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UNIVERSITI SAINS MALAYSIA

First Semester Examination  
2009/2010 Academic Session

November 2009

**MAT 203 – Vector Calculus**  
**[Kalkulus Vektor]**

Duration : 3 hours  
[Masa : 3 jam]

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Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **all three** [3] questions.

**Arahan:** Jawab **semua tiga** [3] soalan.]

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

1. Instruction: Write the choice of your answers in the exam answer script

- (a) If  $u = \langle 1, 2, 2 \rangle$ ,  $v = \langle -6, 2, 3 \rangle$  find the component of  $u$  in the direction of  $v$ .
- (A) 0 (B)  $\frac{4}{7}$  (C)  $\frac{2}{7}$   
 (D) 1 (E) none of the above
- (b) Find the equation of plane through the point  $3, 2, 1$  with normal vector  $2i - j + xk$ .
- (A)  $2x - y + 4z = 8$  (B)  $3x - y = 4$  (C)  $4x - 5y + z = 1$   
 (D)  $2x + 4z = 7$  (E) none of the above
- (c) Find a vector parallel to the line of intersection of the planes  $2x + 3y - 4z = 4$ , and  $3x + 2y + 2z = 5$ .
- (A)  $\langle 16, 14, 5 \rangle$  (B)  $\langle 16, -14, 5 \rangle$   
 (C)  $\langle 14, -16, -5 \rangle$  (D)  $\langle 3, 2, -1 \rangle$   
 (E) none of the above.
- (d) Calculate the distance between the two planes  $5x - 2y + 2z = 12$  and  $-10x + 4y - 4z = 8$ .
- (A)  $\frac{4}{\sqrt{33}}$  (B)  $\frac{16}{\sqrt{33}}$  (C)  $\frac{2}{\sqrt{11}}$   
 (D)  $4\sqrt{33}$  (E) none of the above.
- (e) Find the area of the triangle which has its vertices at the points  $P = 1, 1, 1$ ,  $Q = 2, 3, 3$  and  $R = 4, 1, 2$ .
- (A)  $4\sqrt{3}$  (B)  $\sqrt{65}$  (C)  $\frac{1}{2}$   
 (D)  $\frac{4}{3}$  (E) none of the above.
- (f) The domain of the function  $f(x, y) = \frac{1}{x-y}$  is
- (A)  $D = x, y : x \neq y$  (B)  $D = x, y : x = y$   
 (C)  $D = x, y : x = 0$  (D)  $D = x, y : y = 0$   
 (E) none of the above.
- (g) The limit  $\lim_{x, y \rightarrow 0, 0} \frac{x^2 - y^2}{x^2 + y^2}$
- (A) 3 (B) 2 (C) 4  
 (D) does not exist (E) none of the above.

1. Arahan: Tuliskan jawapan pilihan anda di dalam kertas jawapan peperiksaan.

- (a) Jika  $u = \langle 1, 2, 2 \rangle$ ,  $v = \langle -6, 2, 3 \rangle$  dapatkan komponen  $u$  di dalam arah  $v$ .
- (A) 0 (B)  $\frac{4}{7}$  (C)  $\frac{2}{7}$   
 (D) 1 (E) tiada satu pun jawapan di atas.
- (b) Dapatkan persamaan satah yang melalui titik  $3, 2, 1$  dengan vektor normal  $2i - j + xk$ .
- (A)  $2x - y + 4z = 8$  (B)  $3x - y = 4$  (C)  $4x - 5y + z = 1$   
 (D)  $2x + 4z = 7$  (E) tiada satu pun jawapan di atas.
- (c) Dapatkan vektor yang selari drngan garis pertindanan dua satah  $2x + 3y - 4z = 4$ , dan  $3x + 2y + 2z = 5$ .
- (B)  $\langle 16, 14, 5 \rangle$  (B)  $\langle 16, -14, 5 \rangle$   
 (C)  $\langle 14, -16, -5 \rangle$  (D)  $\langle 3, 2, -1 \rangle$   
 (E) tiada satu pun jawapan di atas.
- (d) Dapatkan jarak diantara dua satah  $5x - 2y + 2z = 12$  dan  $-10x + 4y - 4z = 8$ .
- (A)  $\frac{4}{\sqrt{33}}$  (B)  $\frac{16}{\sqrt{33}}$  (C)  $\frac{2}{\sqrt{11}}$   
 (D)  $4\sqrt{33}$  (E) tiada satu pun jawapan di atas.
- (e) Cari luas segitiga yang mempunyai vertek pada titik-titik  $P = 1, 1, 1$ ,  $Q = 2, 3, 3$  dan  $R = 4, 1, 2$ .
- (A) 4'03 (B)  $\sqrt{65}$  (C)  $\frac{1}{2}$   
 (D)  $\frac{4}{3}$  (E) tiada satu pun jawapan di atas.
- (f) Domain bagi fungsi  $f(x, y) = \frac{1}{x-y}$  adalah
- (A)  $D = x, y : x \neq y$  (B)  $D = x, y : x = y$   
 (C)  $D = x, y : x = 0$  (D)  $D = x, y : y = 0$   
 (E) tiada satu pun jawapan di atas.
- (g) had  $\lim_{x, y \rightarrow 0, 0} \frac{x^2 - y^2}{x^2 + y^2}$  adalah
- (A) 3 (B) 2 (C) 4  
 (D) tidak wujud (E) tiada satu pun jawapan di atas.

