.00*	.972	.000	5.10	10.90
	2	1	-4.20*	.905	.001	-6.90	-1.50
		3	.80	1.064	1.000	-2.38	3.98
		4	3.80*	1.064	.014	.62	6.98
	3	1	-5.00*	.972	.000	-7.90	-2.10
		2	-.80	1.064	1.000	-3.98	2.38
		4	3.00	1.122	.096	-.35	6.35
	4	1	-8.00*	.972	.000	-10.90	-5.10
		2	-3.80*	1.064	.014	-6.98	-.62
		3	-3.00	1.122	.096	-6.35	.35
X2	1	2	4.80*	.767	.000	2.51	7.09
		3	6.00*	.824	.000	3.54	8.46
		4	11.00*	.824	.000	8.54	13.46
	2	1	-4.80*	.767	.000	-7.09	-2.51
		3	1.20	.903	1.000	-1.49	3.89
		4	6.20*	.903	.000	3.51	8.89
	3	1	-6.00*	.824	.000	-8.46	-3.54
		2	-1.20	.903	1.000	-3.89	1.49
		4	5.00*	.952	.000	2.16	7.84
	4	1	-11.00*	.824	.000	-13.46	-8.54
		2	-6.20*	.903	.000	-8.89	-3.51
		3	-5.00*	.952	.000	-7.84	-2.16
X3	1	2	5.55*	.856	.000	3.00	8.10
		3	4.75*	.919	.000	2.01	7.49
		4	8.75*	.919	.000	6.01	11.49
	2	1	-5.55*	.856	.000	-8.10	-3.00
		3	-.80	1.007	1.000	-3.80	2.20
		4	3.20*	1.007	.033	.20	6.20
	3	1	-4.75*	.919	.000	-7.49	-2.01
		2	.80	1.007	1.000	-2.20	3.80
		4	4.00*	1.061	.009	.83	7.17
	4	1	-8.75*	.919	.000	-11.49	-6.01
		2	-3.20*	1.007	.033	-6.20	-.20
		3	-4.00*	1.061	.009	-7.17	-.83

Based on observed means.

*. The mean difference is significant at the .05 level.

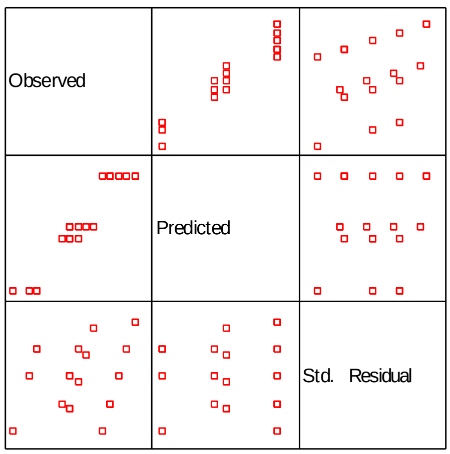
Observed * Predicted * Std. Residual Plots

