

UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Tambahan  
Sidang 1993/94

Jun 1994

MAT420 - Persamaan Pembezaan Separa

Masa: [3 jam]

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Jawab **SEMUA** soalan.

1. (a) Cari bentuk berkanun dan penyelesaian am bagi persamaan:

$$x^2.u_{xx} + 2xy.u_{xy} + y^2.u_{yy} + xy.u_x + y^2.u_y = 0$$

(70/100)

- (b) Cari siri sinus Fourier bagi fungsi

$$f(x) = \begin{cases} x & , \quad 0 < x < \frac{1}{2} \\ 1-x & , \quad \frac{1}{2} < x < 1 \end{cases}$$

(30/100)

2. (a) Cari penyelesaian bagi masalah:

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 \quad , \quad 0 < r < 1 \quad , \quad 0 \leq \theta \leq 2\pi$$

$$u(1, \theta) = f(\theta) \quad , \quad 0 \leq \theta \leq 2\pi$$

$$|u(r, \theta)| \leq M \quad , \quad 0 \leq r < 1 \quad , \quad 0 \leq \theta \leq 2\pi$$

dan tulis penyelesaian ini dalam bentuk suatu kamiran.

(70/100)

...2/-

(b) Selesaikan:

$$\nabla^2 u = 0, \quad 1 < r < 2, \quad 0 \leq \theta \leq 2\pi$$

$$u(1, \theta) = 2 \sin \theta, \quad 0 \leq \theta \leq 2\pi$$

$$u(2, \theta) = 3 \sin \theta, \quad 0 \leq \theta \leq 2\pi$$

$$\text{di mana } \nabla^2 u = u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta}$$

(30/100)

3. Selesaikan:

$$u_t = k u_{xx} + h, \quad 0 < x < \ell, \quad t > 0$$

$$u(0, t) = 1, \quad t > 0$$

$$u(\ell, t) = 2, \quad t > 0$$

$$u(x, 0) = 0, \quad 0 < x < \ell$$

di mana  $h, k, \ell$  ialah pemalar positif.

(100/100)

4. (a) Cari fungsi Green bagi masalah:

$$\nabla^2 u = h \text{ dalam } \Omega$$

$$u = f \text{ pada } \Gamma$$

jika

$$(i) \quad \Omega = \{(x, y): x > 0, y > 0\}$$

$$(ii) \quad \Omega = \{(x, y): 0 < x < 1, 0 < y < 1\}$$

dan  $\Gamma$  ialah sempadan  $\Omega$ .

(50/100)

...3/-

(b) Selesaikan:

$$u_{tt} = c^2 \cdot u_{xx} \quad , \quad x > 0 \quad , \quad t > 0$$

$$u(0,t) = f(t) \quad , \quad t > 0$$

$$u(x,0) = 0 \quad , \quad u_t(x,0) = 0 \quad , \quad x > 0$$

$$\lim_{x \rightarrow \infty} u(x,t) = 0$$

(50/100)

5. (a) Cari jelmaan Fourier bagi fungsi

$$f(x) = e^{-ax^2}$$

di mana  $a > 0$ .

(30/100)

(b) Selesaikan:

$$u_t = u_{xx} \quad , \quad -\infty < x < \infty \quad , \quad t > 0$$

$$u(x,0) = e^{-x^2} \quad , \quad -\infty < x < \infty$$

$$|u(x,t)| \leq M \quad , \quad -\infty < x < \infty \quad , \quad t > 0$$

(70/100)

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