UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang 1992/93

Oktober/November 1992

MSG 466 Analisis Multivariat

Masa : [3 jam]

Jawab **SEMUA** soalan; semua soalan mesti dijawab dalam Bahasa Malaysia.
Terdapat **4** soalan.

1. (a) Diberikan matriks data

$$\underset{\sim}{\mathbf{X}} = \begin{pmatrix} 1 & 4 & 4 \\ 2 & 1 & 0 \\ 5 & 6 & 4 \end{pmatrix}$$

dan gabungan-gabungan linear

$$\underset{\sim}{\mathbf{b}}'\underset{\sim}{\mathbf{X}} = (1 \quad 1 \quad 1)\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}, \text{ dan}$$

$$\underset{\sim}{\mathbf{c}}'\underset{\sim}{\mathbf{X}} = (1 \quad 2 \quad -3)\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}.$$

Nilaikan min, varians dan kovarians sampel bagi $\underset{\sim}{\mathbf{b}}'\underset{\sim}{\mathbf{X}}$ dan $\underset{\sim}{\mathbf{c}}'\underset{\sim}{\mathbf{X}}$.

(b) Cari anggaran kebolehjadian maksimum bagi vektor min $\underset{\sim}{\mu}$ dan matriks kovarians $\underset{\sim}{\Sigma}$ berdasarkan sampel rawak

$$\underset{\sim}{\mathbf{X}} = \begin{pmatrix} 3 & 4 & 5 & 4 \\ 6 & 4 & 7 & 7 \end{pmatrix}$$

dari suatu populasi normal bivariat.

...2/-

(c) Katakan $\underset{\sim}{X}$ tertabur $N_3(\underset{\sim}{\mu}, \underset{\sim}{\Sigma})$ dengan $\underset{\sim}{\mu}' = (2, -3, 1)$ dan $\underset{\sim}{\Sigma} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$

(i) Carikan taburan $3X_1 - 2X_2 + X_3$.

(ii) Labelkan semula pembolehubah, jika diperlukan, dan carikan suatu vektor 2 x 1 $\underset{\sim}{a}$ supaya X_2 dan $X_2 - \underset{\sim}{a}' \begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ adalah tak bersandar.

(d) (i) Nyatakan bentuk statistik Hotelling T^2 untuk menguji hipotesis null bahawa vektor min $\underset{\sim}{\mu}$ sama dengan suatu vektor $\underset{\sim}{\mu}_0$ yang berdasarkan suatu sampel yang terdiri daripada n cerapan dari taburan $N(\underset{\sim}{\mu}, \underset{\sim}{\Sigma})$.

Berikan fungsi T^2 yang mempunyai taburan-F di bawah hipotesis null dan nyatakan darjah kebebasan terlibat.

(ii) T^2 dapat dihitung sebagai

$$T^2 = \frac{|\underset{\sim}{S} + n(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu}_0)(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu}_0)'|}{|\underset{\sim}{S}|} - 1$$

di mana $\underset{\sim}{\bar{X}}$ dan $\underset{\sim}{S}$ ialah penganggar saksama bagi $\underset{\sim}{\mu}$ dan $\underset{\sim}{\Sigma}$, masing-masingnya.

Prestasi (markah dari jumlah 100) yang dicapai oleh satu kumpulan 10 pelajar di dalam tiga peperiksaan ijazah yang berlainan memberi suatu vektor min $\underset{\sim}{\bar{X}}' = (54, 48, 56)$ dan matriks kovarians

$$\underset{\sim}{S} = \begin{pmatrix} 36 & 4 & 16 \\ 4 & 20 & 12 \\ 16 & 12 & 24 \end{pmatrix}$$

Andaikan normaliti trivariat dan ujikan hipotesis bahawa populasi min $\underset{\sim}{\mu}' = (50, 50, 50)$ melawan hipotesis alternatif bahawa $\underset{\sim}{\mu}' \neq (50, 50, 50)$.

...3/-

(iii) Jika keputusan-keputusan pada tiga peperiksaan yang sama diambil oleh kumpulan pelajar yang lain tersedia, nyatakan bagaimanakah anda dapat menguji hipotesis bahawa min-min bagi dua taburan normal trivariat adalah sama. Berikan juga anggapan-anggapan yang anda fikirkan perlu dibuat.

(100/100)

2. (a) Tulis matriks yang sepadan dengan bentuk kuadratik

$$2X_1^2 - 2X_1X_2 + X_2^2 + 4X_1X_3 - 3X_3^2 .$$

Kemudian, tentukan sama ada matriks itu tentu positif.

(b) Katakan $\underset{\sim}{X}_1, \underset{\sim}{X}_2, \underset{\sim}{X}_3$ dan $\underset{\sim}{X}_4$ adalah vektor-vektor rawak tak bersandar yang tertabur $N_p(\underset{\sim}{\mu}, \underset{\sim}{\Sigma})$.

(i) Cari taburan sut bagi vektor-vektor rawak

$$\underset{\sim}{V}_1 = -\frac{1}{4}\underset{\sim}{X}_1 + \frac{1}{4}\underset{\sim}{X}_2 - \frac{1}{4}\underset{\sim}{X}_3 + \frac{1}{4}\underset{\sim}{X}_4 \quad \text{dan}$$

$$\underset{\sim}{V}_2 = \frac{1}{4}\underset{\sim}{X}_1 - \frac{1}{4}\underset{\sim}{X}_2 - \frac{1}{4}\underset{\sim}{X}_3 + \frac{1}{4}\underset{\sim}{X}_4 .$$

(ii) Cari fungsi ketumpatan tercantum bagi $(\underset{\sim}{V}_1, \underset{\sim}{V}_2)$ dengan $\underset{\sim}{V}_1, \underset{\sim}{V}_2$ tertakrif seperti di dalam (i).

(c) H. Bumpus (1898) mengkaji morfologi burung-burung pipit yang dikutip selepas suatu angin ribut yang kencang. Beliau mengambil 8 sukatan morfologi pada setiap burung dan juga menimbang burung tersebut. Di sini, kita hanya mempertimbangkan 5 pembolehubah bagi burung betina sahaja. Pembolehubah-pembolehubah yang berkaitan ialah:

X_1 = jumlah panjang (total length)
X_2 = "alar extent"
X_3 = "length of beak and head"
X_4 = "length of humerus"
X_5 = "length of keel of sternum" .