Dependent Variable: X1

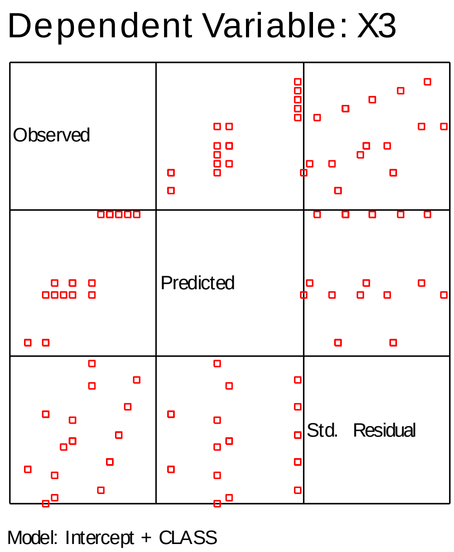


Model: Intercept + CLASS

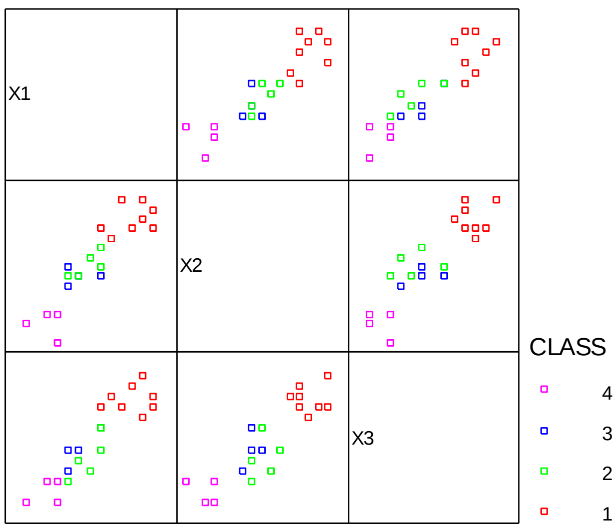
Dependent Variable: X2



Model: Intercept + CLASS


Dependent Variable: X3
Observed
Predicted
Std. Residual
Model: Intercept + CLASS

Graph


X1
X2
X3
CLASS
4
3
2
1

OUTPUT C

Discriminant

Group Statistics

KAWASA		Mean	Std. Deviation	Valid N (listwise) Unweighted	Weighted
Mountain	X1	123.444	4.8247	9	9.000
	X2	95.222	31.7166	9	9.000
	X3	139.667	9.2871	9	9.000
	X4	79.911	3.7106	9	9.000
	X5	32.733	1.5158	9	9.000
	X6	66.100	2.4459	9	9.000
	X7	47.511	3.5044	9	9.000
	X8	42.611	2.1310	9	9.000
	X9	17.756	.6729	9	9.000
Arctic	X1	113.938	3.4150	16	16.000
	X2	99.063	3.7854	16	16.000
	X3	140.375	5.8977	16	16.000
	X4	80.762	2.4383	16	16.000
	X5	33.081	1.9132	16	16.000
	X6	66.194	2.6639	16	16.000
	X7	45.781	2.9208	16	16.000
	X8	39.981	2.6883	16	16.000
	X9	17.756	.6460	16	16.000
Total	X1	117.360	6.0614	25	25.000
	X2	97.680	18.6497	25	25.000
	X3	140.120	7.1141	25	25.000
	X4	80.456	2.9119	25	25.000
	X5	32.956	1.7557	25	25.000
	X6	66.160	2.5361	25	25.000
	X7	46.404	3.1849	25	25.000
	X8	40.928	2.7732	25	25.000
	X9	17.756	.6417	25	25.000

Tests of Equality of Group Means

	Wilks' Lambda	F	df1	df2	Sig.
X1	.410	33.154	1	23	.000
X2	.990	.236	1	23	.631
X3	.998	.055	1	23	.817
X4	.979	.482	1	23	.495
X5	.991	.219	1	23	.644
X6	1.000	.008	1	23	.932
X7	.929	1.752	1	23	.199
X8	.784	6.331	1	23	.019
X9	1.000	.000	1	23	.998

Summary of Canonical Discriminant Functions

Eigenvalues

Function	Eigenvalue	% of Variance	Cumulative %	Canonical Correlation
1	14.594[a]	100.0	100.0	.967

a. First 1 canonical discriminant functions were used in the analysis.

Wilks' Lambda

Test of Function(s)	Wilks' Lambda	Chi-square	df	Sig.
1	.064	50.818	9	.000

ardized Canonical Discriminant Function Coeffi

	Function
	1
X1	2.650
X2	.125
X3	-.544
X4	-1.054
X5	-.421
X6	-.119
X7	.284
X8	.666
X9	-1.376

Structure Matrix

	Function
	1
X1	.314
X8	.137
X7	.072
X4	-.038
X2	-.027
X5	-.026
X3	-.013
X6	-.005
X9	.000

Pooled within-groups correlations between discrim
variables and standardized canonical discriminan
Variables ordered by absolute size of correlation w

nical Discriminant Function Coeffici

	Function
	1
X1	.669
X2	.007
X3	-.075
X4	-.358
X5	-.236
X6	-.046
X7	.091
X8	.266
X9	-2.099
(Constant	-6.807

Unstandardized coeffic

nctions at Group Centroi

	Function
KAWASA	1
Mountain	4.886
Arctic	-2.748

Unstandardized canonical discri
functions evaluated at group m

Classification Statistics

ssification Function Coefficie

	KAWASAN	
	Mountain	Arctic
X1	7.916	2.811
X2	.301	.250
X3	-3.404	-2.832
X4	4.754	7.486
X5	-6.719	-4.918
X6	5.836	6.188
X7	-.672	-1.363
X8	6.149	4.122
X9	8.063	24.090
(Constant	-725.430	-665.304

Fisher's linear discriminant fur

Classification Results[b,c]

		KAWASA	Predicted Group Membership		Total
			Mountain	Arctic	
Original	Count	Mountain	9	0	9
		Arctic	0	16	16
	%	Mountain	100.0	.0	100.0
		Arctic	.0	100.0	100.0
Cross-validate[a]	Count	Mountain	8	1	9
		Arctic	0	16	16
	%	Mountain	88.9	11.1	100.0
		Arctic	.0	100.0	100.0

a. Cross validation is done only for those cases in the analys validation, each case is classified by the functions derive cases other than that case.

b. 100.0% of original grouped cases correctly classified.

c. 96.0% of cross-validated grouped cases correctly classifie

General Linear Model

Multivariate Tests[b]

