UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2009/2010

Jun 2010

MAT 102 – Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

*[**Arahan:** Jawab **semua sepuluh** [10] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Find the limit if it exists. Otherwise show that the limit does not exist.

(a) $\lim_{x\to 0}\frac{1-\sqrt{1-x^2}}{x}$

(b) $\lim_{x\to\infty} x^{e^{-x}}$

(c) $\lim_{x\to -\infty} e^{\pi+c}$, where c is a constant

(d) $\lim_{(x,y,z)\to(0,0,0)}\frac{x^2+2y^2+3z^2}{x^2+y^2+z^2}$

[24 marks]

2. A sequence is an ordered list of numbers whereas a series is the sum of a list of numbers. Consider the sequence $\left\{1+\frac{(-1)^n}{n+2}\right\}_{n=1}^{\infty}$.

(a) List the first four terms of the sequence.

(b) Is this sequence monotonic? Give your reason.

(c) Does the sequence converge? Give your reason.

(d) Does the series $\sum_{n=1}^{\infty}\left[1+\frac{(-1)^n}{n+2}\right]$ converge or diverge? Give your reason.

[16 marks]

3. Determine whether each of the following series converges.

(a) $\sum_{n=1}^{\infty}\frac{5(2^{n-1})}{3^n}$

(b) $\sum_{n=1}^{\infty}\frac{(n+1)^2}{n(n+2)}$

(c) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3^n}{(n+1)!}$

[18 marks]

1. *Cari had jika wujud. Sebaliknya tunjukkan bahawa had tersebut tidak wujud.*

(a) $\lim_{x \to 0} \frac{1-\sqrt{1-x^2}}{x}$

(b) $\lim_{x \to \infty} x^{e^{-x}}$

(c) $\lim_{x \to -\infty} e^{\pi + c}$, dengan c ialah pemalar

(d) $\lim_{(x,y,z) \to (0,0,0)} \frac{x^2+2y^2+3z^2}{x^2+y^2+z^2}$

[24 markah]

2. *Jujukan adalah satu senarai nombor bertertib sedangkan siri ialah jumlah satu senarai nombor. Pertimbangkan jujukan* $\left\{1+\frac{(-1)^n}{n+2}\right\}_{n=1}^{\infty}$.

(a) *Senaraikan empat sebutan pertama jujukan tersebut.*

(b) *Adakah jujukan itu berekanada? Beri alasan anda.*

(c) *Adakah jujukan itu menumpu? Beri alasan anda.*

(d) *Adakah siri* $\sum_{n=1}^{\infty}\left[1+\frac{(-1)^n}{n+2}\right]$ *menumpu atau mencapah? Beri alasan anda.*

[16 markah]

3. *Tentukan sama ada setiap siri berikut menumpu.*

(a) $\sum_{n=1}^{\infty} \frac{5(2^{n-1})}{3^n}$

(b) $\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$

© $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n}{(n+1)!}$

[18 markah]

4. By using the fact that $\frac{1}{1+x^2}=\sum_{n=0}^{\infty}(-1)^n x^{2n}$ for $|x|<1$, find the power series representation for $\frac{x}{1+4x^2}$.

[6 marks]

5. Determine whether each of the integrals converges.

(a) $\int_3^{\infty}\frac{1}{x-1}dx$

(b) $\int_2^{3}\frac{1}{x-1}dx$

© $\int_1^{2}\frac{1}{x-1}dx$

[20 marks]

6. Is the function

$$f(x,y)=\begin{cases}\frac{x^2-y^2}{x^2+y^2} & \text{if } (x,y)\neq(0,0),\\ 0 & \text{if } (x,y)=(0,0),\end{cases}$$

continuous at $(0,0)$? Give your reason.

[10 marks]

4. *Dengan menggunakan fakta bahawa* $\frac{1}{1+x^2}=\sum_{n=0}^{\infty}(-1)^n x^{2n}$ *untuk* $|x|<1$, *cari perwakilan siri kuasa untuk* $\frac{x}{1+4x^2}$.

