UNIVERSITI SAINS MALAYSIA

First Semester Examination
2009/2010 Academic Session

November 2009

MAT 101 - Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>SEVEN</u> pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>TUJUH</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

*[**Arahan:**   Jawab **semua sepuluh** [10] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Find $f'\ 1$ for the following f

    (i) $f\ x\ =x^2\sin\ 2\pi x$

    (ii) $f\ 2x+1\ =x^2+2x+1$

    [6 marks]

2. Using the definition of derivative, find $f'\ 1$ for $f\ x\ =x^2\sqrt{x}$

    [6 marks]

3. (a)   Let $f\ x\ =2x+1$. Given $\varepsilon>0$, find in terms of ε a value of $\delta>0$ such that if $|x-2|<\delta$ then $|f\ x\ -5|<\varepsilon$.

    [3 marks]

    (b)   Using $\varepsilon-\delta,$ prove that $\lim\limits_{x\to1}x^2+2=3$

    [5 marks]

4. Find the following limits

    (i) $\lim\limits_{x\to2}\dfrac{x^3-3x^2+4x-4}{x^2-5x+6}$

    (ii) $\lim\limits_{x\to1}\dfrac{x^2+5x-6}{\sqrt{x^2+1}-\sqrt{3x-1}}$

    (iii) $\lim\limits_{x\to1}\dfrac{|x|-1}{1+x}$

    [9 marks]

5. Suppose $f:\mathbf{R}\to\mathbf{R}$ is an even function

    (i)   Prove that for any $x\in\mathbf{R}$, and $h\neq0$

    $$\frac{f\ -x+h\ -f\ -x}{h}=\frac{f\ x-h\ -f\ x}{h}$$

    (ii)   If f is differentiable on $\mathbf{R}$, show that $f':\mathbf{R}\to\mathbf{R}$ is an odd function.

    [7 marks]

1. Cari $f'\ 1$ untuk f yang berikut

    (i) $f\ x\ =x^2 \sin\ 2\pi x$

    (ii) $f\ 2x+1\ =x^2+2x+1$

    [6 markah]

2. Menggunakan takrif terbitan cari $f'\ 1$ untuk $f\ x\ =x^2\sqrt{x}$

    [6 markah]

3. (a) Andai $f\ x\ =2x+1$. Diberi $\varepsilon>0$, cari dalam sebutan ε suatu nilai $\delta>0$ supaya jika $|x-2|<\delta$ maka $|f\ x\ -5|<\varepsilon$.

    [3 markah]

    (b) Menggunakan $\varepsilon-\delta$, buktikan bahawa $\displaystyle\operatorname*{had}_{x\to 1} x^2+2=3$

    [5 markah]

4. Cari nilai had berikut

    (i) $\displaystyle\operatorname*{had}_{x\to 2}\frac{x^3-3x^2+4x-4}{x^2-5x+6}$

    (ii) $\displaystyle\operatorname*{had}_{x\to 1}\frac{x^2+5x-6}{\sqrt{x^2+1}-\sqrt{3x-1}}$

    (iii) $\displaystyle\operatorname*{had}_{x\to -1}\frac{|x|-1}{1+x}$

    [9 markah]

5. Andaikan $f:\mathbf{R}\to\mathbf{R}$ adalah suatu fungsi genap

    (i)   Buktikan bahawa untuk $x\in\mathbf{R}$, dan $h\neq 0$

    $$\frac{f\ -x+h\ -f\ -x}{h}=\frac{f\ x-h\ -f\ x}{h}$$

    (ii) Jika f terbezahan pada $\mathbf{R}$, tunjukkan bahawa $f':\mathbf{R}\to\mathbf{R}$ adalah suatu fungsi ganjil.

    [7 markah]

6. (a) Compute the first derivative of the following functions with respect to x

     (i) $f\ x\ =\ x^2+1^{\ 2}\sin\ x^2+1$

     (ii) $g\ x\ =\dfrac{\ell n\ x}{x^2+1},\quad x>0$

                                                   [6 marks]

   (b) The function f on the interval $0,1$ is given by $f\ x\ =x^x\ 1-x^{\ 1-x}$.

     (i) Find c so that $f'\ c\ =0$

     (ii) Show that $f\ c$ is a local minimum.

                                                   [6 marks]

7. (a) Show that $\dfrac{x-1}{x\ x+1^{\ 2}}=\dfrac{1}{x+1}-\dfrac{1}{x}+\dfrac{2}{x+1^{\ 2}}.$

     Hence determine $\displaystyle\int\dfrac{x-1}{x\ x+1^{\ 2}}dx$

                                                   [4 marks]

   (b) Determine $\displaystyle\int e^{\sqrt{x}}dx$

                                                   [4 marks]

   (c) If $f:\mathbf{R}\to\mathbf{R}$ is an odd function , show that $\displaystyle\int_{-p}^{0}f\ x\ dx=-\int_{0}^{p}f\ x\ dx$

                                                   [4 marks]

8. (a) Let f be a twice differentiable function on a,b and

$$g\ x\ =f\ x\ -f'\ c\ x-c\ -f\ c\ ,c\in\ a,b$$

     (i) Show that c is a critical number of g.