Effect		Value	F	Hypothesis df	Error df	Sig.
Intercept	Pillai's Trace	1.000	474.611[a]	9.000	13.000	.000
	Wilks' Lambda	.000	474.611[a]	9.000	13.000	.000
	Hotelling's Trac	867.038	474.611[a]	9.000	13.000	.000
	Roy's Largest Ro	867.038	474.611[a]	9.000	13.000	.000
KAWASAN	Pillai's Trace	.934	20.553[a]	9.000	13.000	.000
	Wilks' Lambda	.066	20.553[a]	9.000	13.000	.000
	Hotelling's Trac	14.229	20.553[a]	9.000	13.000	.000
	Roy's Largest Ro	14.229	20.553[a]	9.000	13.000	.000
JANTINA	Pillai's Trace	.872	9.860[a]	9.000	13.000	.000
	Wilks' Lambda	.128	9.860[a]	9.000	13.000	.000
	Hotelling's Trac	6.826	9.860[a]	9.000	13.000	.000
	Roy's Largest Ro	6.826	9.860[a]	9.000	13.000	.000
KAWASAN * JANT	Pillai's Trace	.676	3.009[a]	9.000	13.000	.035
	Wilks' Lambda	.324	3.009[a]	9.000	13.000	.035
	Hotelling's Trac	2.083	3.009[a]	9.000	13.000	.035
	Roy's Largest Ro	2.083	3.009[a]	9.000	13.000	.035

a. Exact statistic

b. Design: Intercept+KAWASAN+JANTINA+KAWASAN * JANTINA

Levene's Test of Equality of Error Variances[a]

	F	df1	df2	Sig.
X1	.770	3	21	.523
X2	5.624	3	21	.005
X3	.295	3	21	.829
X4	.368	3	21	.777
X5	.869	3	21	.473
X6	1.355	3	21	.284
X7	.960	3	21	.430
X8	1.511	3	21	.241
X9	2.384	3	21	.098

Tests the null hypothesis that the error varianc dependent variable is equal across groups.

a. Design: Intercept+KAWASAN+JANTINA+K * JANTINA

Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	X1	781.160[a]	3	260.387	54.355	.000
	X2	414.840[b]	3	138.280	.366	.778
	X3	656.740[c]	3	218.913	8.240	.001
	X4	114.184[d]	3	38.061	8.949	.001
	X5	18.072[e]	3	6.024	2.263	.111
	X6	45.731[f]	3	15.244	2.947	.056
	X7	84.149[g]	3	28.050	3.698	.028
	X8	46.081[h]	3	15.360	2.329	.104
	X9	4.629[i]	3	1.543	6.169	.004
Intercept	X1	288702.893	1	288702.893	60266.011	.000
	X2	199546.023	1	199546.023	528.259	.000
	X3	399217.984	1	399217.984	15027.026	.000
	X4	132340.906	1	132340.906	31115.446	.000
	X5	22217.909	1	22217.909	8345.227	.000
	X6	90096.981	1	90096.981	17417.364	.000
	X7	44564.120	1	44564.120	5874.718	.000
	X8	35334.611	1	35334.611	5358.020	.000
	X9	6473.130	1	6473.130	25879.374	.000
KAWASAN	X1	385.878	1	385.878	80.551	.000
	X2	10.226	1	10.226	.027	.871
	X3	40.419	1	40.419	1.521	.231
	X4	14.176	1	14.176	3.333	.082
	X5	1.920	1	1.920	.721	.405
	X6	.807	1	.807	.156	.697
	X7	2.532	1	2.532	.334	.570
	X8	28.528	1	28.528	4.326	.050
	X9	.102	1	.102	.409	.529
JANTINA	X1	260.545	1	260.545	54.388	.000
	X2	55.675	1	55.675	.147	.705
	X3	625.578	1	625.578	23.547	.000
	X4	107.144	1	107.144	25.191	.000
	X5	17.345	1	17.345	6.515	.019
	X6	45.199	1	45.199	8.738	.008
	X7	33.572	1	33.572	4.426	.048
	X8	4.179	1	4.179	.634	.435
	X9	4.610	1	4.610	18.431	.000
KAWASAN * JANTINA	X1	23.009	1	23.009	4.803	.040
	X2	325.617	1	325.617	.862	.364
	X3	160.709	1	160.709	6.049	.023
	X4	22.681	1	22.681	5.333	.031
	X5	2.049	1	2.049	.770	.390
	X6	1.920	1	1.920	.371	.549
	X7	52.760	1	52.760	6.955	.015
	X8	3.964	1	3.964	.601	.447
	X9	.615	1	.615	2.459	.132
Error	X1	100.600	21	4.790		
	X2	7932.600	21	377.743		
	X3	557.900	21	26.567		
	X4	89.318	21	4.253		
	X5	55.909	21	2.662		
	X6	108.629	21	5.173		
	X7	159.301	21	7.586		
	X8	138.489	21	6.595		
	X9	5.253	21	.250		
Total	X1	345216.000	25			
	X2	246882.000	25			
	X3	492055.000	25			
	X4	162032.700	25			
	X5	27226.430	25			
	X6	109583.000	25			
	X7	54076.730	25			
	X8	42062.100	25			
	X9	7891.770	25			
Corrected Total	X1	881.760	24			
	X2	8347.440	24			
	X3	1214.640	24			
	X4	203.502	24			
	X5	73.982	24			
	X6	154.360	24			
	X7	243.450	24			
	X8	184.570	24			
	X9	9.882	24			

