UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang 1992/93

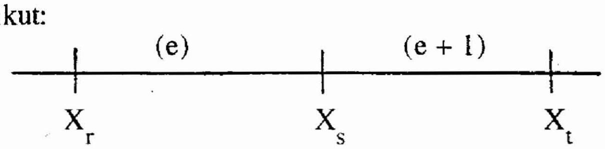
Oktober/November 1992

MSG442 - Kaedah Unsur Terhingga

Masa: [3 jam]

Jawab **semua** soalan.

1. (a) Katakan $N_r^{(e)}$, $N_s^{(e)}$, $N_s^{(e+1)}$, $N_t^{(e+1)}$ ialah fungsi bentuk linear bagi selang berikut:



(Di sini $X_t - X_s = X_s - X_r = L$, $r = s - 1$ dan $t = s + 1$.)

Jika

$$W_s(x) = \begin{cases} N_s^{(e)}(x) & , X_r \le x \le X_s \\ N_s^{(e+1)}(x) & , X_s \le x \le X_t \end{cases}$$

dan

$$\phi(x) = \begin{cases} N_r^{(e)}\Phi_r + N_s^{(e)}\Phi_s & , X_r \le x \le X_s \\ N_s^{(e+1)}\Phi_s + N_t^{(e+1)}\Phi_t & , X_s \le x \le X_t \end{cases}$$

tunjukkan bahawa

$$-\int_{X_r}^{X_t} W_s(x)\left[D\frac{d^2\phi}{dx^2} + Q\right] dx$$

$$= \frac{D}{L}\left[-\Phi_{s-1} + 2\Phi_s - \Phi_{s+1}\right] - QL$$

di mana D dan Q ialah pemalar.

(50/100)

...2/-

(b) Cari penyelesaian hampiran bagi

$$\frac{d^2\phi}{dx^2} + 10 = 0$$

$\phi(0) = 1, \quad \phi(3) = 4.$

dengan menggunakan kaedah Galerkin dan membahagikan selang [0,3] kepada tiga selang yang sama.

(20/100)

(c) Katakan N_i, N_j, N_k ialah fungsi bentuk bagi unsur segitiga linear dan

$$\phi = N_i\Phi_i + N_j\Phi_j + N_k\Phi_k$$

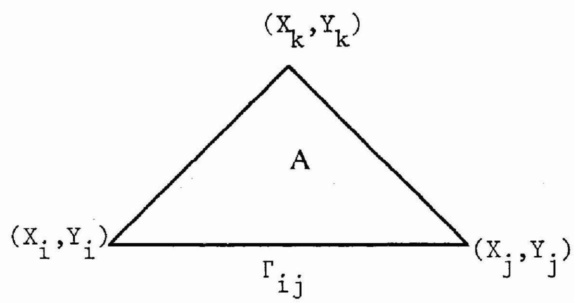
Cari

(i) $\int_A N_i\phi \, dA$

(ii) $\int_A \frac{\partial N_i}{\partial x} \cdot \frac{\partial \phi}{\partial x} \, dA$

(iii) $\int_{\Gamma_{ij}} N_i \, N_j \, d\Gamma$

di mana A dan Γ_{ij} diberi seperti berikut:



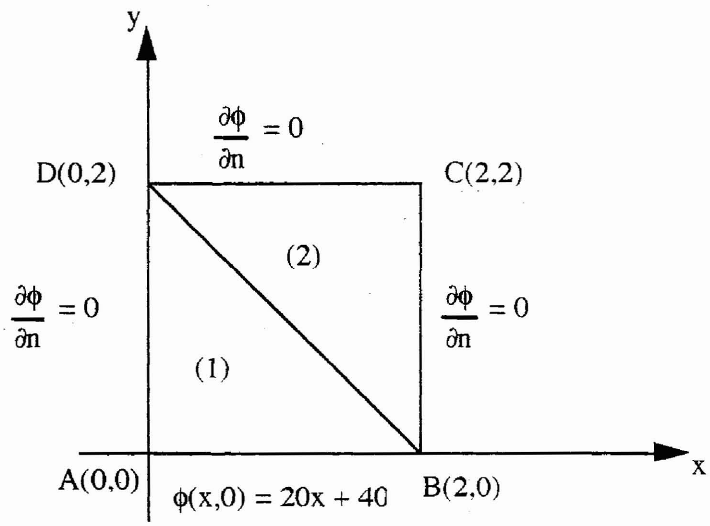
(30/100)

...3/-

2. (a) Dengan menggunakan kaedah unsur terhingga dan penyegitigaan seperti ditunjukkan di dalam gambar rajah, selesaikan masalah haba:

$$k\frac{\partial^2\phi}{\partial x^2} + k\frac{\partial^2\phi}{\partial y^2} - 2h(\phi - \phi_f) = 0$$

di dalam rantau ABCD berikut:



dengan syarat:

$\phi(x,0) = 20x + 40$, $0 \le x \le 2$

$\frac{\partial\phi}{\partial n} = 0$ pada BC, CD dan DA.