- (h) Find the length  $L$  of  $\vec{r}(t) = \cos t, \sin t, 2t$  from  $t=0$  to  $t=2\pi$   
 (A) 2 (B)  $3\sqrt{\pi}$  (C)  $2\sqrt{5}\pi$   
 (D)  $4\sqrt{5}\pi$  (E) none of the above.
- (i) Find the work done by  $F$  along any path  $C$  from the point  $P(1,1,1)$  to the point  $Q(1,2,3)$   
 (A) 26 (B) 20 (C) 43  
 (D) 30 (E) none of the above.
- (j) Find  $\nabla\phi$  if  $\phi(x,y,z) = e^x + 3xyz$  at  $(0,1,2)$   
 (A)  $3i+3k$  (B)  $2i+3j$  (C)  $7i$   
 (D)  $4i+3k$  (E) none of the above.
- (k) Compute  $\iint_E e^{-4x^2-9y^2} dx dy$  where  $E$  is the ellipse  $4x^2 + 9y^2 \leq 25$   
 (A)  $\frac{\pi}{3}$  (B)  $\frac{\pi}{3} 1-e^{-25}$   
 (C)  $\frac{\pi}{2} 1-e^{-5}$  (D)  $2\pi 1-e^{-20}$   
 (E) none of the above.
- (l) Evaluate the triple integral  $\int_0^2 \int_{-1}^x \int_0^y 6z+1 dz dy dx$   
 (A)  $\frac{4}{5}$  (B)  $\frac{19}{3}$  (C) 1  
 (D)  $\frac{5}{8}$  (E) none of the above.
- (m) Let  $z = \ln(x+2y^2-2z^2)$ . Compute  $\frac{\partial z}{\partial x}$  at  $x=1, y=1$  and  $z=1$   
 (A) -1 (B)  $\frac{1}{5}$  (C)  $-\frac{1}{5}$   
 (D) 0 (E) none of the above.
- (n) Evaluate  $\int_0^1 x^2 \sqrt{1-x^2} dx$   
 (A)  $\frac{\pi}{10}$  (B)  $\frac{\pi}{16}$  (C)  $2\pi$   
 (D) 0 (E) none of the above.

- (h) Dapatkan panjang  $L$  bagi  $\vec{r}(t) = \cos t, \sin t, 2t$  dari  $t=0$  sehingga  $t=2\pi$ .
- (A) 2 (B)  $3\sqrt{\pi}$  (C)  $2\sqrt{5}\pi$   
 (D)  $4\sqrt{5}\pi$  (E) tiada satu pun jawapan di atas.
- (i) Dapatkan kerja yang dilakukan  $F$  disepanjang laluan  $C$  bermula dari titik  $P(1,1,1)$  sehingga titik  $Q(1,2,3)$
- (A) 26 (B) 20 (C) 43  
 (D) 30 (E) tiada satu pun jawapan di atas.
- (j) Dapatkan  $\nabla\phi$  jika  $\phi(x,y,z) = e^x + 3xyz$  pada titik  $(0,1,2)$
- (A)  $3i+3k$  (B)  $2i+3j$  (C)  $7i$   
 (D)  $4i+3k$  (E) tiada satu pun jawapan di atas.
- (k) Dapatkan  $\iint_E e^{-4x^2-9y^2} dx dy$ .  $E$  adalah sebuah ellipse  $4x^2 + 9y^2 \leq 25$
- (A)  $\frac{\pi}{3}$  (B)  $\frac{\pi}{3} 1-e^{-25}$   
 (C)  $\frac{\pi}{2} 1-e^{-5}$  (D)  $2\pi 1-e^{-20}$   
 (E) tiada satu pun jawapan di atas.
- (l) Nilaikan  $\int_0^2 \int_{-1}^x \int_0^y 6z+1 dz dy dx$
- (A)  $\frac{4}{5}$  (B)  $\frac{19}{3}$  (C) 1  
 (D)  $\frac{5}{8}$  (E) tiada satu pun jawapan di atas.
- (m) Biarkan  $z = \ln x + 2y^2 - 2z^2$ . Dapatkan  $\frac{\partial z}{\partial x}$  pada  $x=1, y=1$  dan  $z=1$
- (A) -1 (B)  $\frac{1}{5}$  (C)  $-\frac{1}{5}$   
 (D) 0 (E) tiada satu pun jawapan di atas.
- (n) Nilaikan  $\int_0^1 x^2 \sqrt{1-x^2} dx$
- (A)  $\frac{\pi}{10}$  (B)  $\frac{\pi}{16}$  (C)  $2\pi$   
 (D) 0 (E) tiada satu pun jawapan di atas.

