UNIVERSITI SAINS MALAYSIA

First Semester Examination
2009/2010 Academic Session

November 2009

MAT 111 – Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>SEVEN</u> pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>TUJUH</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u>: Answer **<u>all five</u>** [5] questions.

*[**<u>Arahan</u>**: Jawab **<u>semua lima</u>** [5] soalan.]*

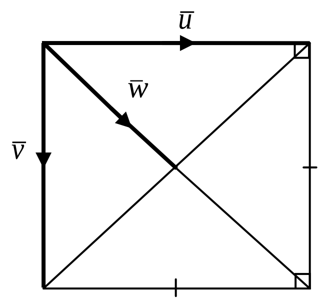
In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Give the definition of the inner product $u \cdot v$ of two vectors u and v.

(b) If $u = a,b,c$ and $v = \alpha,\beta,\gamma$, write the inner product $u \cdot v$ of u and v.

(c) If u and v are unit vectors, find the value of $u \cdot v$ and $u \cdot w$.



[100 marks]

2. (a) Find a linear transformation $T:\mathbb{R}^2 \rightarrow \mathbb{R}^3$ that satisfies $(2,-1)T = (1,-1,1)$ and $(1,1)T = (0,1,0)$. Hence, find $(-1,2)T$.

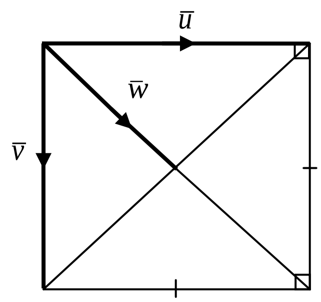
(b) Find a matrix A such that $(u)T = uA$ for the linear transformation T defined in (a).

[100 marks]

1. (a) *Berikan takrif hasildarab terkedalam* $\bar{u} \cdot \bar{v}$ *antara dua vektor* $\bar{u}$ *dan* $\bar{v}$.

(b) *Jika* $\bar{u} = a, b, c$ *dan* $\bar{v} = \alpha, \beta, \gamma$ *, tuliskan hasildarab terkedalam* $\bar{u} \cdot \bar{v}$ *antara* $\bar{u}$ *dan* $\bar{v}$.

(c) *Jika* $\bar{u}$ *dan* $\bar{v}$ *ialah vektor unit, cari nilai-nilai untuk* $\bar{u} \cdot \bar{v}$ *dan* $\bar{u} \cdot \bar{w}$.



[100 markah]

2. (a) *Cari transformasi linear* $T : \mathbb{R}^2 \to \mathbb{R}^3$ *yang mematuhi* $(2,-1)T = (1,-1,1)$ *dan* $(1,1)T = (0,1,0)$. *Seterusnya, cari* $(-1,2)T$.

(b) *Cari matriks* A *sedemikian* $(\bar{u})T = \bar{u}A$ *untuk transformasi linear yang tertakrif di (a).*

[100 markah]

3. The matrix $A=\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 2 & -3 \\ -2 & -1 & 1 \end{bmatrix}$ is reduced to reduced-echelon form B by some appropriate elementary row operations as follows:

$$A=\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 2 & -3 \\ -2 & -1 & 1 \end{bmatrix} \xrightarrow{R_{1,2}(-3)} A_1 \xrightarrow{R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 3 & -5 \\ -2 & -1 & 1 \end{bmatrix} \xrightarrow{R_{1,3}(2)} A_3$$

$$\xrightarrow[R_5]{R_4} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_6} A_7 \xrightarrow{R_8} \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = B$$

(a) Write out the matrices A_1, A_3 and A_7.

(b) Write out the row operations R_2, R_4, R_5, R_6 and R_8.

(c) Find the bases for the row space, the column space and the null space of A.

(d) Find the linear transformation $T:\mathbb{R}^4 \to \mathbb{R}^3$ corresponding to matrix A.

(e) Find the kernel, $\ker T$ and the image, $\operatorname{Im} T$ of T.

(f) Determine the rank, $\operatorname{rank}(T)$ and the nullity, $\operatorname{nul}(T)$ of T.