$k = 2$, $h = 0.1$ dan $\phi_f = 20°C$.

(60/100)

(b) Suatu unsur segiempat mempunyai bucu-bucu pada (0,0), (2,0), (3,3) dan (0,2). Cari transformasi dari koordinat (x,y) ke koordinat asli (ξ,η) dengan menggunakan fungsi bentuk bilinear.

Jika $\phi = N_i\Phi_i + N_j\Phi_j + N_k\Phi_k + N_m\Phi_m$, cari $\frac{\partial\phi}{\partial x}$ dan $\frac{\partial\phi}{\partial y}$ pada $\xi = \eta = 0.5$.

(40/100)

...4/-

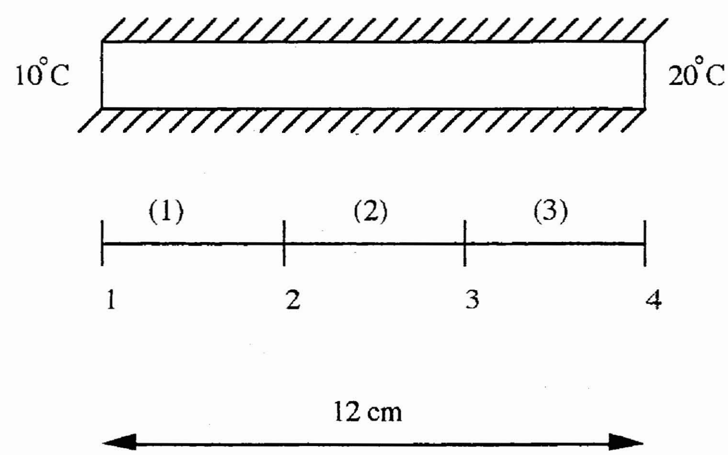
3. (a) Pertimbangkan masalah aliran haba berikut:

$$\frac{\partial^2 \phi}{\partial x^2} = 3\frac{\partial \phi}{\partial t} \quad , \quad 0 < x < 12 \quad , \quad t > 0.$$

$$\phi(x,0) = 50^\circ C \quad , \quad 0 \le x \le 12$$

$$\phi(0,t) = 10^\circ C \quad , \quad t > 0$$

$$\phi(12,t) = 20^\circ C \quad , \quad t > 0$$



Dengan membahagikan selang [0,12] ke tiga bahagian yang sama dan menggunakan kaedah unsur terhingga dengan perumusan konsisten dan skema beza ke depan (iaitu $\theta = 0$), binakan persamaan yang berbentuk:

$$[A]\{\Phi\}_{t+\Delta t} = [P]\{\Phi\}_t + \{F^*\}$$

Cari Φ_2 dan Φ_3 pada masa $\Delta t = 1$ saat.

(30/100)

...5/-

(b) Pertimbangkan penyelesaian persamaan

$$D \frac{\partial^2 \phi}{\partial x^2} + D \frac{\partial^2 \phi}{\partial y^2} = \lambda \frac{\partial \phi}{\partial t}$$

melalui kaedah unsur terhingga dengan perumusan tergumpal. Untuk segitiga A(0,0), B(b,0), C(0,b) , cari syarat atas Δt supaya ayunan berangka dapat dielakkan.

(30/100)

(c) Unsur segiempat kuadratik 8-nod digunakan untuk menyelesaikan persamaan Laplace

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad \text{di dalam } \Omega$$

$$\phi = \phi_o \quad \text{pada sempadan}$$

Terangkan bagaimana matriks unsur dibinakan jika kuadratur Gauss empat titik digunakan.