Semua sukatan adalah di dalam mm. Terdapat 21 burung yang terus hidup (Kumpulan 1) dan 28 yang mati (Kumpulan 2).

...4/-

Statistik-statistik ringkas adalah seperti berikut:

Bagi burung yang terus hidup:

$$\underset{\sim}{\bar{X}}_1 = \begin{pmatrix} 157.381 \\ 241.000 \\ 31.433 \\ 18.500 \\ 20.810 \end{pmatrix}, \quad \underset{\sim}{S}_1^2 = \begin{pmatrix} 11.048 & 9.100 & 1.557 & 0.870 & 1.286 \\ 9.100 & 17.500 & 1.910 & 1.310 & 0.880 \\ 1.557 & 1.910 & 0.531 & 0.189 & 0.240 \\ 0.870 & 1.310 & 0.189 & 0.176 & 0.133 \\ 1.286 & 0.880 & 0.240 & 0.133 & 0.575 \end{pmatrix}$$

$n_1 = 21$

Bagi burung yang mati:

$$\underset{\sim}{\bar{X}}_2 = \begin{pmatrix} 158.429 \\ 241.571 \\ 31.479 \\ 18.446 \\ 20.839 \end{pmatrix}, \quad \underset{\sim}{S}_2^2 = \begin{pmatrix} 15.069 & 17.190 & 2.243 & 1.746 & 2.931 \\ 17.190 & 32.550 & 3.398 & 2.950 & 4.066 \\ 2.243 & 3.398 & 0.728 & 0.470 & 0.559 \\ 1.746 & 2.950 & 0.470 & 0.434 & 0.506 \\ 2.931 & 4.066 & 0.559 & 0.506 & 1.321 \end{pmatrix}$$

$n_2 = 28$

(i) Dapatkan matriks kovarians sampel tergembleng, $\underset{\sim}{S}_p$.

(ii) Dengan mengandaikan songsang S_p adalah:

$$\underset{\sim}{S}_p^{-1} = \begin{pmatrix} 0.2061 & -0.0694 & -0.2395 & 0.0785 & -0.1969 \\ -0.0694 & 0.1234 & -0.0376 & -0.5517 & 0.0277 \\ -0.2395 & -0.0376 & 4.2219 & -3.2624 & -0.0181 \\ 0.0785 & -0.5577 & -3.2624 & 11.4610 & -1.2720 \\ -0.1969 & 0.0277 & -0.0181 & -1.2720 & 1.8068 \end{pmatrix},$$

Ujikan H_0: $\underset{\sim}{\mu}_1 = \underset{\sim}{\mu}_2$ berlawan H_1: $\underset{\sim}{\mu}_1 \neq \underset{\sim}{\mu}_2$ pada aras keertian $\alpha = .05$.

Tulis kesimpulan anda.
Juga, berikan anggapan-anggapan yang telah dibuat.

(100/100)

3. Bagi setiap bahagian yang berikut, tulis suatu perenggan yang menghuraikan kesimpulan-kesimpulan anda. Output-output bagi setiap bahagian dilampirkan pada akhir soalan ini.

(a) Data yang memberi peratusan tenaga buruh di dalam sembilan jenis industri berbeza untuk 26 negara di Europa dikaji.

Mula-mula, suatu analisis komponen prinsipal dijalankan dengan pakej SAS dengan menggunakan prosedur PROC PRINCOMP.

...5/-

(b) Data di dalam soalan (a) kemudiannya dianalisiskan pula melalui suatu analisis faktor dengan menggunakan prosedur SAS, PROC FACTOR.

(c) Sukatan-sukatan dibuat pada tengkorak orang lelaki negara Egypt daripada kawasan bandar Thebes. Terdapat lima sampel yang terdiri daripada 30 tengkorak daripada setiap zaman, iaitu dari zaman "early predynastic (circa 4000 BC)", zaman "late predynastic (circa 3300 BC)", zaman "12th and 13th dynasties (circa 1850 BC)", zaman "Ptolemaic (circa 200 BC)", dan zaman "Roman (circa AD150)".

Empat sukatan adalah tersedia bagi setiap tengkorak, iaitu,

X_1 = "maximun breadth"
X_2 = "basibregmatic height"
X_3 = "basialveolar length"
dan X_4 = "nasal height".

Semua sukatan adalah dalam mm.
Data tersebut dianalisiskan melalui prosedur, PROC DISCRIM, daripada pakej SAS.

(d) Penggalian tempat-tempat pra-sejarah di timor utara negara Thailand telah mengeluarkan suatu siri tulang anjing yang meliputi suatu kala daripada kira-kira 3500 BC ke sehingga masa kini. Keturunan anjing pra-sejarah tidak diketahui dengan pastinya. Mungkin anjing tersebut diturunkan daripada "golden jackal" (Canis au rens) atau serigala. Tetapi, serigala tidak berasal dari Thailand, dan punca-punca asli terdekat adalah barat negara China (Canis lupus chanco) atau Subbenua India (Canis lupus pallipes). Untuk mengkelaskan keturunan anjing pra-sejarah, sukatan-sukatan rahang (mandible) dibuat pada spesimen-spesimen yang tersedia. Sukatan-sukatan ini kemudian dibandingkan dengan sukatan-sukatan yang sepadan pada "golden jackal", serigala Cina dan serigala India. Perbandingan-perbandingan dijadi lebih berguna dengan mempertimbangkan juga anjing "dingo", yang mungkin diasalkan dari India, anjing "cuon" (Cuon alpinus) yang asli di Asia Tenggara, dan anjing kampung moden dari Thailand.

Enam sukatan rahang itu ialah:

X_1 = "breadth of mandible"
X_2 = "height of mandible below 1st molar"
X_3 = "length of 1st molar"
X_4 = "breadth of 1st molar"
X_5 = "length from 1st to 3rd molars inclusive"
X_6 = "length from 1st to 4th premolars inclusive".

Daripada data asal, untuk semua spesi, matriks jarak diperolehi dan kemudian prosedur-prosedur, PROC CLUSTER dan PROC TREE, daripada SAS dijalankan.

