UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2009/2010

Jun 2010

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

*[**Arahan:** Jawab **semua sembilan** [9] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. List down all the standard bases for the following vector spaces. Justify your answer.

(a) $\square^5$ over $\square$.

(b) $\square^5$ over $\square$.

(c) $M_{3\times 2}(\square)$ over $\square$.

(d) $P_2(\square)$ over $\square$.

[20 marks]

2. Determine whether the following sets of vectors are linearly independent over the respective field. Justify your answer.

(a) $V_1 = \{1, 1-x, 1-x^2\}$.

(b) $V_2 = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right\}$.

(c) $V_3 = \{(1,i),(2i,1),(1-i,0)\}$ over $\square$.

(d) $V_4 = \{(1,i),(2i,1),(1-i,0)\}$ over $\square$.

[40 marks]

3. Solve the following linear system.

$$\begin{aligned} 2x+y-2z+3w &= 1 \\ 3x+2y-z+2w &= 4 \\ 3x+3y+3z-3w &= 5 \end{aligned}$$

[10 marks]

1. *Senaraikan semua asas piawai bagi ruang vektor berikut. Hujahkan jawapan anda.*

   (a) $\mathbb{R}^5$ *atas* $\mathbb{R}$.

   (b) $\mathbb{R}^5$ *atas* $\mathbb{Q}$.

   (c) $M_{3\times 2}(\mathbb{R})$ *atas* $\mathbb{R}$.

   (d) $P_2(\mathbb{R})$ *atas* $\mathbb{R}$.

   [20 markah]

2. *Tentukan sama ada set-set vektor berikut adalah tak bersandar linear atas medan masing-masing. Hujahkan jawapan anda.*

   (a) $V_1 = \{1, 1-x, 1-x^2\}$.

   (b) $V_2 = \left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right\}$.

   (c) $V_3 = \{(1,i),(2i,1),(1-i,0)\}$ *atas* $\mathbb{R}$.

   (d) $V_4 = \{(1,i),(2i,1),(1-i,0)\}$ *atas* $\mathbb{C}$.

   [40 markah]

3. *Selesaikan sistem linear berikut.*

$$\begin{aligned} 2x + y - 2z + 3w &= 1 \\ 3x + 2y - z + 2w &= 4 \\ 3x + 3y + 3z - 3w &= 5 \end{aligned}$$

[10 markah]

4. Let

$$A=\begin{pmatrix} 1+i & 0 & 1+2i \\ 1-i & 0 & 2-3i \\ 1-2i & 2+3i & 0 \end{pmatrix}.$$

Using row reduction method, evaluate the determinant of *A*.

[20 marks]

5. Let $T:\mathbb{R}^3 \to P_2(\mathbb{R})$ be a linear transformation such that

$$T(a_1,a_2,a_3)=a_1+(a_1+a_2)x+(a_1+a_2+a_3)x^2.$$

Determine whether *T* is an isomorphism.

[20 marks]

6. Let $T:P_3(\mathbb{R})\to P_2(\mathbb{R})$ be a linear transformation. Give the matrix representation of *T* relative to the standard basis.

[10 marks]

7. Let $S:\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$S(x,y,z)=(-x,-z,y).$$

(a) Determine whether *S* can be diagonalised or not.
(b) Find the Jordan canonical form for *S*.
(c) Compute S^k, k is a positive integer.

[40 marks]

8. Let *T* be the linear operator on $\mathbb{R}^3$ defined by

$$T(x,y,z)=(2x+(1+2i)y-4iz,(3-2i)x+iz,(2i-1)x+(1+3i)y-3iz).$$

Find $T^*(x,y,z)$, where T^* is the adjoint of *T*.

[20 marks]

4. *Biar*

$$A=\begin{pmatrix} 1+i & 0 & 1+2i \\ 1-i & 0 & 2-3i \\ 1-2i & 2+3i & 0 \end{pmatrix}.$$

Dengan menggunakan kaedah penurunan baris, nilaikan penentu bagi A.

[20 markah]

5. *Biar* $T:\mathbb{R}^3 \to P_2(\mathbb{R})$ *suatu transformasi linear sedemikian hingga*

$$T(a_1,a_2,a_3)=a_1+(a_1+a_2)x+(a_1+a_2+a_3)x^2.$$

Tentukan sama ada T adalah suatu isomorfisma.

[20 markah]

6. *Biar* $T:P_3(\mathbb{R})\to P_2(\mathbb{R})$ *suatu transformasi linear. Berikan matriks perwakilan T terhadap asas piawai.*

[10 markah]

7. *Biar* $S:\mathbb{R}^3 \to \mathbb{R}^3$ *suatu transformasi linear sedemikian hingga*

$$S(x,y,z)=(-x,-z,y).$$

(a) Tentukan sama ada S boleh dipepenjurukan.
(b) Dapatkan bentuk berkanun Jordan bagi S.
(c) Hitung S^k, k *suatu integer positif.*

[40 markah]

8. *Biar T suatu operator linear pada* $\mathbb{R}^3$ *ditakrifkan sebagai*

$$T(x,y,z)=(2x+(1+2i)y-4iz,(3-2i)x+iz,(2i-1)x+(1+3i)y-3iz).$$

Dapatkan $T^*(x,y,z), T^*$ adalah adjoint bagi *T*.

[20 markah]

9. Let S and T be linear operators on V a vector space. Suppose S^* dan T^* are the respective adjoint of S and T, Show that

(a) $S^* + T^* = (S+T)^*$.

(b) $(ST)^* = T^* S^*$.

(c) $(T^*)^* = T$.

[20 marks]

9. *Biar S dan T linear operator pada V suatu ruang vektor. Biar* S^* dan T^* *adalah adjoint bagi S dan T masing-masing, tunjukkan bahawa*

(a) $S^* + T^* = (S+T)^*.$

(b) $(ST)^* = T^* S^*.$

(c) $(T^*)^* = T.$

[20 markah]

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