a. R Squared = .886 (Adjusted R Squared = .870)
b. R Squared = .050 (Adjusted R Squared = -.086)
c. R Squared = .541 (Adjusted R Squared = .475)
d. R Squared = .561 (Adjusted R Squared = .498)
e. R Squared = .244 (Adjusted R Squared = .136)
f. R Squared = .296 (Adjusted R Squared = .196)
g. R Squared = .346 (Adjusted R Squared = .252)
h. R Squared = .250 (Adjusted R Squared = .142)
i. R Squared = .468 (Adjusted R Squared = .393)

Estimated Marginal Means

1. KAWASAN

Estimates

Dependent Varia	KAWASA	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
X1	Mountain	121.917	.774	120.307	123.526
	Arctic	113.317	.565	112.141	114.492
X2	Mountain	97.083	6.872	82.793	111.373
	Arctic	98.483	5.018	88.047	108.919
X3	Mountain	136.917	1.822	133.127	140.706
	Arctic	139.700	1.331	136.932	142.468
X4	Mountain	78.808	.729	77.292	80.325
	Arctic	80.457	.532	79.349	81.564
X5	Mountain	32.325	.577	31.125	33.525
	Arctic	32.932	.421	32.056	33.808
X6	Mountain	65.508	.804	63.836	67.181
	Arctic	65.902	.587	64.680	67.123
X7	Mountain	46.558	.974	44.533	48.583
	Arctic	45.862	.711	44.383	47.341
X8	Mountain	42.317	.908	40.429	44.205
	Arctic	39.978	.663	38.599	41.357
X9	Mountain	17.542	.177	17.174	17.909
	Arctic	17.682	.129	17.413	17.950

Pairwise Comparisons

Dependent Varia	(I) KAWASA	(J) KAWASA	Mean Difference (I-J)	Std. Error	Sig.[a]	5% Confidence Interval fo Difference[a]	
						Lower Bound	Upper Bound
X1	Mountain	Arctic	8.600*	.958	.000	6.607	10.593
	Arctic	Mountain	-8.600*	.958	.000	-10.593	-6.607
X2	Mountain	Arctic	-1.400	8.509	.871	-19.095	16.295
	Arctic	Mountain	1.400	8.509	.871	-16.295	19.095
X3	Mountain	Arctic	-2.783	2.257	.231	-7.476	1.909
	Arctic	Mountain	2.783	2.257	.231	-1.909	7.476
X4	Mountain	Arctic	-1.648	.903	.082	-3.526	.229
	Arctic	Mountain	1.648	.903	.082	-.229	3.526
X5	Mountain	Arctic	-.607	.714	.405	-2.092	.879
	Arctic	Mountain	.607	.714	.405	-.879	2.092
X6	Mountain	Arctic	-.393	.996	.697	-2.464	1.677
	Arctic	Mountain	.393	.996	.697	-1.677	2.464
X7	Mountain	Arctic	.697	1.206	.570	-1.811	3.204
	Arctic	Mountain	-.697	1.206	.570	-3.204	1.811
X8	Mountain	Arctic	2.338*	1.124	.050	.000	4.676
	Arctic	Mountain	-2.338*	1.124	.050	-4.676	.000
X9	Mountain	Arctic	-.140	.219	.529	-.595	.315
	Arctic	Mountain	.140	.219	.529	-.315	.595

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

2. JANTINA

Estimates

Dependent Varia	JANTINA	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
X1	Males	121.150	.565	119.975	122.325
	Females	114.083	.774	112.474	115.693
X2	Males	96.150	5.018	85.714	106.586
	Females	99.417	6.872	85.127	113.707
X3	Males	143.783	1.331	141.016	146.551
	Females	132.833	1.822	129.044	136.623
X4	Males	81.898	.532	80.791	83.006
	Females	77.367	.729	75.850	78.883
X5	Males	33.540	.421	32.664	34.416
	Females	31.717	.577	30.517	32.916
X6	Males	67.177	.587	65.955	68.398
	Females	64.233	.804	62.561	65.906
X7	Males	47.478	.711	45.999	48.957
	Females	44.942	.974	42.917	46.967
X8	Males	41.595	.663	40.216	42.974
	Females	40.700	.908	38.812	42.588
X9	Males	18.082	.129	17.813	18.350
	Females	17.142	.177	16.774	17.509