(100/100)

...6/-

Output bagi Soalan 3(a)

Principal Component Analysis

26 Observations
9 Variables

Simple Statistics

	AGR	MIN	MAN	PS	CON
Mean	19.13076923	1.253846154	27.00769231	0.9076923077	8.165384615
Std	15.54656925	0.970043615	7.00776273	0.3762159773	1.645586171

	SER	FIN	SPS	TC
Mean	12.95769231	4.000000000	20.02307692	6.546153846
Std	4.57525283	2.806563735	6.82954216	1.391468510

Principal Component Analysis

Covariance Matrix

	AGR	MIN	MAN	
AGR	241.6958154	0.5398769	-73.1138462	agriculture
MIN	0.5398769	0.9409846	3.0263692	mining
MAN	-73.1138462	3.0263692	49.1087385	manufacturing
PS	-2.3398462	0.1479692	1.0159385	power supplies
CON	-13.7720923	-0.0408615	5.7022769	construction
SER	-52.4210462	-1.7600308	6.5351385	service industries
FIN	-9.5920000	-1.2052000	-3.0648000	finance
SPS	-79.2911385	-1.8616923	7.3786154	social and personal services
TC	-12.2206769	0.2114154	3.4196308	transport and communications

Principal Component Analysis

Covariance Matrix

	PS	CON	SER	
AGR	-2.3398462	-13.7720923	-52.4210462	agriculture
MIN	0.1479692	-0.0408615	-1.7600308	mining
MAN	1.0159385	5.7022769	6.5351385	manufacturing
PS	0.1415385	0.0370769	0.3475385	power supplies
CON	0.0370769	2.7079538	2.6804769	construction
SER	0.3475385	2.6804769	20.9329385	service industries
FIN	0.1160000	0.0752000	4.6940000	finance
SPS	0.3402154	1.7784308	17.8786154	social and personal services
TC	0.1964308	0.8876615	1.1940308	transport and communications

...7/-

Principal Component Analysis

Covariance Matrix

	FIN	SPS	TC	
AGR	-9.5920000	-79.2911385	-12.2206769	agriculture
MIN	-1.2052000	-1.8616923	0.2114154	mining
MAN	-3.0648000	7.3786154	3.4196308	manufacturing
PS	0.1160000	0.3402154	0.1964308	power supplies
CON	0.0752000	1.7784308	0.8876615	construction
SER	4.6940000	17.8786154	1.1940308	service industries
FIN	7.8768000	2.0632000	-0.9604000	finance
SPS	2.0632000	46.6426462	5.3964923	social and personal services
TC	-0.9604000	5.3964923	1.9361846	transport and communications

Principal Component Analysis

Total Variance = 371.9836

Eigenvalues of the Covariance Matrix

	Eigenvalue	Difference	Proportion	Cumulative
PRIN1	303.458	259.756	0.815784	0.81578
PRIN2	43.702	28.494	0.117483	0.93327
PRIN3	15.207	9.568	0.040882	0.97415
PRIN4	5.639	3.196	0.015160	0.98931
PRIN5	2.443	1.397	0.006569	0.99588
PRIN6	1.046	0.625	0.002812	0.99869
PRIN7	0.421	0.356	0.001131	0.99982
PRIN8	0.065	0.063	0.000175	0.99999
PRIN9	0.002		0.000005	1.00000

Principal Component Analysis

Eigenvectors

	PRIN1	PRIN2	PRIN3	
AGR	-.891758	0.006827	0.118467	agriculture
MIN	-.001923	-.092347	0.079379	mining
MAN	0.271271	-.770269	0.184679	manufacturing
PS	0.008388	-.012016	-.006768	power supplies
CON	0.049594	-.068989	-.077313	construction
SER	0.191798	0.234417	-.579613	service industries
FIN	0.031129	0.130082	-.469970	finance
SPS	0.298046	0.566777	0.597745	social and personal services
TC	0.045364	0.009888	0.159415	transport and communications

...8/-

Principal Component Analysis

Eigenvectors

	PRIN4	PRIN5	PRIN6	
AGR	-.096767	-.180044	0.152626	agriculture
MIN	-.010156	0.001122	-.456361	mining
MAN	-.010401	-.336000	0.200931	manufacturing
PS	0.018142	0.002460	-.230864	power supplies
CON	-.082926	0.724262	0.558357	construction
SER	-.607609	-.265863	0.021572	service industries
FIN	0.781193	-.121062	0.055282	finance
SPS	0.048337	-.235916	0.247861	social and personal services
TC	-.037835	0.434890	-.545939	transport and communications

Principal Component Analysis

Eigenvectors

	PRIN7	PRIN8	PRIN9	
AGR	-.091621	0.068678	0.335411	agriculture
MIN	0.766470	0.290464	0.323961	mining
MAN	-.161983	0.074118	0.337463	manufacturing
PS	0.062937	-.909183	0.339898	power supplies
CON	0.194295	-.004458	0.325327	construction
SER	-.087935	0.104436	0.336653	service industries
FIN	-.079977	0.122755	0.334362	finance
SPS	-.004544	0.052137	0.332364	social and personal services
TC	-.567476	0.223814	0.334215	transport and communications

...9/-

Output bagi Soalan 3(b):

Initial Factor Method: Principal Components

Prior Communality Estimates: ONE

Eigenvalues of the Correlation Matrix: Total = 9 Average = 1

	1	2	3	4	5
Eigenvalue	3.487151	2.130173	1.098958	0.994483	0.543218
Difference	1.356978	1.031216	0.104475	0.451265	0.159790
Proportion	0.3875	0.2367	0.1221	0.1105	0.0604
Cumulative	0.3875	0.6241	0.7463	0.8568	0.9171

	6	7	8	9
Eigenvalue	0.383428	0.225754	0.136790	0.000046
Difference	0.157674	0.088964	0.136744	
Proportion	0.0426	0.0251	0.0152	0.0000
Cumulative	0.9597	0.9848	1.0000	1.0000

3 factors will be retained by the MINEIGEN criterion.

