UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2009/2010

Jun 2010

MAA 102 – Calculus for Science Students II
[Kalkulus untuk Pelajar Sains II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>FIVE</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>LIMA</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini].

<u>Instructions</u>:   Answer **<u>all ten</u>** [10] questions.

*[**<u>Arahan:</u>**   Jawab **<u>semua sepuluh</u>** [10] soalan].*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Consider the sequence, defined by $a_n = \left(\dfrac{2}{3}\right)^n$, $n \geq 1$.

   (a) Write the first five terms of the sequence.

   (b) Determine whether the sequence converges or diverges.

   (c) Let $b_n = \dfrac{a_{n+1}}{a_n}$. Find the limit of b_n when $n \to \infty$.

   [5 marks]

2. Test the convergence of the series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \dfrac{\sqrt[3]{n}}{n+4}$ (b) $\displaystyle\sum_{n=1}^{\infty} \dfrac{-4^n}{n!}$

   [10 marks]

3. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \dfrac{x-2^n}{n3^n}$.

   [12 marks]

4. Find the coefficient of $x-3^2$ in the Taylor polynomial $T_2 \, x$ for the function x^3 at the number 3.

   [6 marks]

5. Show that $\displaystyle\int_1^{\infty} \dfrac{|\sin x|}{x^2}\, dx$ converges.

   [5 marks]

6. (a) Find the domain of the function $f \, x, y = \dfrac{\sqrt{x+y+1}}{x-1}$.

   (b) Let $z = \dfrac{x-y}{x+y}$, where $x = u^2 + v^2 + w$ and $y = uvw$. Draw the tree diagram to write out the chain Rule to find $\dfrac{\partial z}{\partial u}$ at the point $u, v, w = 1, -1, 1$.

   [13 marks]

   …3/-

1. *Pertimbangkan jujukan yang ditakrif oleh* $a_n = \left(\dfrac{2}{3}\right)^n$, $n \geq 1$.

    (a)   *Tulis lima sebutan pertama bagi jujukan.*

    (b)   *Tentukan samada jujukan menumpu atau mencapah.*

    (c)   *Biar* $b_n = \dfrac{a_{n+1}}{a_n}$ *. Dapatkan had bagi* b_n *apabila* $n \to \infty$.

    [5 markah]

2. *Uji penumpuan siri:*

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n+4}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{-4^{\,n}}{n!}$

    [10 markah]

3. *Dapatkan jejari dan selang penumpuan bagi siri kuasa* $\displaystyle\sum_{n=1}^{\infty} \frac{x-2^{\,n}}{n3^n}$.

    [12 markah]

4. *Dapatkan koefisien bagi* $x-3^{\,2}$ *dalam polinomial Taylor* $T_2\,x$ *bagi fungsi* x^3 *pada nombor 3.*

    [6 markah]

5. *Tunjukkan bahawa* $\displaystyle\int_1^{\infty} \frac{|\sin x|}{x^2}\,dx$ *menumpu.*

    [5 markah]

6. *(a)*   *Dapatkan domain bagi fungsi* $f\,x,y = \dfrac{\sqrt{x+y+1}}{x-1}$.

    (b)   *Biar* $z = \dfrac{x-y}{x+y}$, *di mana* $x = u^2 + v^2 + w^2$ *dan* $y = u\,v$. *Lukis gambarajah pokok untuk menuliskan petua rantai untuk mendapatkan* $\dfrac{\partial z}{\partial u}$ *pada titik* $u,v,w = 1,-1,1$.

    [13 markah]

7. A certain function $f(x, y)$ has directional derivative 8 in the direction $\underset{\sim}{v_1} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ and 1 in the direction $\underset{\sim}{v_2} = 12\underset{\sim}{i} + 5\underset{\sim}{j}$ at the point $P_0(-1, 2)$. Find the directional derivative of f at P_0 in the direction of $\underset{\sim}{v} = 3\underset{\sim}{i} - 5\underset{\sim}{j}$.

[6 marks]

8. Find and classify the critical points of the function $f\ x, y\ = 4xy - x^4 - y^4$.

[15 marks]

9. Evaluate the integrals.

(a) $\displaystyle\int_0^{\pi/4} \int_x^{\pi/4} \frac{\sin y}{y}\, dy\, dx$.

(b) $\displaystyle\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9 - x^2 - y^2}}\, dx\, dy$.

[10 marks]

10. (a) Show that the differential equation $\dfrac{dy}{dx} = \dfrac{-2xy + y^2 + 2x}{x^2 + 2xy - 1}$ is exact. Hence, find the particular solution when $y\ 1\ = 3$.

(b) According to the Newton's Law of Cooling, the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. A hard boiled egg at $96°C$ is put in a sink of $16°C$ water. After 7 minutes, the egg's temperature is $35°C$. When will its temperature be $18°C$?

[18 marks]

7. *Suatu fungsi $f(x, y)$ mempunyai terbitan berarah 8 dalam arah $\underset{\sim}{v}_1 = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ dan 1 dalam arah $\underset{\sim}{v}_2 = 12\underset{\sim}{i} + 5\underset{\sim}{j}$ pada titik $P_0(-1, 2)$. Cari terbitan berarah f pada P_0 dalam arah $\underset{\sim}{v} = 3\underset{\sim}{i} - 5\underset{\sim}{j}$.*

[6 markah]

8. *Dapatkan dan kelaskan titik-titik genting bagi fungsi $f\ x, y\ = 4xy - x^4 - y^4$.*

[15 markah]

9. *Nilaikan kamiran.*

(a) $\displaystyle\int_0^{\pi/4} \int_x^{\pi/4} \frac{\sin y}{y}\, dy\, dx$.

(b) $\displaystyle\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9 - x^2 - y^2}}\, dx\, dy$.

[10 markah]

10. (a) *Tunjukkan bahawa persamaan pembezaan $\dfrac{dy}{dx} = \dfrac{-2xy + y^2 + 2x}{x^2 + 2xy - 1}$ adalah tepat. Seterusnya, dapatkan penyelesaian khusus bila $y\ 1\ = 3$.*

(b) *Menurut Hukum Penyejukan Newton, kadar penyejukan suatu objek berkadaran dengan perbezaan suhu antara objek dan persekitarannya. Sebiji telur rebus bersuhu $96°C$ dimasukkan ke dalam sink yang airnya bersuhu $16°C$. Selepas 7 minit, suhu telur adalah $35°C$. Bilakah suhu telur menjadi $18°C$?*

[18 markah]

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