...33/-

Pairwise Comparisons

Dependent Varia	(I) JANTIN	(J) JANTIN	Mean Difference (I-J)	Std. Error	Sig.[a]	5% Confidence Interval fo Difference[a]	
						ower Bound	Jpper Bound
X1	Males	Females	7.067*	.958	.000	5.074	9.059
	Females	Males	-7.067*	.958	.000	-9.059	-5.074
X2	Males	Females	-3.267	8.509	.705	-20.962	14.428
	Females	Males	3.267	8.509	.705	-14.428	20.962
X3	Males	Females	10.950*	2.257	.000	6.257	15.643
	Females	Males	-10.950*	2.257	.000	-15.643	-6.257
X4	Males	Females	4.532*	.903	.000	2.654	6.409
	Females	Males	-4.532*	.903	.000	-6.409	-2.654
X5	Males	Females	1.823*	.714	.019	.338	3.309
	Females	Males	-1.823*	.714	.019	-3.309	-.338
X6	Males	Females	2.943*	.996	.008	.873	5.014
	Females	Males	-2.943*	.996	.008	-5.014	-.873
X7	Males	Females	2.537*	1.206	.048	.029	5.044
	Females	Males	-2.537*	1.206	.048	-5.044	-.029
X8	Males	Females	.895	1.124	.435	-1.443	3.233
	Females	Males	-.895	1.124	.435	-3.233	1.443
X9	Males	Females	.940*	.219	.000	.485	1.395
	Females	Males	-.940*	.219	.000	-1.395	-.485

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

3. KAWASAN * JANTINA

Dependent Varia	KAWASA	JANTINA	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
X1	Mountain	Males	126.500	.894	124.642	128.358
		Females	117.333	1.264	114.705	119.961
	Arctic	Males	115.800	.692	114.361	117.239
		Females	110.833	.894	108.975	112.692
X2	Mountain	Males	91.500	7.935	74.999	108.001
		Females	102.667	11.221	79.331	126.002
	Arctic	Males	100.800	6.146	88.019	113.581
		Females	96.167	7.935	79.666	112.667
X3	Mountain	Males	145.167	2.104	140.791	149.543
		Females	128.667	2.976	122.478	134.855
	Arctic	Males	142.400	1.630	139.010	145.790
		Females	137.000	2.104	132.624	141.376
X4	Mountain	Males	82.117	.842	80.366	83.868
		Females	75.500	1.191	73.024	77.976
	Arctic	Males	81.680	.652	80.324	83.036
		Females	79.233	.842	77.482	80.984
X5	Mountain	Males	33.550	.666	32.165	34.935
		Females	31.100	.942	29.141	33.059
	Arctic	Males	33.530	.516	32.457	34.603
		Females	32.333	.666	30.948	33.719
X6	Mountain	Males	67.283	.929	65.352	69.214
		Females	63.733	1.313	61.003	66.464
	Arctic	Males	67.070	.719	65.574	68.566
		Females	64.733	.929	62.802	66.664
X7	Mountain	Males	49.417	1.124	47.078	51.755
		Females	43.700	1.590	40.393	47.007
	Arctic	Males	45.540	.871	43.729	47.351
		Females	46.183	1.124	43.845	48.522
X8	Mountain	Males	43.200	1.048	41.020	45.380
		Females	41.433	1.483	38.350	44.517
	Arctic	Males	39.990	.812	38.301	41.679
		Females	39.967	1.048	37.786	42.147
X9	Mountain	Males	18.183	.204	17.759	18.608
		Females	16.900	.289	16.300	17.500
	Arctic	Males	17.980	.158	17.651	18.309
		Females	17.383	.204	16.959	17.808