[100 marks]

3. *Matriks* $A=\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 2 & -3 \\ -2 & -1 & 1 \end{bmatrix}$ *diturunkan ke bentuk eselon terturun* B *melalui beberapa operasi baris permulaan yang sesuai seperti berikut:*

$$A=\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 2 & -3 \\ -2 & -1 & 1 \end{bmatrix} \xrightarrow{R_{1,2}(-3)} A_1 \xrightarrow{R_2} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 3 & -5 \\ -2 & -1 & 1 \end{bmatrix} \xrightarrow{R_{1,3}(2)} A_3$$

$$\xrightarrow[R_5]{R_4} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & -5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_6} A_7 \xrightarrow{R_8} \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & -\frac{5}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = B$$

(a) *Tuliskan matriks-matriks* A_1, A_3 *dan* A_7.

(b) *Tuliskan operasi-operasi baris* R_2, R_4, R_5, R_6 *dan* R_8.

(c) *Cari asas-asas untuk ruang baris, ruang lajur dan ruang nol untuk* A.

(d) *Cari transformasi linear* $T:\mathbb{R}^4 \to \mathbb{R}^3$ *bersepadan dengan matriks* A.

(e) *Cari inti,* $\ker T$ *dan imej,* $\operatorname{Im} T$ *untuk* T.

(f) *Tentukan pangkat,* $\operatorname{rank}(T)$ *dan kenolan,* $\operatorname{null}(T)$ *untuk* T.

[100 markah]

4. (a) Let W be a subspace of $\mathbb{R}^n$ and let $W^{\perp}$ be the set of all vectors orthogonal to W. Show that $W^{\perp}$ is a subspace of $\mathbb{R}^n$.

(b) Show that if $\vec{v}$ is both in W and $W^{\perp}$, then $\vec{v} = \vec{0}$.

(c) Suppose that a vector $\vec{x}$ is orthogonal to vectors $\vec{u}$ and $\vec{v}$. Show that $\vec{x}$ is orthogonal to every $\vec{w}$ in the linear span $L(\vec{u}, \vec{v})$ of $\vec{u}$ and $\vec{v}$.

[100 marks]

5. Let $M = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and consider the standard basis $A = e_1, e_2$ of $\mathbb{R}^2$. Then the corresponding linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $(x, y)T = (x + 2y, 4x + 3y)$.

(a) Write out the matrix $T_{A,A}$.

(b) Find the eigen values of T. Hence, find the corresponding eigen vectors.

(c) Find the matrix C such that $C^{-1}MC$ is a diagonal matrix.

[100 marks]

4. (a) *Andaikan* W *suatu subruang untuk* $\mathbb{R}^n$ *dan* $W^{\perp}$ *set semua vektor ortogon terhadap* W. *Tunjukkan bahawa* $W^{\perp}$ *ialah subruang untuk* $\mathbb{R}^n$.

(b) *Tunjukkan bahawa jika* $\vec{v}$ *terletak dalam* W *dan* $W^{\perp}$, *maka* $\vec{v} = \vec{0}$.

(c) *Andaikan vektor* $\vec{x}$ *ortogon terhadap vektor-vektor* $\vec{u}$ *dan* $\vec{v}$. *Tunjukkan bahawa* $\vec{x}$ *ortogon terhadap semua* $\vec{w}$ *dalam rentangan linear* $L(\vec{u},\vec{v})$ *untuk* $\vec{u}$ *dan* $\vec{v}$.

[100 markah]

5. *Andaikan* $M = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ *dan pertimbangkan asas piawai* $A = e_1, e_2$ *untuk* $\mathbb{R}^2$. *Maka transformasi linear* $T : \mathbb{R}^2 \to \mathbb{R}^2$ *yang sepadan ditakrifkan oleh* $(x, y)T = (x + 2y, 4x + 3y)$.

(a) *Tuliskan matriks* $T_{A,A}$.

(c) *Cari nilai-nilai eigen untuk* T. *Seterusnya cari vektor-vektor eigen yang sepadan.*

(c) *Cari matriks* C *sedemikian* $C^{-1}MC$ *ialah matriks pepenjuru.*

[100 markah]

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