UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2009/2010

Jun 2010

MAT 202 - Introduction to Analysis
[Pengantar Analisis]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>SEVEN</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>TUJUH</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u>: Answer **<u>all three</u>** [3] questions.

*[**<u>Arahan:</u>** Jawab **<u>semua tiga</u>** [3] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) State the completeness axiom for $\mathbb{R}$

[$\mathbb{R}$ = set of all real numbers]

[5 marks]

(b) Find the supremum and infimum for the following sets if they exist.

(i) $A = \left\{ \frac{n}{n+1} | n \in \mathbb{N} \right\}$

[$\mathbb{N}$ = set of all natural numbers]

[10 marks]

(ii) $A = \{ r \in Q : r^2 < 4 \}$

[Q= set of all rational numbers]

[10 marks]

(c) Prove that $\sqrt{2}+\sqrt{3}$ is not rational

[10 marks]

(d) Define a countable set.

Let *B* be a set of all circles with a centre at a point $(a, 0)$ and radius r where *r* is a rational number and *a* a natural number. Determine whether *B* is accountable set.

[15 marks]

1. (a) *Nyatakan aksiom kelengkapan* $\mathbb{R}$

[$\mathbb{R}$ *= set semua nombor nyata]*

[5 markah]

(b) *Cari supremum dan infimum untuk set berikut jika wujud.*

(i) $A = \left\{ \frac{n}{n+1} \middle| n \in \mathbb{N} \right\}$

[$\mathbb{N}$ *= set semua nombor asli]*

[10 markah]

(ii) $A = \{ r \in Q : r^2 < 4 \}$

[Q *= set semua nombor nisbah]*

[10 markah]

(c) *Buktikan bahawa* $\sqrt{2} + \sqrt{3}$ *bukan nombor nisbah*

[10 markah]

(d) *Takrifkan set terbilangkan.*

Biar B set semua bulatan degnan pusatnya pada titik $(a, 0)$ *dan jejari r dimana r adalah nombor nisbah dan a nombor asli. Tentukan samada B adalah terbilangkan.*

[15 markah]

2. (a) Let $a_n = \left\{ \dfrac{1-2n^2}{4n+3n^2} : n \in \square \right\}$

(i) Determine the limit of a_n .

(ii) Use the definition to verity the convergence of a_n .

(iii) Find the smallest integer $\square$ such that
$\left| \dfrac{1-2n^2}{4n+3n^2} - \left(\dfrac{-2}{3} \right) \right| < 0.01$ for all $n \geq \square$.

[20 marks]

(b) Let a_n and b_n be convergent sequences and suppose that x_n is a sequence such that $a_n \leq x_n \leq b_n$ for all n. If the sequences a_n and b_n both converge to L, then show that the sequence x_n converges to L.

[20 marks]

(c) Using the definition of Cauchy sequence, determine whether the sequence $\left\{ \dfrac{2n^2+1}{n^2} \right\}$ is Cauchy.

[15 marks]

(d) Given a set $A = 2,5 \cap Q$.
[Q = set of all rational numbers]

Find the interior points, accumulation/limit point and the isolated points.
Furthermore, determine whether *A* is open or closed or neither.

[25 marks]

2. (a) *Biar* $a_n = \left\{ \dfrac{1-2n^2}{4n+3n^2} : n \in □ \right\}$

(i) *Tentukan had* a_n

(ii) *Gunakan takrifan untuk menentusahkan penumpuan* a_n .

(iii) *Cari integer terkecil* □ *supaya*

$\left| \dfrac{1-2n^2}{4n+3n^2} - \left(\dfrac{-2}{3} \right) \right| < 0.01$ *for all* $n \geq □$.

[20 markah]

(b) *Biar* a_n *dan* b_n *jujukan menumpu dan andaikan bahawa* x_n *adalah satu jujukan supaya* $a_n \leq x_n \leq b_n$ *untuk semua* n. *Jika jujukan* a_n *dan* b_n *kedua-duanya menumpu ke* L, *tunjukkan bahawa jujukan* x_n *menumpu ke* L.

[20 markah]

(c) *Gunakan takrifan jujukan Cauchy, tentukan samada jujukan* $\left\{ \dfrac{2n^2+1}{n^2} \right\}$ *adalah Cauchy.*

[15 markah]

(d) *Diberi set* $A = 2,5 \cap Q$.

[Q *= set semua nombor nisbah]*

Cari titik pedalaman, titik had dan titik terpencil.
Kemudian tentukan samada A adalah terbuka atau tertutup atau bukan kedua-duanya.

[25 markah]

3. (a) Given $T = \{(-n,n) : n \in \mathbb{N}\}$, an open covering of a set $A \subseteq \mathbb{R}$, where *A* is a compact set, show that *A* is bounded.

[20 marks]

(b) Given a set $A = (0,1)$ and a collection

$$\mathfrak{I} = \left\{ \left(\frac{1}{3}, 1\right), \left(\frac{1}{4}, \frac{1}{2}\right), \left(\frac{1}{5}, \frac{1}{3}\right), \ldots \left(\frac{1}{n}, \frac{1}{n-2}\right), \ldots \right\}.$$

(i) Determine whether $\mathfrak{I}$ is an open covering for *A*. Give reason.

(ii) Use the definition of compactness (in terms of open covering) to show that A is not compact.

[20 marks]

(c) Given a function $f(x) = \dfrac{1}{x^2}$, show that this function is uniformly continuous on (c, ∞) for any fixed $c > 0$ but not uniformly continuous on $(0, \infty)$.

[25 marks]

(d) Let $\{f_n\}$ be a sequence of functions which are bounded on a set A, and let the sequence converge to function f on *A*. Denote $R_n = \sup\limits_{x \in A} |f_n(x) - f(x)|$. Then, show that $\{f_n\}$ converges uniformly on *A* if and only if $\lim\limits_{n \to \infty} R_n = 0$.

[15 marks]

3. (a) *Diberi* $T = \{(-n,n) : n \in \mathbb{N}\}$ *, suatu tudung terbuka bagi set* $A \subseteq \mathbb{R}$*, dimana A adalah set padat, tunjukkan bahawa A terbatas.*

[20 markah]

(b) *Diberi set* $A = (0,1)$ *dan pungutan*

$$\Im = \left\{ \left(\frac{1}{3},1\right), \left(\frac{1}{4},\frac{1}{2}\right), \left(\frac{1}{5},\frac{1}{3}\right), \ldots, \left(\frac{1}{n},\frac{1}{n-2}\right), \ldots \right\}.$$

(i) *Tentukan samada* $\Im$ *adalah tudung terbuka bagi A. Beri alasan.*

(ii) *Gunakan takrifkan kepadatan (dalam sebutan tudung terbuka) untuk menunjukan bahawa A bukan padat.*

[20 markah]

(c) *Diberi fungsi* $f(x) = \dfrac{1}{x^2}$*, tunjukkan bahawa fungsi ini selanjar secara seragam pada* $[c,\infty)$ *untuk titip tetap* $c > 0$ *tetapi tidak selanjar secara seragam pada* $(0,\infty)$ *.*

[25 markah]

(d) *Biar* $\{f_n\}$ *jujukan fungsi yang terbatas pada set A, dan biar jujukan ini menumpu kepada fungsi f pada A. Tandakan* $R_n = \sup_{x \in A} |f_n(x) - f(x)|$*. Kemudian tunjukkan bahawa* $\{f_n\}$ *menumpu secara seragam pada A jika dan hanya jika* $\operatorname{had}_{n \to \infty} R_n = 0.$

[15 markah]

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