OUTPUT D

Factor Analysis

Correlation Matrix

		X1	X2	X3	X4	X5	X6	X7	X8	X9
Correlation	X1	1.000	-.152	.444	.363	.355	.462	.487	.454	.463
	X2	-.152	1.000	-.218	.092	-.022	-.190	-.322	-.101	-.154
	X3	.444	-.218	1.000	.759	.653	.752	.689	.324	.538
	X4	.363	.092	.759	1.000	.692	.741	.462	.173	.367
	X5	.355	-.022	.653	.692	1.000	.757	.243	.139	.317
	X6	.462	-.190	.752	.741	.757	1.000	.579	.230	.485
	X7	.487	-.322	.689	.462	.243	.579	1.000	.529	.396
	X8	.454	-.101	.324	.173	.139	.230	.529	1.000	.313
	X9	.463	-.154	.538	.367	.317	.485	.396	.313	1.000
Sig. (1-tailed)	X1		.234	.013	.037	.041	.010	.007	.011	.010
	X2	.234		.148	.330	.458	.182	.058	.316	.232
	X3	.013	.148		.000	.000	.000	.000	.057	.003
	X4	.037	.330	.000		.000	.000	.010	.204	.036
	X5	.041	.458	.000	.000		.000	.121	.253	.061
	X6	.010	.182	.000	.000	.000		.001	.135	.007
	X7	.007	.058	.000	.010	.121	.001		.003	.025
	X8	.011	.316	.057	.204	.253	.135	.003		.064
	X9	.010	.232	.003	.036	.061	.007	.025	.064	

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.738
Bartlett's Test of Sphericity	Approx. Chi-Square	110.823
	df	36
	Sig.	.000

Anti-image Matrices

		X1	X2	X3	X4	X5	X6	X7	X8	X9
Anti-image Covariance	X1	.597	.025	.038	-.017	-.057	-.011	-.071	-.143	-.157
	X2	.025	.717	.078	-.188	-.013	.067	.064	-.071	-.016
	X3	.038	.078	.190	-.101	-.086	.022	-.114	.022	-.118
	X4	-.017	-.188	-.101	.267	-.030	-.071	-.001	.035	.045
	X5	-.057	-.013	-.086	-.030	.261	-.136	.143	-.070	.086
	X6	-.011	.067	.022	-.071	-.136	.209	-.098	.069	-.089
	X7	-.071	.064	-.114	-.001	.143	-.098	.264	-.172	.083
	X8	-.143	-.071	.022	.035	-.070	.069	-.172	.609	-.084
	X9	-.157	-.016	-.118	.045	.086	-.089	.083	-.084	.589
Anti-image Correlation	X1	.873[a]	.038	.113	-.043	-.144	-.032	-.180	-.237	-.264
	X2	.038	.461[a]	.211	-.430	-.030	.173	.148	-.107	-.025
	X3	.113	.211	.769[a]	-.447	-.387	.113	-.510	.064	-.352
	X4	-.043	-.430	-.447	.806[a]	-.115	-.299	-.004	.087	.113
	X5	-.144	-.030	-.387	-.115	.665[a]	-.581	.545	-.175	.220
	X6	-.032	.173	.113	-.299	-.581	.774[a]	-.419	.192	-.254
	X7	-.180	.148	-.510	-.004	.545	-.419	.647[a]	-.429	.210
	X8	-.237	-.107	.064	.087	-.175	.192	-.429	.695[a]	-.141
	X9	-.264	-.025	-.352	.113	.220	-.254	.210	-.141	.765[a]

a. Measures of Sampling Adequacy (MSA)

Communalities

	Initial	Extraction
X1	1.000	.527
X2	1.000	.356
X3	1.000	.822
X4	1.000	.831
X5	1.000	.772
X6	1.000	.826
X7	1.000	.720
X8	1.000	.543
X9	1.000	.447

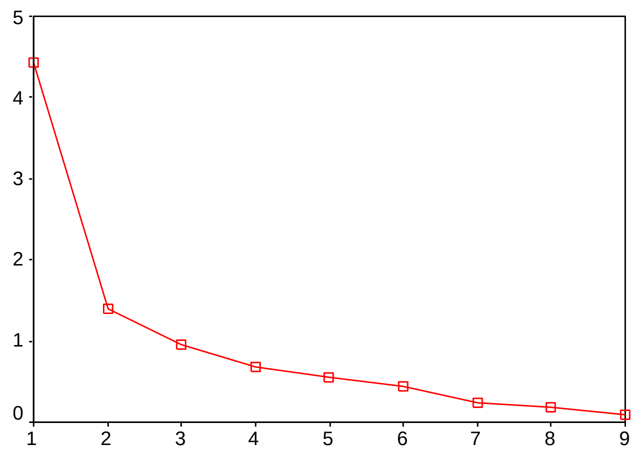
Extraction Method: Principal Compone

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	4.439	49.317	49.317	4.439	49.317	49.317	3.520	39.115	39.115
2	1.405	15.613	64.930	1.405	15.613	64.930	2.323	25.815	64.930
3	.961	10.682	75.612						
4	.683	7.586	83.198						
5	.557	6.192	89.391						
6	.441	4.896	94.287						
7	.236	2.623	96.910						
8	.184	2.046	98.956						
9	.094	1.044	100.000						

Extraction Method: Principal Component Analysis.