[6 markah]

5. *Tentukan sama ada setiap kamiran berikut menumpu.*

(a) $\int_3^{\infty}\frac{1}{x-1}dx$

(b) $\int_2^{3}\frac{1}{x-1}dx$

© $\int_1^{2}\frac{1}{x-1}dx$

[20 markah]

6. *Adakah fungsi*

$$f(x,y)=\begin{cases}\frac{x^2-y^2}{x^2+y^2} & \text{if } (x,y)\neq(0,0),\\ 0 & \text{if } (x,y)=(0,0),\end{cases}$$

selanjar pada $(0,0)$ *? Beri alasan anda.*

[10 markah]

7. Suppose that $f(x,y)$ is differentiable at $(1,1)$ and its total derivative is $f'(1,1)=\langle 2,1\rangle$.

(a) What are the partial derivatives f_x and f_y of f at $(1,1)$?

(b) Given that the directional derivative $D_{\mathbf{v}} f(1,1)$ is $\frac{1}{2}\sqrt{5}$, find the angle between $f'(1,1)$ and $\mathbf{v}$.

[10 marks]

8. If $w=x^2+yz$ and $x=3t^2+1,\ y=2t-4,\ z=t^3$, find $\dfrac{dw}{dt}$ in term of t.

[7 marks]

9. A region is bounded by the graphs of $2y=16-x^2$ and $x+2y-4=0$. Sketch the region and set up the double integral that represents the area.

[10 marks]

10. *True* or *False*

(a) If the sequence $\{a_n\}$ is convergent, then $\lim\limits_{n\to\infty} a_n = 0$.

(b) If (a,b) is a critical point of $f:\square\times\square\to\square$, then $f_x(a,b)=0$.

© If (a,b) is a critical point of $f:\square\times\square\to\square$ and $f_{xx}(a,b)f_{yy}(a,b)<\left[f_{xy}(a,b)\right]^2$, then f has a saddle point at (a,b).

(d) If we want to find the maximum and minimum values of a function $f(x,y,z)$, we can always use the Lagrange multiplier method.

(e) If $\lim\limits_{(a,y)\to(a,b)} f(x,y)$ and $\lim\limits_{(x,b)\to(a,b)} f(x,y)$ exist, then $\lim\limits_{(x,y)\to(a,b)} f(x,y)$ exists.

[10 marks]

7. *Andaikan* $f(x,y)$ *terbezakan pada* $(1,1)$ *dan terbitan seluruhnya ialah* $f'(1,1) = \langle 2,1 \rangle$.

(a) *Apakah terbitan separa* f_x *dan* f_y *untuk* f *pada* $(1,1)$ *?*

(b) *Diberi terbitan berarah* $D_{\vec{v}} f(1,1)$ *ialah* $\frac{1}{2}\sqrt{5}$, *cari sudut di antara* $f'(1,1)$ *dan* $\vec{v}$.

[10 markah]

8. *Jika* $w = x^2 + yz$ *dan* $x = 3t^2 + 1$, $y = 2t - 4$, $z = t^3$, *cari* $\dfrac{dw}{dt}$.

[7 markah]

9. *Suatu rantau dibatasi oleh graf-graf* $2y = 16 - x^2$ *dan* $x + 2y - 4 = 0$. *Lakarkan rantau tersebut dan bentukkan kamiran berganda yang mewakili luas rantau tersebut.*

[10 markah]

10. *Benar atau Palsu*

(a) *Jika jujukan* $\{a_n\}$ *menumpu, maka* $\lim\limits_{n \to \infty} a_n = 0$.

(b) *Jika* (a,b) *ialah titik genting untuk* $f : \square \times \square \to \square$ *, maka* $f_x(a,b) = 0$.

© *Jika* (a,b) *ialah titik genting untuk* $f : \square \times \square \to \square$ *dan* $f_{xx}(a,b) f_{yy}(a,b) < \left[f_{xy}(a,b) \right]^2$, *maka* f *mempunyai titik pelana pada* (a,b).

(d) *Jika kita ingin mencari nilai-nilai maksimum dan minimum untuk suatu fungsi* $f(x,y,z)$ *, kita sentiasa boleh menggunakan kaedah pendarab Lagrange.*

(e) *Jika* $\lim\limits_{(a,y) \to (a,b)} f(x,y)$ *dan* $\lim\limits_{(x,b) \to (a,b)} f(x,y)$ *wujud, maka* $\lim\limits_{(x,y) \to (a,b)} f(x,y)$ *wujud.*

[10 markah]

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