
UNIVERSITI SAINS MALAYSIA

First Semester Examination
2009/2010 Academic Session

November 2009

MAT 222 – Differential Equations II
[Persamaan Pembezaan II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

Arahan: Jawab **semua empat** [4] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Consider the system of differential equations below where x, y and z are functions in t :

$$x' = \frac{15}{7}x + \frac{4}{7}y - \frac{8}{7}z, \quad y' = \frac{6}{7}x + \frac{17}{7}y - \frac{6}{7}z, \quad z' = -\frac{3}{7}x + \frac{2}{7}y + \frac{10}{7}z.$$

- (i) Write the system above as a matrix equation for a suitable vector \underline{u} .
- (ii) Obtain the eigen values of the 3×3 matrix above.
- (iii) Hence, or otherwise, deduce the general solution for x .
- (iv) Solve for x given $x(0) = 3, x'(0) = 7$ and $x''(0) = 19$.
- (v) Given $y - 3z = e^{2t}$, obtain the solutions for y and z .

[100 marks]

2. Consider the second order partial differential equation $u_{xx} + 6u_{xy} + 9u_{yy} = 0$ where $u = u(x, y)$.

- (i) Obtain the general solution for the given equation.
- (ii) Hence, solve the given partial differential equation using the initial conditions $u(0, y) = e^y$ and $u(x, 0) = e^{-3x} + x^2$.

[100 marks]

3. Consider the function below:

$$f(x) = \frac{1}{2} x + |x| \quad \text{for } a < x < \pi.$$

- (a) Given $a = -\pi$, find the Fourier series for $f(x)$.
- (b) Given $a = 0$, find the Fourier Cosine series for $f(x)$.
- (c) Given $a = 0$, find the Fourier Sine series for $f(x)$.
- (d) Compare the value of $f\left(\frac{\pi}{2}\right)$ as determined by each of the three Fourier series above.

[100 marks]

1. Pertimbangkan sistem persamaan pembezaan berikut di mana x, y dan z merupakan fungsi-fungsi dalam t :

$$x' = \frac{15}{7}x + \frac{4}{7}y - \frac{8}{7}z, \quad y' = \frac{6}{7}x + \frac{17}{7}y - \frac{6}{7}z, \quad z' = -\frac{3}{7}x + \frac{2}{7}y + \frac{10}{7}z.$$

- (i) Tuliskan persamaan di atas sebagai suatu persamaan matriks bagi suatu vektor \underline{u} yang sesuai.
- (ii) Dapatkan nilai-nilai eigen bagi matriks 3×3 di atas.
- (iii) Dengan itu, atau dengan cara lain, deduksikan penyelesaian am bagi x .
- (iv) Dapatkan penyelesaian bagi x diberikan $x(0) = 3, x'(0) = 7$ dan $x''(0) = 19$.
- (v) Diberikan $y - 3z = e^{2t}$, dapatkan penyelesaian bagi y dan z .

[100 markah]

2. Pertimbangkan persamaan pembezaan separa peringkat kedua $u_{xx} + 6u_{xy} + 9u_{yy} = 0$ di mana $u = u(x, y)$.

- (i) Dapatkan penyelesaian am bagi persamaan pembezaan yang diberi.
- (ii) Dengan itu, selesaikan persamaan pembezaan separa di atas dengan menggunakan syarat awal $u(0, y) = e^y$ dan $u(x, 0) = e^{-3x} + x^2$.

[100 markah]

3. Pertimbangkan fungsi berikut:

$$f(x) = \frac{1}{2} x + |x| \quad \text{bagi } a < x < \pi.$$

- (a) Diberi $a = -\pi$, cari siri Fourier bagi $f(x)$.
- (b) Diberi $a = 0$, cari siri Kosinus Fourier bagi $f(x)$.
- (c) Diberi $a = 0$, cari siri Sinus Fourier bagi $f(x)$.
- (d) Bandingkan nilai-nilai $f\left(\frac{\pi}{2}\right)$ seperti yang ditentukan oleh setiap daripada tiga siri Fourier di atas.

[100 markah]

4. (a) Solve the following heat equation for $v = v(x, t)$:

$$v_t = kv_{xx}, 0 < x < l, t > 0,$$

given the initial conditions:

$$v(x, 0) = f(x), 0 < x < l, \text{ and}$$

$$v(0, t) = v(l, t) = 0, t > 0.$$

- (b) Consider the heat equation below for $u = u(x, t)$:

$$u_t = u_{xx} + 2, 0 < x < \pi, t > 0,$$

with the initial conditions:

$$u(x, 0) = x(\pi - x), 0 < x < \pi, \text{ and}$$

$$u(0, t) = u(\pi, t) = 0, t > 0.$$

- (i) Find a quadratic function $g(x)$, and suitable values for k and l such that the substitution $u(x, t) = v(x, t) + g(x)$ would transform this equation to the form in part (a).
- (ii) Hence, or otherwise, solve this equation.

[100 marks]

4. (a) Selesaikan persamaan haba berikut bagi $v = v(x, t)$:

$$v_t = kv_{xx}, 0 < x < l, t > 0,$$

diberikan syarat awal:

$$v(x, 0) = f(x), 0 < x < l, \text{ dan}$$

$$v(0, t) = v(l, t) = 0, t > 0.$$

(b) Pertimbangkan persamaan haba berikut bagi $u = u(x, t)$:

$$u_t = u_{xx} + 2, 0 < x < \pi, t > 0,$$

dengan syarat awal:

$$u(x, 0) = x(\pi - x), 0 < x < \pi, \text{ dan}$$

$$u(0, t) = u(\pi, t) = 0, t > 0.$$

(i) Cari suatu fungsi kuadratik $g(x)$, dan nilai-nilai yang sesuai bagi k dan l supaya gantian $u(x, t) = v(x, t) + g(x)$ dapat mengubah bentuk persamaan ini kepada bentuk persamaan dalam (a).

(ii) Dengan itu, atau dengan cara lain, selesaikan persamaan ini.

[100 markah]