Scree Plot



Component Number

Component Matrix[a]

	Component 1	Component 2
X3	.902	
X6	.879	
X4	.792	.452
X7	.750	
X5	.726	.495
X1	.660	
X9	.646	
X8	.481	-.558
X2		.550

Extraction Method: Principal Compone

a. 2 components extracted.

Rotated Component Matrix[a]

	Component 1	Component 2
X4	.910	
X5	.878	
X6	.861	
X3	.805	.417
X7	.408	.744
X8		.731
X1		.615
X2		-.587
X9	.443	.501

Extraction Method: Principal Component
Rotation Method: Varimax with Kaiser No

a. Rotation converged in 3 iterations.

General Linear Model

Between-Subjects Factors

		Value Label	N
KAWASA	1	Mountain	9
	2	Arctic	16
JANTINA	1	Males	16
	2	Females	9

Descriptive Statistics

	KAWASA	JANTINA	Mean	Std. Deviation	N
REGR factor score 1 for analysis 1	Mountain	Males	4263832	.60444274	6
		Females	1.51748	.43868540	3
		Total	2215724	1.10503842	9
	Arctic	Males	4960125	.78803160	10
		Females	4943289	.92454075	6
		Total	1246345	.95011340	16
	Total	Males	4699015	.70398296	16
		Females	3353804	.91872589	9
		Total	0000000	1.00000000	25
REGR factor score 2 for analysis 1	Mountain	Males	1265140	1.10327886	6
		Females	1055985	.32091582	3
		Total	7158098	1.07983203	9
	Arctic	Males	4149991	.76545841	10
		Females	3820496	.66164876	6
		Total	4026430	.70551664	16
	Total	Males	1630683	1.16248143	16
		Females	2898992	.56432700	9
		Total	0000000	1.00000000	25

x's Test of Equality of Covariance Matrices[a]

Box's M	9.216
F	.768
df1	9
df2	505.253
Sig.	.646

Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across g

a. Design: Intercept+KAWASAN+JANTINA+KAWASA JANTINA

Bartlett's Test of Sphericity[a]

Likelihood Ratio	.601
Approx. Chi-Squ	.813
df	2
Sig.	.666

Tests the null hypothesis that the residual cova matrix is proportional to an identity matrix.

a. Design: Intercept+KAWASAN+JANTINA+K * JANTINA

Multivariate Tests[c]

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	Pillai's Trace	.114	1.282[b]	2.000	20.000	.299	.114	2.564	.246
	Wilks' Lambda	.886	1.282[b]	2.000	20.000	.299	.114	2.564	.246
	Hotelling's Trace	.128	1.282[b]	2.000	20.000	.299	.114	2.564	.246
	Roy's Largest Ro	.128	1.282[b]	2.000	20.000	.299	.114	2.564	.246
KAWASAN	Pillai's Trace	.275	3.800[b]	2.000	20.000	.040	.275	7.599	.623
	Wilks' Lambda	.725	3.800[b]	2.000	20.000	.040	.275	7.599	.623
	Hotelling's Trace	.380	3.800[b]	2.000	20.000	.040	.275	7.599	.623
	Roy's Largest Ro	.380	3.800[b]	2.000	20.000	.040	.275	7.599	.623
JANTINA	Pillai's Trace	.555	12.457[b]	2.000	20.000	.000	.555	24.914	.990
	Wilks' Lambda	.445	12.457[b]	2.000	20.000	.000	.555	24.914	.990
	Hotelling's Trace	1.246	12.457[b]	2.000	20.000	.000	.555	24.914	.990
	Roy's Largest Ro	1.246	12.457[b]	2.000	20.000	.000	.555	24.914	.990
KAWASAN * JANT	Pillai's Trace	.235	3.072[b]	2.000	20.000	.069	.235	6.143	.527
	Wilks' Lambda	.765	3.072[b]	2.000	20.000	.069	.235	6.143	.527
	Hotelling's Trace	.307	3.072[b]	2.000	20.000	.069	.235	6.143	.527
	Roy's Largest Ro	.307	3.072[b]	2.000	20.000	.069	.235	6.143	.527

a. Computed using alpha = .05

b. Exact statistic

c. Design: Intercept+KAWASAN+JANTINA+KAWASAN * JANTINA

Levene's Test of Equality of Error Variances[a]

	F	df1	df2	Sig.
REGR factor sco 1 for analysis	.404	3	21	.752
REGR factor sco 2 for analysis	1.092	3	21	.374

Tests the null hypothesis that the error variance of the d variable is equal across groups.