Initial Factor Method: Principal Components

Factor Pattern

	FACTOR1	FACTOR2	FACTOR3	
AGR	-0.97812	0.07822	-0.05103	agriculture
MIN	-0.00247	0.90170	0.21082	mining
MAN	0.64891	0.51820	0.15773	manufacturing
PS	0.47752	0.38107	0.58819	power supplies
CON	0.60724	0.07486	-0.16073	construction
SER	0.70759	-0.51108	0.12066	service industries
FIN	0.13888	-0.66218	0.61574	finance
SPS	0.72344	-0.32331	-0.32697	social and personal services
TC	0.68500	0.29569	-0.39323	transport and communications

Initial Factor Method: Principal Components

Variance explained by each factor

FACTOR1	FACTOR2	FACTOR3
3.487151	2.130173	1.098958

Final Communality Estimates: Total = 6.716282

AGR	MIN	MAN	PS	CON
0.965447	0.857506	0.714499	0.719212	0.400174

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SER	FIN	SPS	TC
0.776446	0.836900	0.734813	0.711285

Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2	3
1	0.90315	0.37218	0.21400
2	-0.10616	0.67658	-0.72868
3	-0.41599	0.63539	0.65056

Rotation Method: Varimax

Rotated Factor Pattern

	FACTOR1	FACTOR2	FACTOR3	
AGR	-0.87047	-0.34354	-0.29951	agriculture
MIN	-0.18565	0.74310	-0.52042	mining
MAN	0.46544	0.69234	-0.13612	manufacturing
PS	0.14614	0.80928	0.20717	power supplies
CON	0.60734	0.17452	-0.02916	construction
SER	0.64313	-0.00577	0.60233	service industries
FIN	-0.06041	-0.00509	0.91281	finance
SPS	0.82372	-0.15725	0.17769	social and personal services
TC	0.75085	0.20515	-0.32469	transport and communications

Rotation Method: Varimax

Variance explained by each factor

FACTOR1	FACTOR2	FACTOR3
3.058592	1.901820	1.755870

Final Communality Estimates: Total = 6.716282

AGR	MIN	MAN	PS	CON
0.965447	0.857506	0.714499	0.719212	0.400174

SER	FIN	SPS	TC
0.776446	0.836900	0.734813	0.711285

...11/-

Output bagi Soalan 3(c):

Discriminant Analysis

150 Observations	149 DF Total
4 Variables	145 DF Within Classes
5 Classes	4 DF Between Classes

Class Level Information

PERIOD	Frequency	Weight	Proportion	Prior Probability
12th and 13th dynasties	30	30.0000	0.200000	0.200000
Early predynastic (circ	30	30.0000	0.200000	0.200000
Late predynastic (circa	30	30.0000	0.200000	0.200000
Ptolemic period (circa	30	30.0000	0.200000	0.200000
Roman period (circa AD	30	30.0000	0.200000	0.200000

Discriminant Analysis Within-Class Covariance Matrices

PERIOD = 12th and 13th dynasties DF = 29

Variable	X1	X2	
X1	12.11954023	0.78620690	maximum breadth (mm)
X2	0.78620690	24.78620690	basibregmatic height (mm)
X3	-0.77471264	3.59310345	basialveolar length (mm)
X4	0.89885057	-0.08965517	nasal height (mm)

Variable	X3	X4	
X1	-0.77471264	0.89885057	maximum breadth (mm)
X2	3.59310345	-0.08965517	basibregmatic height (mm)
X3	20.72298851	1.67011494	basialveolar length (mm)
X4	1.67011494	12.59885057	nasal height (mm)

Discriminant Analysis Within-Class Covariance Matrices

PERIOD = Early predynastic (circ DF = 29

Variable	X1	X2	
X1	26.30919540	4.15172414	maximum breadth (mm)
X2	4.15172414	19.97241379	basibregmatic height (mm)
X3	0.45402299	-0.79310345	basialveolar length (mm)
X4	7.24597701	0.39310345	nasal height (mm)

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Variable	X3	X4	
X1	0.45402299	7.24597701	maximum breadth (mm)
X2	-0.79310345	0.39310345	basibregmatic height (mm)
X3	34.62643678	-1.91954023	basialveolar length (mm)
X4	-1.91954023	7.63678161	nasal height (mm)

Discriminant Analysis Within-Class Covariance Matrices

PERIOD = Late predynastic (circa DF = 29

Variable	X1	X2	
X1	23.13678161	1.01034483	maximum breadth (mm)
X2	1.01034483	21.59655172	basibregmatic height (mm)
X3	4.76781609	3.36551724	basialveolar length (mm)
X4	1.84252874	5.62413793	nasal height (mm)

Variable	X3	X4	
X1	4.76781609	1.84252874	maximum breadth (mm)
X2	3.36551724	5.62413793	basibregmatic height (mm)
X3	18.89195402	0.19080460	basialveolar length (mm)
X4	0.19080460	8.73678161	nasal height (mm)

Discriminant Analysis Within-Class Covariance Matrices

PERIOD = Ptolemic period (circa DF = 29

Variable	X1	X2	
X1	15.36206897	-5.53448276	maximum breadth (mm)
X2	-5.53448276	26.35517241	basibregmatic height (mm)
X3	-2.17241379	8.11034483	basialveolar length (mm)
X4	2.05172414	6.14827586	nasal height (mm)

Variable	X3	X4	
X1	-2.17241379	2.05172414	maximum breadth (mm)
X2	8.11034483	6.14827586	basibregmatic height (mm)
X3	21.08505747	5.32873563	basialveolar length (mm)
X4	5.32873563	7.96436782	nasal height (mm)

Discriminant Analysis Within-Class Covariance Matrices

PERIOD = Roman period (circa AD DF = 29

Variable	X1	X2	
X1	28.62643678	-0.22988506	maximum breadth (mm)
X2	-0.22988506	24.71264368	basibregmatic height (mm)
X3	-1.87931034	11.72413793	basialveolar length (mm)
X4	-1.99425287	2.14942529	nasal height (mm)

...13/-

Variable	X3	X4	
X1	-1.87931034	-1.99425287	maximum breadth (mm)
X2	11.72413793	2.14942529	basibregmatic height (mm)
X3	25.56896552	0.39655172	basialveolar length (mm)
X4	0.39655172	13.82643678	nasal height (mm)

Discriminant Analysis

Pooled Within-Class Covariance Matrix DF = 145

Variable	X1	X2	
X1	21.11080460	0.03678161	maximum breadth (mm)
X2	0.03678161	23.48459770	basibregmatic height (mm)
X3	0.07908046	5.20000000	basialveolar length (mm)
X4	2.00896552	2.84505747	nasal height (mm)

Variable	X3	X4	
X1	0.07908046	2.00896552	maximum breadth (mm)
X2	5.20000000	2.84505747	basibregmatic height (mm)
X3	24.17908046	1.13333333	basialveolar length (mm)
X4	1.13333333	10.15264368	nasal height (mm)

Discriminant Analysis Pooled Covariance Matrix Information

Covariance Matrix Rank	Natural Log of the Determinant of the Covariance Matrix
4	11.6052991