     (ii) If $f''\ c\ \le0,$ show by first derivative test that $g\ c$ is a local maximum.

                                                   [6 marks]

   (b) If $f\ x\ =\displaystyle\int_{2}^{x^2}\dfrac{\ell n\ t}{1+\ell n\ t}dt$, show that $f'\ \sqrt{e}\ =\sqrt{e}.$

                                                   [4 marks]

6.   (a)   *Hitung terbitan pertama terhadap x untuk fungsi yang berikut*

          (i) $f(x) = (x^2+1)^2 \sin(x^2+1)$

          (ii) $g(x) = \dfrac{\ell n\, x}{x^2+1}, \quad x > 0$

                                           [6 markah]

    (b)   *Fungsi f pada selang $(0,1)$ diberi sebagai $f(x) = x^x (1-x)^{1-x}$.*

          (i)   *Cari c supaya $f'(c) = 0$*

          (ii)   *Tunjukkan bahawa $f(c)$ minimum setempat.*

                                           [6 markah]

7.   (a)   *Tunjukan bahawa* $\dfrac{x-1}{x(x+1)^2} = \dfrac{1}{x+1} - \dfrac{1}{x} + \dfrac{2}{(x+1)^2}.$

          Seterusnya tentukan $\displaystyle\int \dfrac{x-1}{x(x+1)^2}$

                                           [4 markah]

    (b)   *Tentukan* $\displaystyle\int e^{\sqrt{x}}\, dx$

                                           [4 markah]

    (c)   *Jika $f: \mathbf{R} \to \mathbf{R}$ suatu fungsi ganjil, tunjukkan bahawa*

          $\displaystyle\int_{-p}^{0} f(x)\, dx = -\int_{0}^{p} f(x)\, dx$

                                           [4 markah]

8.   (a)   *Andai f sebagai suatu fungsi yang terbeza dua kali pada (a,b) dan*

          $g(x) = f(x) - f'(c)(x-c) - f(c), c \in (a,b)$

          (i)   *Tunjukan c suatu nombor genting g.*

          (ii)   *Jika $f''(c) \leq 0$, tunjukkan dengan ujian terbitan pertama bahawa $g(c)$ ialah maksimum setempat.*

                                           [6 markah]

    (b)   *Jika $f(x) = \displaystyle\int_{2}^{x^2} \dfrac{\ell n\, t}{1+\ell n\, t}\, dt$, tunjukan bahawa $f'(\sqrt{e}) = \sqrt{e}$.*

                                           [4 markah]

9. A region bounded by the curve $y = x^3 + e^x$, $x-axis$, $y-axis$ and the line $x = u$ whose area is denoted by $A\ u$ is rotated through $360°$ about the $x-axis$.

   (i)   Find $A\ u$.

   (ii) State in integral form the volume of the rotated solid.

   (iii) If the area $A\ u$ is increasing at 1 unit^2/sec as the line $x = u$ moves to the right then find the rate at which volume of the rotated solid is increasing when $u = 1$.

   [12 marks]

10. (a) Given that $f\ x = 1 + \dfrac{1}{x^2 - 4}$, $|x| \neq 2$.

   (i)   Find $f'\ x$ and $f''\ x$.

   (ii) Show that f has one stationary point and classify it.

   (iii) Find the equations of one horizontal and two vertical asymptotes.

   (iv) Sketch the graph of f.

   [12 marks]

   (b) Evaluate the definite integrals

   (i) $\displaystyle\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$

   (ii) $\displaystyle\int_0^1 x\sin\ \pi x\ dx$

   [6 marks]

9. Suatu rantau yang dibatasi oleh lengkung $y = x^3 + e^x$, $paksi-x$, $paksi-y$ dan garis $x = u$ yang luasnya di tanda dengan $A\ u$ di putar $360°$ pada $paksi-x$.

   (i) Cari $A\ u$.

   (ii) Nyatakan dalam bentuk kamiran isipadu bungkah putaran.

   (iii) Jika luas $A\ u$ menokok pada kadar 1 $unit^2$/saat apabila garis $x = u$ bergerak ke kanan, maka cari kadar perubahan isipadu bungkah putaran apabila $u = 1$.

   [12 markah]

10. (a) Diberi $f\ x = 1 + \dfrac{1}{x^2 - 4}$, $|x| \neq 2$.

   (i) Cari $f'\ x$ dan $f''\ x$.

   (ii) Tunjukkan f hanya mempunyai satu titik pegun dan tentukan jenisnya.

   (iii) Cari persamaan satu asimptot mendatar dan dua asimptot tegak.

   (iv) Lakar graf f.

   [12 markah]

   (b) Nilaikan kamiran tentu

   (i) $\int_1^2 \left(x + \dfrac{1}{x} \right)^2 dx$

   (ii) $\int_0^1 x\sin\ \pi x\ dx$

   [6 markah]

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