- (o) Find the directional derivative of  $f(x, y) = xy^2 + x^3y$  at the point  $(1, 2)$ , in the direction of  $v = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ .
- (A)  $\frac{15}{\sqrt{2}}$       (B)  $\frac{5}{\sqrt{2}}$       (C)  $\frac{3}{\sqrt{5}}$   
 (D)  $5\sqrt{2}$       (E) none of the above.
2. (a) Show that the function  $f(x, y) = \sin(x+y) + \cos(x-y)$  satisfies the wave equation  $\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2} = 0$ .
- (b) Find the equation of the tangent plane to the surface  $x^2 + y^2 + z^2 = 9$  at the point  $(2, 2, -1)$ .
- (c) Find all local maxima and minima of  $f(x, y) = x^2 + xy + y^2 - 3x$ .
3. (a) If  $S$  is the entire  $x, y$  plane, evaluate the integral  $I = \iint_S e^{-x^2-y^2} ds$  by transforming the integral into polar co-ordinates.
- (b) A co-ordinate system  $(u, k, w)$  is related to Cartesian co-ordinates  $(x_1, x_2, x_3)$  by
- $$x_1 = ukw, x_2 = uk(1-w^2)^{\frac{1}{2}},$$
- $$x_3 = \frac{u^2 - k^2}{2}$$
- (i) Find the scale factors  $h_u, h_k, h_w$ .
- (ii) Find the volume element in the  $(u, k, w)$  system.
- (c) Verify Stokes theorem for the given surface  $S$  defined by  $x^2 + y^2 + 5z = 1, z \geq 0$ , oriented by upward normal,  $\vec{F} = xz\vec{i} + yz\vec{j} + (x^2 + y^2)\vec{k}$ .

- (o) Dapatkan terbitan berarah bagi  $f(x, y) = xy^2 + x^3y$  pada titik  $(1, 2)$ , di dalam arah  $v = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ .
- (A)  $\frac{15}{\sqrt{2}}$       (B)  $\frac{5}{\sqrt{2}}$       (C)  $\frac{3}{\sqrt{5}}$   
 (D)  $5\sqrt{2}$       (E) tiada satu pun jawapan di atas.
2. (a) Tunjukkan bahawa fungsi  $f(x, y) = \sin x + y + \cos x - y$  memenuhi persamaan gelombang  $\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2} = 0$ .
- (b) Dapatkan persamaan satah tangen bagi permukaan  $x^2 + y^2 + z^2 = 9$  pada titik  $(2, 2, -1)$ .
- (c) Dapatkan semua titik maksimum tempatan dan minimum tempatan bagi  $f(x, y) = x^2 + xy + y^2 - 3x$ .
3. (a) Jika  $S$  adalah satah  $x, y$  nilaikan  $I = \iint_S e^{-x^2-y^2} ds$ , dengan menukar bentuk kamiran ini kepada bentuk koordinat polar.
- (b) Sistem koordinat  $u, k, w$  adalah berkait dengan koordinat kartesian  $x_1, x_2, x_3$  dengan
- $$x_1 = ukw, x_2 = uk \sqrt{1-w^2},$$
- $$x_3 = \frac{u^2 - k^2}{2}$$
- (i) Cari faktor skalar  $hu, hk, hw$ .
- (ii) Cari elemen ketumpatan di dalam sistem  $u, k, w$ .
- (c) Tentusahkan theorem stokes bagi permukaan  $S$  yang ditakrifkan sebagai  $x^2 + y^2 + 5z = 1, z \geq 0$ , dengan orientasi normal yang berarah keluar,  $\vec{F} = xz\mathbf{i} + yz\mathbf{j} + (x^2 + y^2)\mathbf{k}$ .