a. Design: Intercept+KAWASAN+JANTINA+KAWASAN

Tests of Between-Subjects Effects

Source	Dependent Variab	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Corrected Model	REGR factor score 1 for analysis 2	11.926[b]	3	3.975	6.914	.002	.497	20.741	.950
	REGR factor score 2 for analysis 2	10.246[c]	3	3.415	5.214	.008	.427	15.643	.869
Intercept	REGR factor score 1 for analysis 2	1.548	1	1.548	2.692	.116	.114	2.692	.347
	REGR factor score 2 for analysis 2	.065	1	.065	.100	.755	.005	.100	.060
KAWASAN	REGR factor score 1 for analysis 2	1.558	1	1.558	2.709	.115	.114	2.709	.349
	REGR factor score 2 for analysis 2	4.311	1	4.311	6.582	.018	.239	6.582	.687
JANTINA	REGR factor score 1 for analysis 2	11.230	1	11.230	19.531	.000	.482	19.531	.988
	REGR factor score 2 for analysis 2	1.876	1	1.876	2.864	.105	.120	2.864	.365
KAWASAN * JANT	REGR factor score 1 for analysis 2	1.186	1	1.186	2.063	.166	.089	2.063	.278
	REGR factor score 2 for analysis 2	2.087	1	2.087	3.187	.089	.132	3.187	.399
Error	REGR factor score 1 for analysis 2	12.074	21	.575					
	REGR factor score 2 for analysis 2	13.754	21	.655					
Total	REGR factor score 1 for analysis 2	24.000	25						
	REGR factor score 2 for analysis 2	24.000	25						
Corrected Total	REGR factor score 1 for analysis 2	24.000	24						
	REGR factor score 2 for analysis 2	24.000	24						

a. Computed using alpha = .05

b. R Squared = .497 (Adjusted R Squared = .425)

c. R Squared = .427 (Adjusted R Squared = .345)

Between-Subjects SSCP Matrix

			REGR factor score 1 for analysis 2	REGR factor score 2 for analysis 2
Hypothesis	Intercept	REGR factor score 1 for analysis	1.548	-.318
		REGR factor score 2 for analysis	-.318	.065
	KAWASAN	REGR factor score 1 for analysis	1.558	-2.591
		REGR factor score 2 for analysis	-2.591	4.311
	JANTINA	REGR factor score 1 for analysis	11.230	4.589
		REGR factor score 2 for analysis	4.589	1.876
	KAWASAN * JANT	REGR factor score 1 for analysis	1.186	1.573
		REGR factor score 2 for analysis	1.573	2.087
Error		REGR factor score 1 for analysis	12.074	-2.437
		REGR factor score 2 for analysis	-2.437	13.754

Based on Type III Sum of Squares

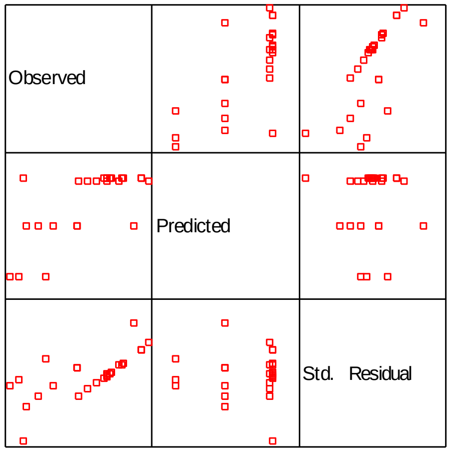
Residual SSCP Matrix

		REGR factor score 1 for analysis 2	REGR factor score 2 for analysis 2
Sum-of-Squares and Cross-Produc	REGR factor score 1 for analysis	12.074	-2.437
	REGR factor score 2 for analysis	-2.437	13.754
Covariance	REGR factor score 1 for analysis	.575	-.116
	REGR factor score 2 for analysis	-.116	.655
Correlation	REGR factor score 1 for analysis	1.000	-.189
	REGR factor score 2 for analysis	-.189	1.000

Based on Type III Sum of Squares

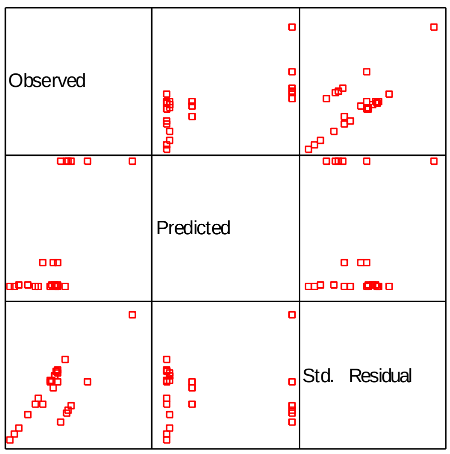
Observed * Predicted * Std. Residual Plots

Dependent Variable: REGR factor s



Model: Intercept + KAWASAN + JANTINA + KAWASAN*JANTINA

Dependent Variable: REGR factor s



Model: Intercept + KAWASAN + JANTINA + KAWASAN*JANTINA

- ooo O ooo -