UNIVERSITI SAINS MALAYSIA

First Semester Examination
2009/2010 Academic Session

November 2009

MAA 111 – Algebra for Science Students
[Aljabar untuk Pelajar Sains]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

*[**Arahan:*** *Jawab **semua sembilan** [9] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Solve the system below using the Gauss elimination method.

$$\begin{aligned} 2x_1 + x_2 + 5x_3 + x_4 &= 5 \\ x_1 + x_2 - 3x_3 - 4x_4 &= -1 \\ 3x_1 + 6x_2 - 2x_3 + x_4 &= 8 \\ 2x_1 + 2x_2 + 2x_3 - 3x_4 &= 2 \end{aligned}$$

[10 marks]

2. Solve the simultaneous equations using Cramer's Rule.

$$\begin{aligned} 3x_1 - x_2 + 5x_3 &= -2 \\ -4x_1 + x_2 + 7x_3 &= 10 \\ 2x_1 + 4x_2 - x_3 &= 3 \end{aligned}$$

[8 marks]

3. Determine a basis and dimension for the null space of

$$A = \begin{bmatrix} 7 & 2 & -2 & -4 & 3 \\ -3 & -3 & 0 & 2 & 1 \\ 4 & -1 & -8 & 0 & 20 \end{bmatrix}$$

[10 marks]

4. Consider the matrix $B = \begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$. Is B diagonalizable? If yes, find P such that $P^{-1}BP$ is diagonal.

[14 marks]

5. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(x, y, z) = (x + 2y - z,\ y + z,\ x + y - 2z).$$

(a) Find a basis and the dimension of the image of T.
(b) Find a basis and the dimension of the kernel of T.
(c) From your result in (a) and (b), verify the Dimension Theorem.

[10 marks]

1. *Selesaikan sistem di bawah menggunakan kaedah penghapusan Gauss.*

$$\begin{aligned} 2x_1 + x_2 + 5x_3 + x_4 &= 5 \\ x_1 + x_2 - 3x_3 - 4x_4 &= -1 \\ 3x_1 + 6x_2 - 2x_3 + x_4 &= 8 \\ 2x_1 + 2x_2 + 2x_3 - 3x_4 &= 2 \end{aligned}$$

[10 markah]

2. *Selesaikan persamaan serentak dengan menggunakan Petua Cramer.*

$$\begin{aligned} 3x_1 - x_2 + 5x_3 &= -2 \\ -4x_1 + x_2 + 7x_3 &= 10 \\ 2x_1 + 4x_2 - x_3 &= 3 \end{aligned}$$

[8 markah]

3. *Tentukan suatu asas dan dimensi ruang nol bagi*

$$A = \begin{bmatrix} 7 & 2 & -2 & -4 & 3 \\ -3 & -3 & 0 & 2 & 1 \\ 4 & -1 & -8 & 0 & 20 \end{bmatrix}$$

[10 markah]

4. *Pertimbangkan matriks* $B = \begin{bmatrix} 4 & 0 & 1 \\ -1 & -6 & -2 \\ 5 & 0 & 0 \end{bmatrix}$. *Adakah* B *terpepenjurukan? Jika ya, cari P sedemikian* $P^{-1}BP$ *adalah pepenjuru.*

[14 markah]

5. *Biar* $T : \mathbb{R}^3 \to \mathbb{R}^3$ *suatu transformasi linear ditakrifkan sebagai*

$$T(x, y, z) = (x + 2y - z,\ y + z,\ x + y - 2z).$$

(a) Cari suatu asas dan dimensi imej bagi T.
(b) Cari suatu asas dan dimensi inti bagi T.
(c) Daripada keputusan anda dalam (a) dan (b), tentusahkan Teorem Dimensi.

[10 markah]

6. By inspection, determine whether the following sets of vectors are linearly independent or linearly dependent. Justify your answer.

(a) $\langle -2,3,2\rangle$, $\langle 0,0,0\rangle$, $\langle 0,1,1\rangle$

(b) $\langle -2,2\rangle$, $\langle 0,1\rangle$, $\langle 4,3\rangle$, $\langle 7,-1\rangle$

(c) $\langle 8,1,0\rangle$, $\langle 2,2,3\rangle$

[6 marks]

7. If you have 4 vectors in $\mathbb{R}^4$ which are linearly independent, can you always conclude they span $\mathbb{R}^4$? Justify your answer.

[2 marks]

8. Define the linear combination of vectors in a vector space.

[2 marks]

9. Consider the vectors $v_1 = (1,1,1)$, $v_2 = (0,1,1)$ and $v_3 = (0,0,1)$ which form a basis of $\mathbb{R}^3$. Use the Gram-Schmidt method to find the orthonormal basis of $\mathbb{R}^3$.

[8 marks]

6. *Dengan pemeriksaan, tentukan sama ada vektor-vektor berikut adalah tak bersandar linear atau bersandar linear. Jelaskan jawapan anda.*

   (a) $-2,3,2$, $0,0,0$, $0,1,1$

   (b) $-2,2$, $0,1$, $4,3$, $7,-1$

   (c) $8,1,0$, $2,2,3$

   [6 markah]

7. *Jika anda mempunyai 4 vektor tak bersandar linear dalam* $\square^4$*, bolehkah anda sentiasa membuat kesimpulan bahawa vektor tersebut merentang* $\square^4$*? Jelaskan jawapan anda.*

   [2 markah]

8. *Takrifkan gabungan linear vektor dalam suatu ruang vektor.*

   [2 markah]

9. *Pertimbangkan vektor* $v_1 = (1,1,1)$, $v_2 = (0,1,1)$ *dan* $v_3 = (0,0,1)$ *yang merupakan suatu asas bagi* $\square^3$*. Guna kaedah Gram-Schmidt untuk mencari suatu asas ortonormal bagi* $\square^3$*.*

   [8 markah]

- ooo O ooo -