(40/100)

- ooo00ooo -

Unsur Linear 1-D

$$\left[k^{(e)}\right] = \frac{D}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Unsur Segitiga Linear

$$N_i = [a_i + b_i x + c_i y]/(2A), \quad N_j = [a_j + b_j x + c_j y]/(2A)$$
$$N_k = [a_k + b_k x + c_k y]/(2A)$$

dengan

$$2A = \begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix}$$

dan

$$a_i = X_j Y_k - X_k Y_j, \quad b_i = Y_j - Y_k, \quad c_i = X_k - X_j$$
$$a_j = X_k Y_i - X_i Y_k, \quad b_j = Y_k - Y_i, \quad c_j = X_i - X_k$$
$$a_k = X_i Y_j - X_j Y_i, \quad b_k = Y_i - Y_j, \quad c_k = X_j - X_i$$

$$\left[k_D^{(e)}\right] = \frac{D_x}{4A}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_i b_j & b_j^2 & b_j b_k \\ b_i b_k & b_j b_k & b_k^2 \end{bmatrix} + \frac{D_y}{4A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_i c_j & c_j^2 & c_j c_k \\ c_i c_k & c_j c_k & c_k^2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QA}{3}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{dll.}$$

$$\int_A L_1^a L_2^b L_3^c \, dA = \frac{a!\,b!\,c!}{(a+b+c+2)!} 2A$$

Unsur Segiempat Tepat Bilinear

$$N_i = \frac{1}{4}(1-\xi)(1-\eta), \quad N_j = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_k = \frac{1}{4}(1+\xi)(1+\eta), \quad N_m = \frac{1}{4}(1-\xi)(1+\eta)$$

$$N_i = \left(1 - \frac{s}{2b}\right)\left(1 - \frac{t}{2a}\right), \quad N_j = \frac{s}{2b}\left(1 - \frac{t}{2a}\right)$$

$$N_k = \frac{st}{4ab}, \quad N_m = \frac{t}{2a}\left(1 - \frac{s}{2b}\right)$$

$$\left[k_D^{(e)}\right] = \frac{D_x a}{6b}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{D_y b}{6a}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QA}{4}\begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

Unsur Kuadratik 1-D

$$N_1 = \frac{1}{2}\xi(\xi-1), \quad N_2 = -(\xi+1)(\xi-1), \quad N_3 = \frac{1}{2}\xi(\xi+1)$$

Unsur Segitiga Kuadratik 6-Nod

$$N_1 = L_1(2L_1-1), \quad N_2 = 4L_1L_2,$$

$$N_3 = L_2(2L_2-1), \quad N_4 = 4L_2(1-L_1-L_2)$$

$$N_5 = 1 - 3(L_1+L_2) + 2(L_1+L_2)^2, \quad N_6 = 4L_1(1-L_1-L_2)$$

Unsur Segiempat Kuadratik 8-Nod

$N_1 = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta), \quad N_2 = \frac{1}{2}(1-\xi^2)(1-\eta)$

$N_3 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1), \quad N_4 = \frac{1}{2}(1-\eta^2)(1+\xi)$

$N_5 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1), \quad N_6 = \frac{1}{2}(1-\xi^2)(1+\eta)$

$N_7 = -\frac{1}{4}(1-\xi)(1+\eta)(\xi-\eta+1), \quad N_8 = \frac{1}{2}(1-\eta^2)(1-\xi)$

Kuadratur Gauss-Legendre

$n=1$	$\xi_i = 0.0$	$W_i = 2.0$
$n=2$	$\xi_i = \pm 0.577350$	$W_i = 1.0$
$n=3$	$\xi_i = 0.0$	$W_i = 8/9$
	$\xi_i = \pm 0.774597$	$W_i = 5/9$
$n=4$	$\xi_i = \pm 0.861136$	$W_i = 0.347855$
	$\xi_i = \pm 0.339981$	$W_i = 0.652145$

Untuk Domain Segitiga

n	Titik	L_1	L_2	W_i
2	*a*	1/3	1/3	1/2
3	*a*	1/2	0	1/6
	b	1/2	1/2	1/6
	c	0	1/2	1/6

Masalah Bersandarkan Masa

$$\left([C] + \theta\Delta t[K]\right)\{\Phi\}_b = \left([C] - (1-\theta)\Delta t[K]\right)\{\Phi\}_a + \Delta t\left((1-\theta)\{F\}_a + \theta\{F\}_b\right)$$

Perumusan Konsisten

$$\left[c^{(e)}\right] = \frac{\lambda L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \qquad \left[c^{(e)}\right] = \frac{\lambda A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\left[c^{(e)}\right] = \frac{\lambda A}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}$$

$$\Delta t > \frac{\lambda L^2}{6D\theta}, \qquad \Delta t < \frac{\lambda L^2}{12D(1-\theta)}$$

Perumusan Tergumpal

$$\left[c^{(e)}\right] = \frac{\lambda L}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \left[c^{(e)}\right] = \frac{\lambda A}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[c^{(e)}\right] = \frac{\lambda A}{4}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Delta t < \frac{\lambda L^2}{4D(1-\theta)}$$