Discriminant Analysis

Pairwise Generalized Squared Distances Between Groups

$$D^2(i|j) = (\bar{X}_i - \bar{X}_j)' \, COV^{-1} \, (\bar{X}_i - \bar{X}_j)$$

Generalized Squared Distance to PERIOD

From PERIOD	12th and 13th dynasties	Early predynastic (circ
12th and 13th dynasties	0	0.90307
Early predynastic (circ	0.90307	0
Late predynastic (circa	0.72894	0.09103
Ptolemic period (circa	0.44311	1.88113
Roman period (circa AD	0.91087	2.69682

...14/-

Discriminant Analysis

Pairwise Generalized Squared Distances Between Groups

Generalized Squared Distance to PERIOD

From PERIOD	Late predynastic (circa	Ptolemic period (circa
12th and 13th dynasties	0.72894	0.44311
Early predynastic (circ	0.09103	1.88113
Late predynastic (circa	0	1.59401
Ptolemic period (circa	1.59401	0
Roman period (circa AD	2.17569	0.21929

Discriminant Analysis

Pairwise Generalized Squared Distances Between Groups

Generalized Squared Distance to PERIOD

From PERIOD	Roman period (circa AD
12th and 13th dynasties	0.91087
Early predynastic (circ	2.69682
Late predynastic (circa	2.17569
Ptolemic period (circa	0.21929
Roman period (circa AD	0

Discriminant Analysis

Multivariate Statistics and F Approximations

S=4 M=-0.5 N=70

Statistic	Value	F	Num DF	Den DF	Pr > F
Wilks' Lambda	0.66358580	3.9009	16	434.4548	0.0001
Pillai's Trace	0.35330557	3.5120	16	580	0.0001
Hotelling-Lawley Trace	0.48181908	4.2310	16	562	0.0001
Roy's Greatest Root	0.42509538	15.4097	4	145	0.0001

NOTE: F Statistic for Roy's Greatest Root is an upper bound.

Discriminant Analysis Linear Discriminant Function

$$\text{Constant} = -.5\ \bar{X}_j'\ \text{COV}^{-1}\ \bar{X}_j \qquad \text{Coefficient Vector} = \text{COV}^{-1}\ \bar{X}_j$$

...15/-

PERIOD

	12th and 13th dynasties	Early predynastic (circ
CONSTANT	-923.73320	-912.91843
X1	6.15066	6.00123
X2	4.80899	4.76743
X3	2.81891	2.95687
X4	2.10128	2.12381

Discriminant Analysis   Linear Discriminant Function

PERIOD

	Late predynastic (circa	Ptolemic period (circa
CONSTANT	-913.74166	-921.81038
X1	6.05150	6.18499
X2	4.73160	4.73805
X3	2.96168	2.76466
X4	2.09382	2.25832

PERIOD

	Roman period (circa AD	Label
CONSTANT	-912.51338	nasal height (mm)
X1	6.22088	maximum breadth (mm)
X2	4.66502	basibregmatic height (mm)

Discriminant Analysis   Linear Discriminant Function

PERIOD

	Roman period (circa AD	Label
X3	2.73952	basialveolar length (mm)
X4	2.21539	nasal height (mm)

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

Generalized Squared Distance Function:

$$D_j^2(X) = (X-\bar{X}_j)' \, COV^{-1} (X-\bar{X}_j)$$

Posterior Probability of Membership in each PERIOD:

2   2

...16/-

$$Pr(j|X) = \exp(-.5\ D_j(X)) / \mathrm{SUM}_k \exp(-.5\ D_k(X))$$

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

Number of Observations and Percent Classified into PERIOD:

From PERIOD	12th and 13th dynasties	Early predynastic (circ
12th and 13th dynasties	15 50.00	4 13.33
Early predynastic (circ	4 13.33	12 40.00
Late predynastic (circa	5 16.67	10 33.33

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

Number of Observations and Percent Classified into PERIOD:

From PERIOD	12th and 13th dynasties	Early predynastic (circ
Ptolemic period (circa	7 23.33	3 10.00
Roman period (circa AD	4 13.33	2 6.67
Total Percent	35 23.33	31 20.67

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

Number of Observations and Percent Classified into PERIOD:

From PERIOD	Late predynastic (circa	Ptolemic period (circa
12th and 13th dynasties	4 13.33	2 6.67

...17/-

Early predynastic (circ	8 26.67	4 13.33
Late predynastic (circa	8 26.67	4 13.33

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

Number of Observations and Percent Classified into PERIOD:

From PERIOD	Late predynastic (circa	Ptolemic period (circa
Ptolemic period (circa	3 10.00	5 16.67
Roman period (circa AD	4 13.33	9 30.00
Total Percent	27 18.00	24 16.00

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

Number of Observations and Percent Classified into PERIOD:

From PERIOD	Roman period (circa AD	Total
12th and 13th dynasties	5 16.67	30 100.00
Early predynastic (circ	2 6.67	30 100.00
Late predynastic (circa	3 10.00	30 100.00

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

...18/-

Number of Observations and Percent Classified into PERIOD:

From PERIOD	Roman period (circa AD	Total
Ptolemic period (circa	12 40.00	30 100.00
Roman period (circa AD	11 36.67	30 100.00
Total Percent	33 22.00	150 100.00

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

Error Count Estimates for PERIOD:

	12th and 13th dynasties	Early predynastic (circ	Late predynastic (circa
Rate	0.5000	0.6000	0.7333
Priors	0.2000	0.2000	0.2000

Discriminant Analysis

Classification Summary for Calibration Data: WORK.SKULL

Resubstitution Summary using Linear Discriminant Function

Error Count Estimates for PERIOD:

	Ptolemic period (circa	Roman period (circa AD	Total
Rate	0.8333	0.6333	0.6600
Priors	0.2000	0.2000	

...19/-

Output bagi Soalan 3(d):

Single Linkage Cluster Analysis

Mean Distance Between Observations = 3.312381

Number of Clusters	Clusters Joined		Frequency of New Cluster	Normalized Minimum Distance	Tie
6	Modern dog	Prehistoric dog	2	0.217366	
5	CL6	Cuon	3	0.416619	
4	CL5	Dingo	4	0.507188	
3	CL4	Golden jackal	5	0.624928	
2	Chinese wolf	Indian wolf	2	0.697384	
1	CL3	CL2	7	0.715497	

