UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2009/2010

Jun 2010

MAT 518 – Numerical Methods for Differential Equations
[Kaedah Berangka untuk Persamaan Pembezaan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Consider the heat equation $u_t = u_{xx}$.
   (a) Write the Forward Time Centered Space Scheme

   (b) Analyze the stability using the Fourier methods

   (c) Analyze the consistency of the scheme

[100 marks]

2. (a) Consider the equation $u_t = u_{xx} + u_{yy}$. Describe the main steps in the ADI method of solution.
   (Your answer should be approximately two pages long)

[100 marks]

3. (a) Decide the convergence or divergence of Jacobi and Gauss-Seidel iterations for the linear solution Ax = b if

$$A = \begin{bmatrix} 5 & 3 & 4 \\ 3 & 6 & 4 \\ 4 & 4 & 5 \end{bmatrix}.$$

   (b) Consider the Laplace equation

$$\nabla^2 u = 0, \qquad 0 < y < \frac{\pi}{2},\ 0 < x < \frac{\pi}{2},$$

   with values of u defined at the boundary. Suppose that the five-point difference formula is used to discretise the equation with mesh size h, show that the spectral radius of the SOR iteration is $\rho(L_\omega) = \dfrac{1 - \sin 2h}{1 + \sin 2h}$.

   (c) Assuming $\Delta x = \Delta y = h$, discretise the following Poisson equation using the centred difference formula:

$$-u_{xx} - u_{yy} + 4u_x = f(x, y), \qquad (x, y) \in (0,1) x (0,1)$$

$$u(x,0) = u(x,1) = 0, x \in (0,1)$$
$$u(0, y) = u(1, y) = 0, y \in (0,1)$$

   with h = $\frac{1}{2}$. Generate the linear system which arises from this discretisation using row-wise natural ordering.

[100 marks]

1. *Pertimbang persamaan haba* $u_t = u_{xx}$.

   (a) *Tulis skema beza masa kedepan, ruang ke pusat*

   (b) *Analisis stabili menggunakan kaedah Fourier*

   (c) *Analisis kekomisteraan skema*

[100 markah]

2. (a) *Pertimbang persamaan* $u_t = u_{xx} + u_{yy}$. *Huraikan langkah-langkah penting bagi kaedah penyelesaian ADI*
   (Jawapan anda seharusnya sekitarnya dua muka surat panjang)

[100 markah]

3. (a) *Tentukan penumpuan atau pencapahan lelaran Jacobi dan Gauss-Seidel bagi penyelesaian sistem linear Ax = b jika*

$$A = \begin{bmatrix} 5 & 3 & 4 \\ 3 & 6 & 4 \\ 4 & 4 & 5 \end{bmatrix}.$$

   (b) *Pertimbangkan persamaan Laplace*

$$\nabla^2 u = 0, \qquad 0 < y < \frac{\pi}{2},\ 0 < x < \frac{\pi}{2},$$

   dengan nilai u tertakrif pada sempadan. Katakan rumus beza lima-titik digunakan untuk mendiskretkan persamaan ini dengan saiz mesy h, tunjukkan bahawa jejari spektrum bagi lelaran SOR ialah $\rho(L_\omega) = \dfrac{1 - \sin 2h}{1 + \sin 2h}$.

   (c) *Dengan menganggapkan* $\Delta x = \Delta y = h$, *diskretkan persamaan Poisson berikut dengan menggunakan rumus beza ketengah:*

$$-u_{xx} - u_{yy} + 4u_x = f(x, y), \qquad (x, y) \in (0,1)x(0,1)$$

$$u(x,0) = u(x,1) = 0, x \in (0,1)$$
$$u(0, y) = u(1, y) = 0, y \in (0,1)$$

   dengan $h = \frac{1}{2}$. *Janakan sistem linear yang terhasil daripada pendiskretan ini dengan menggunakan tertib baris biasa.*

[100 markah]

4. (a) Consider the following elliptic problem

$$\nabla^2 u = 0, \qquad 0 < x, y < 1$$

with values of u defined at the boundary.

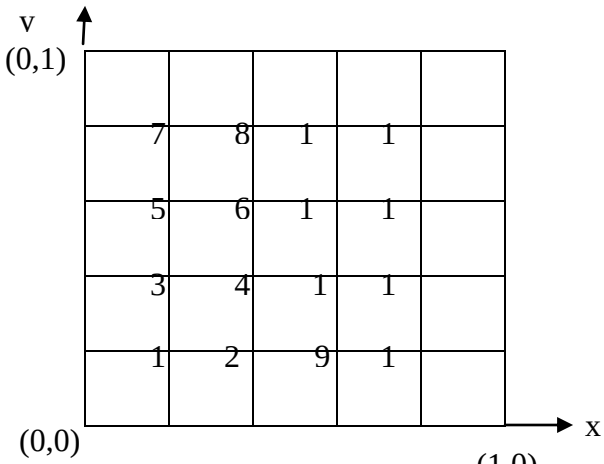


Figure 1

i. Suppose that the five point difference formula is used to discretise the partial differential equation using the ordering as shown in Figure 1. If the resulted block matrix system is solved using the two line SOR (S.2.L.O.R) method, what is the estimated optimum relaxation parameter ω_b and the spectral radius of the S.2.L.O.R iteration matrix $\rho(L_{\omega b})$?

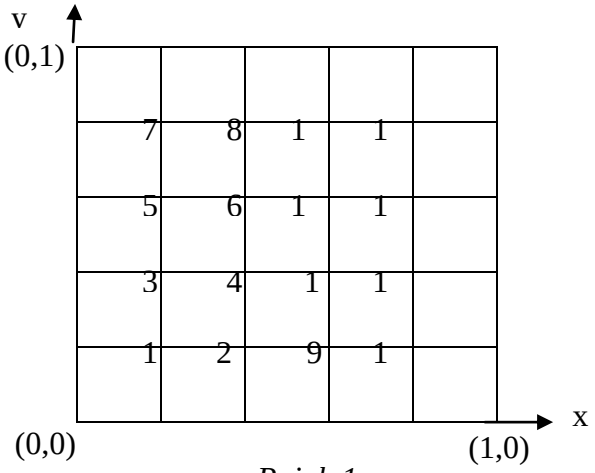
ii. What is the approximate theoretical number of iterations you would expect to get if the two line S.O.R. method is used for mesh size n = 21 and tolerance $\varepsilon = 10^{-7}$.

[100 marks]

4. (a) *Pertimbangkan masalah eliptik berikut*

$$\nabla^2 u = 0, \qquad 0 < x, y < 1$$

dengan nilai u tertakrif pada sempadan.



Rajah 1

i. *Katakan rumus beza terhingga lima titik digunakan untuk mendiskretkan persamaan pembezaan separa ini dengan menggunakan tertib seperti yang ditunjukkan dalam Rajah 1. Sekiranya sistem matriks blok yang terhasil diselesaikan dengan menggunakan kaedah SOR (S.2.L.O.R) dua garis, apakah parameter pengenduran optimum anggaran* ω_b *dan jejari spektrum bagi matriks lelaran S.2.L.O.R dua garis,* $\rho(L_{\omega_b})$ *?*

ii. *Apakah anggaran bilangan lelaran secara teori yang anda dapat jangkakan jika kaedah S.O.R. dua garis digunakan untuk saiz mesy n = 21 dan toleran* $\varepsilon = 10^{-7}$.

[100 markah]

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