Single Linkage Cluster Analysis

Minimum Distance Between Clusters

```
                       0.8    0.7    0.6    0.5    0.4    0.3    0.2    0.1    0
                       +------+------+------+------+------+------+------+------+
S        Modern dog           XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX...............
P                             XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
E   Prehistoric dog           XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX...............
C                             XXXXXXXXXXXXXXXXXXXXX
I              Cuon           XXXXXXXXXXXXXXXXXXXXX.............................
E                             XXXXXXXXXXXXXXX
S             Dingo           XXXXXXXXXXXXXXX...................................
                              XXXXXXX
      Golden jackal           XXXXXXX...........................................
                              X
       Chinese wolf           XX................................................
                              XX
        Indian wolf           XX................................................
```

Average Linkage Cluster Analysis

Root-Mean-Square Distance Between Observations = 3.742944

Number of Clusters	Clusters Joined		Frequency of New Cluster	Normalized RMS Distance	Tie
6	Modern dog	Prehistoric dog	2	0.192362	
5	CL6	Cuon	3	0.403474	
4	CL5	Dingo	4	0.474128	
3	Chinese wolf	Indian wolf	2	0.617161	
2	CL4	Golden jackal	5	0.786647	
1	CL2	CL3	7	1.307933	

...20/-

Average Linkage Cluster Analysis

Average Distance Between Clusters

```
                      1.4     1.2      1       0.8     0.6     0.4     0.2      0
                      +-------+-------+-------+-------+-------+-------+-------+
S        Modern dog       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.........
P                         XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
E   Prehistoric dog       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.........
C                         XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
I              Cuon       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX..................
E                         XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
S             Dingo       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.....................
                          XXXXXXXXXXXXXXXXXXXXXXXXX
      Golden jackal       XXXXXXXXXXXXXXXXXXXXXXXXX...................................
                          X
       Chinese wolf       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX............................
                          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        Indian wolf       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX............................
```

Complete Linkage Cluster Analysis

Mean Distance Between Observations = 3.312381

Number of Clusters	Clusters Joined		Frequency of New Cluster	Normalized Maximum Distance	Tie
6	Modern dog	Prehistoric dog	2	0.217366	
5	CL6	Cuon	3	0.492093	
4	CL5	Dingo	4	0.555492	
3	Chinese wolf	Indian wolf	2	0.697384	
2	CL4	Golden jackal	5	1.041547	
1	CL2	CL3	7	2.321593	

Complete Linkage Cluster Analysis

Maximum Distance Between Clusters

```
                      2.5         2           1.5          1          0.5          0
                      +-----------+-----------+-----------+-----------+-----------+
S        Modern dog       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.....
P                         XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
E   Prehistoric dog       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.....
C                         XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
I              Cuon       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX............
E                         XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
S             Dingo       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.............
                          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      Golden jackal       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX.........................
                          X
       Chinese wolf       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX................
                          XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
        Indian wolf       XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX................
```

2

4. Tulis nota pendek tentang tajuk-tajuk di bawah:

   (a) Komponen prinsipal

   (b) Analisis faktor

   (c) Analisis pembezalayan

   (d) Analisis kelompok

(100/100)

- oooOOooo -

LAMPIRAN

Tatatanda adalah seperti di dalam kuliah.

1. Penguraian spektrum bagi suatu matriks simetrik k x k, $\underset{\sim}{A}$ diberikan oleh

$$\underset{\sim}{A} = \lambda_1 \underset{\sim}{e}_1 \underset{\sim}{e}'_1 + \lambda_2 \underset{\sim}{e}_2 \underset{\sim}{e}'_2 + \ldots + \lambda_k \underset{\sim}{e}_k \underset{\sim}{e}'_k$$

di mana $\lambda_1, \lambda_2, \ldots, \lambda_k$ adalah nilai-nilai eigen $\underset{\sim}{A}$ dan $\underset{\sim}{e}_1, \underset{\sim}{e}_2, \ldots, \underset{\sim}{e}_k$ adalah vektor-vektor eigen terpiawai yang berkaitan.

2. Katakan $\underset{\sim}{X}$ mempunyai $E(\underset{\sim}{X}) = \underset{\sim}{\mu}$ dan Kov $(\underset{\sim}{X}) = \underset{\sim}{\Sigma}$. Maka $\underset{\sim}{c}'\underset{\sim}{X}$ mempunyai min, $\underset{\sim}{c}'\underset{\sim}{\mu}$ dan varians, $\underset{\sim}{c}'\underset{\sim}{\Sigma}\,\underset{\sim}{c}$.

3. f.k.k. normal bivariat:

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{\sigma_{11}\sigma_{22}(1-\rho_{12}^2)}} \times \exp\left\{ -\frac{1}{2(1-\rho_{12}^2)} \left[\left(\frac{x_1 - \mu_1}{\sqrt{\sigma_{11}}}\right)^2 + \left(\frac{x_2 - \mu_2}{\sqrt{\sigma_{22}}}\right)^2 - 2\rho_{12}\left(\frac{x_1 - \mu_1}{\sqrt{\sigma_{11}}}\right)\left(\frac{x_2 - \mu_2}{\sqrt{\sigma_{22}}}\right) \right] \right\}$$

4. f.k.k. normal multivariat:

$$f(\underset{\sim}{x}) = \frac{1}{(2\pi)^{p/2} |\underset{\sim}{\Sigma}|^{1/2}} e^{-(1/2)(\underset{\sim}{x} - \underset{\sim}{\mu})'\underset{\sim}{\Sigma}^{-1}(\underset{\sim}{x} - \underset{\sim}{\mu})}$$

5. Jika $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \underset{\sim}{\Sigma})$, maka $\underset{\sim}{A}\underset{\sim}{X} \sim N_q(\underset{\sim}{A}\underset{\sim}{\mu}, \underset{\sim}{A}\underset{\sim}{\Sigma}\underset{\sim}{A}')$.

...2/-

6. Satu sampel:

(a) $T^2 = n(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu})' \underset{\sim}{S}^{-1}(\underset{\sim}{\bar{X}} - \underset{\sim}{\mu})$

$$\underset{\sim}{\bar{X}} = \frac{1}{n} \sum_{j=1}^{n} \underset{\sim}{X}_j, \quad \underset{\sim}{S} = \frac{1}{n-1} \sum_{j=1}^{n} (\underset{\sim}{X}_j - \underset{\sim}{\bar{X}})(\underset{\sim}{X}_j - \underset{\sim}{\bar{X}})'$$

$$T^2 \sim \frac{(n-1)p}{n-p} F_{p,n-p}$$

(b) Lambda Wilks $\Lambda^{2/n} = |\hat{\underset{\sim}{\Sigma}}| \Big/ |\hat{\underset{\sim}{\Sigma}}_0| = \left[1 + \frac{T^2}{(n-1)}\right]^{-1}$

(c) Selang keyakinan serentak 100(1-α)% bagi $\underset{\sim}{\ell}'\underset{\sim}{\mu}$:

$$\underset{\sim}{\ell}' \underset{\sim}{\bar{X}} \pm \sqrt{\frac{p(n-1)}{n(n-p)} F_{p,n-p}(\alpha) \; \underset{\sim}{\ell}' \underset{\sim}{S} \underset{\sim}{\ell}}$$

(d) Selang keyakinan serentak Bonferroni 100(1-α)% bagi

μ_i, i = 1, ..., p :

$$\bar{X}_i + t_{n-1}\left(\frac{\alpha}{2p}\right) \sqrt{\frac{S_{ii}}{n}}$$

7. Dua sampel tak bersandar:

(a) $$T^2 = \left[\underset{\sim}{\bar{X}}_1 - \underset{\sim}{\bar{X}}_2 - (\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2)\right]' \left[\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\underset{\sim}{S}_p\right]^{-1} \left[\underset{\sim}{\bar{X}}_1 - \underset{\sim}{\bar{X}}_2 - \left(\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2\right)\right]$$

$$T^2 \sim \frac{\left[n_1 + n_2 - 2\right]p}{\left[n_1 + n_2 - p - 1\right]} F_{p,\, n_1 + n_2 - p - 1}$$

...3/-

(b) Selang keyakinan serentak 100(1-α)% bagi $\underset{\sim}{\ell}'(\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2)$:

$$\underset{\sim}{\ell}'\left[\underset{\sim}{\bar{X}}_1 - \underset{\sim}{\bar{X}}_2\right] \pm c \sqrt{\underset{\sim}{\ell}'\left[\frac{1}{n_1} + \frac{1}{n_2}\right] \underset{\sim}{S}_p \underset{\sim}{\ell}}$$

di mana $c^2 = \dfrac{\left[n_1 + n_2 - 2\right]p}{n_1 + n_2 - p - 1} F_{p,\, n_1 + n_2 - p - 1}$

8. <u>MANOVA satu-hala</u>:

(a) $\underset{\sim}{B} = \sum_{\ell=1}^{g} n_\ell (\underset{\sim}{\bar{x}}_\ell - \underset{\sim}{\bar{x}})(\underset{\sim}{\bar{x}}_\ell - \underset{\sim}{\bar{x}})'$

$$\underset{\sim}{W} = \sum_{\ell=1}^{g} \sum_{j=1}^{n_\ell} \left[\underset{\sim}{x}_{\ell j} - \underset{\sim}{\bar{x}}_\ell\right]\left[\underset{\sim}{x}_{\ell j} - \underset{\sim}{\bar{x}}_\ell\right]' = (n_1 - 1)S_1^2 + \ldots + (n_g - 1)S_g^2$$

$$\Lambda^* = \frac{|\underset{\sim}{W}|}{|\underset{\sim}{B} + \underset{\sim}{W}|}$$

(b) Selang keyakinan serentak 100(1-α)% bagi $\tau_{ki} - \tau_{\ell i}$: $\mu_{ij} - \mu_{\ell j}$

$$\bar{X}_{ki} - \bar{X}_{\ell i} \pm t_{n-g}\left[\frac{\alpha}{pg(g-1)}\right]\sqrt{\frac{W_{ii}}{n-g}\left[\frac{1}{n_k} + \frac{1}{n_\ell}\right]},$$

$i = 1, 2, \ldots, p \quad , \quad \ell < k = 1, 2, \ldots, g$

9. Andaikan $\underset{\sim}{E}$ mempunyai d.k. m_E dan H mempunyai d.k. m_H.

Katakan $\Lambda = \dfrac{|\underset{\sim}{E}|}{|\underset{\sim}{E} + \underset{\sim}{H}|}$

...4/-

Maka (1) Untuk p = 1,

$$\left[\frac{1 - \Lambda}{\Lambda}\right] \frac{m_E}{m_H} \sim F_{m_H\,,\,m_E} \quad \text{bagi sebarang } m_H\,.$$

(2) Untuk $m_H = 1$,

$$\left[\frac{1 - \Lambda}{\Lambda}\right] \frac{m_E+1-p}{P} \sim F_{P,\ m_E+1-p} \quad \text{bagi sebarang P.}$$

(3) Untuk p = 2,

$$\left[\frac{1 - \Lambda^{1/2}}{\Lambda^{1/2}}\right] \left[\frac{m_E - 1}{m_H}\right] \sim F_{2m_H, 2(m_E-1)}$$

untuk $m_H \geq 2$.

(4) Untuk $m_H = 2$,

$$\left[\frac{1 - \Lambda^{1/2}}{\Lambda^{1/2}}\right] \left[\frac{m_E + 1 - p}{p}\right] \sim F_{2p,\ 2(m_E+1-p)}$$

untuk $p \geq 2$.

Pembetulan Bartlett: Katakan $n_o = m_E + m_H$.

Bagi m_E besar,

$$-f \log \Lambda \sim X^2_{pm_H}$$

di mana

$$f = m_E - \frac{1}{2}\left[p - m_H + 1\right]$$

$$= n_o - \frac{1}{2}\left[p + m_H + 1\right]$$

10. <u>MANOVA dua-hala:</u>

$$SSP_{\text{faktor 1}} = \sum_{\ell=1}^{g} bn\left[\bar{\underset{\sim}{x}}_{\ell .} - \bar{\underset{\sim}{x}}\right]\left[\bar{\underset{\sim}{x}}_{\ell .} - \bar{\underset{\sim}{x}}\right]$$

...5/-

$$SSP_{faktor\ 2} = \sum_{k=1}^{b} gn\left(\bar{\underset{\sim}{x}}_{.k} - \bar{\underset{\sim}{x}}\right)\left(\bar{\underset{\sim}{x}}_{.k} - \bar{\underset{\sim}{x}}\right)'$$

$$SSP_{\substack{tindakan \\ bersaling}} = \sum_{\ell=1}^{g} \sum_{k=1}^{b} n \left(\bar{\underset{\sim}{x}}_{\ell k} - \bar{\underset{\sim}{x}}_{\ell .} - \bar{\underset{\sim}{x}}_{.k} + \bar{\underset{\sim}{x}}\right)$$

$$\left(\bar{\underset{\sim}{x}}_{\ell k} - \bar{\underset{\sim}{x}}_{\ell .} - \bar{\underset{\sim}{x}}_{.k} + \bar{\underset{\sim}{x}}\right)'$$

$$SSP_{residual} = \sum_{\ell=1}^{g} \sum_{k=1}^{b} \sum_{r=1}^{n} \left(\underset{\sim}{x}_{\ell kr} - \bar{\underset{\sim}{x}}_{\ell k}\right)\left(\underset{\sim}{x}_{\ell kr} - \bar{\underset{\sim}{x}}_{\ell k}\right)'$$

11. Komponen Prinsipal

(a) $\underset{\sim}{Y}_i = \underset{\sim}{e}'_i \underset{\sim}{X}$, $i = 1, 2, \ldots, p.$

$$\rho_{Y_i, X_k} = \frac{e_{ki}\sqrt{\lambda_i}}{\sqrt{\sigma_{kk}}} , \qquad i, k = 1, 2, \ldots, p.$$

(b) $\underset{\sim}{Y}_i = \underset{\sim}{e}'_i \underset{\sim}{Z}$

$$\rho_{Y_i, Z_k} = e_{ki} \sqrt{\lambda_i} , \quad i,k = 1, 2, \ldots, p.$$

12. Analisis Faktor

(a) $\underset{\sim}{X} - \underset{\sim}{\mu} = \underset{\sim}{L}\,\underset{\sim}{F} + \underset{\sim}{\epsilon}$

(b) $Kov(\underset{\sim}{X}) = \underset{\sim}{L}\,\underset{\sim}{L}' + \underset{\sim}{\Psi}$

$Kov(\underset{\sim}{X}, \underset{\sim}{F}) = \underset{\sim}{L}$

...6/-

(c) $h_i^2 = \ell_{i1}^2 + \ell_{i2}^2 + \ldots + \ell_{im}^2$, $i = 1, 2, \ldots, p$.

$\sigma_{ii} = h_i^2 + \psi_i$, $i = 1, 2, \ldots, p$.

(d) Kriterium varimax: Pilih transformasi ortogon T yang menjadikan

$$V = \frac{1}{p} \sum_{j=1}^{m} \left[\sum_{i=1}^{p} \tilde{\ell}_{ij}^{*4} - \left(\sum_{i=1}^{p} \tilde{\ell}_{ij}^{*2} \right)^2 \Big/ p \right]$$

sebesar yang mungkin.

13. <u>Analisis Pembezalayan</u>

(a) $Y = \underset{\sim}{\ell}'\underset{\sim}{X} = (\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2)' \underset{\sim}{\Sigma}^{-1} \underset{\sim}{X}$

$m = \frac{1}{2}\left(\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2\right)' \underset{\sim}{\Sigma}^{-1} \left(\underset{\sim}{\mu}_1 + \underset{\sim}{\mu}_2\right)$

(b) $y = \hat{\underset{\sim}{\ell}}'\underset{\sim}{x} = \left(\bar{\underset{\sim}{x}}_1 - \bar{\underset{\sim}{x}}_2\right)' \underset{\sim}{S}_p^{-1} \underset{\sim}{x}$

$\hat{m} = \frac{1}{2} \left(\bar{\underset{\sim}{x}}_1 - \bar{\underset{\sim}{x}}_2\right)' S_p^{-1} \left(\bar{\underset{\sim}{x}}_1 + \bar{\underset{\sim}{x}}_2\right)$

(c) Petua peruntukan:

Untukkan $\underset{\sim}{x}_o$ kepada $\begin{cases} \pi_1 \text{ jika } y_o \geq \hat{m} \\ \pi_2 \text{ jika } y_o < \hat{m} \end{cases}$

...7/-

(d) $\underset{\sim}{B}_o = \sum_{i=1}^{g} \left(\underset{\sim}{\mu}_i - \bar{\underset{\sim}{\mu}}\right) \left(\underset{\sim}{\mu}_i - \bar{\underset{\sim}{\mu}}\right)'$

$\lambda_1, \ldots, \lambda_s$ nilai eigen dan

$\underset{\sim}{e}_1, \ldots, \underset{\sim}{e}_s$ vektor eigen $\underset{\sim}{\Sigma}^{-1}\underset{\sim}{B}_o$.

$\underset{\sim}{\ell}_i \underset{\sim}{X} = \underset{\sim}{e}_i \underset{\sim}{X}$ pembezalayan ke-i , i = 1, 2, ..., s.

(e) $\hat{\underset{\sim}{B}}_o = \sum_{i=1}^{g} \left(\bar{\underset{\sim}{x}}_i - \bar{\underset{\sim}{x}}\right)\left(\bar{\underset{\sim}{x}}_i - \bar{\underset{\sim}{x}}\right)'$

$\underset{\sim}{W} = \sum_{i=1}^{g} \sum_{j=1}^{n_i} \left(\underset{\sim}{x}_{ij} - \bar{\underset{\sim}{x}}_i\right)\left(\underset{\sim}{x}_{ij} - \bar{\underset{\sim}{x}}_i\right)'$

$\hat{\underset{\sim}{\ell}}_i \underset{\sim}{x} = \hat{\underset{\sim}{e}}_i \underset{\sim}{x}$ pembezalayan sampel ke-i, i = 1, ..., s.

(f) Petua peruntukan:

Untukkan $\underset{\sim}{x}$ kepada π_k jika

$$\sum_{j=1}^{r} \left(\hat{y}_j - \bar{y}_{kj}\right)^2 = \sum_{j=1}^{r} \left[\hat{\underset{\sim}{\ell}}'_j \left(\underset{\sim}{x} - \bar{\underset{\sim}{x}}_k\right) \right]^2$$

$$\leq \sum_{j=1}^{r} \left[\hat{\underset{\sim}{\ell}}'_j \left(\underset{\sim}{x} - \bar{\underset{\sim}{x}}_i\right) \right]^2$$

bagi semua i ≠ k, r ≤ s.

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