# UNIVERSITI SAINS MALAYSIA 

Semester I Examination
Academic Session 2010/2011

November 2010

# EEE 512 - ADVANCED DIGITAL SIGNAL AND IMAGE PROCESSING 

Time: 3 Hours

## INSTRUCTION TO CANDIDATE:

Please ensure that this examination paper contains EIGHT printed pages and SIX questions before answering.

Answer FIVE questions.

Distribution of marks for each question is stated accordingly.

All questions must be answered in English.

1. (a) What are the differences between analog signal, digital signal, sampleddata signal, and quantized boxcar signal?
(20 marks)
(b) Is the system defined by:

$$
y[n]=x[n]+0.5(x[n-1]+x[n+1])
$$

Is the system is a linear system?
(10 marks)

Is the system is a causal system?
(10 marks)

Is the system is a recursive system?
(10 marks)
(c) Determine the convolution sum $y[n]$ of the two sequences:

$$
\begin{aligned}
& \{g[n]\}=\{-2,-1,0,1,2\} \\
& \{h[n]\}=\{1,2,3\}
\end{aligned}{ }^{\uparrow}
$$

(25 marks)
(d) Determine the discrete-time signal $V[n]$ obtained by uniformly sampled a continuous-time signal $v_{a}(t)$ composed of a weighted sum of seven sinusoidal signals of frequencies $20 \mathrm{~Hz}, 40 \mathrm{~Hz}, 70 \mathrm{~Hz}$, $160 \mathrm{~Hz}, 330 \mathrm{~Hz}, 840 \mathrm{~Hz}$, and 920 Hz , at sampling rate of 100 Hz . Given that $v_{a}(t)$ is:

$$
\begin{aligned}
v_{a}(t) & =\cos \left(2 O_{t} t\right)+\cos \left(4 O_{f} t\right)+4 \sin \left(O_{\lambda} t\right)-3 \cos \left(6 \theta_{t} t\right) \\
& \left.-3 \cos \left(3 \theta_{t} t\right)+2 \cos \left(49^{t}\right)+2 \cos 92 \theta t\right)
\end{aligned}
$$

2. (a) What is a canonic structure?
(10 marks)
(b) Find the equivalent realization for the block diagram shown in Figure 1, by using transpose operation.


Figure 1
(15 marks)
(c) Consider the third order IIR transfer function below:

$$
H(z)=\frac{(z+0.1)(z-0.5)}{(z-0.2)\left(z^{2}+0.5 z+0.3\right)}
$$

Draw the corresponding realization of the transfer function using:
(i) Direct form II realization
(ii) Cascade form realization
(iii) Parallel form II realization
3.
(a) The digital filter structure in Figure 2 is implemented using 9-bit signed two's complement fixed-point arithmetic with all products quantized before additions. Draw the linear noise model of the un-scaled system and compute its total output noise power.


Figure 2
(50 marks)
(b) Scale the first-order digital filter structure shown in Figure 3 using the $L_{2}$-norm scaling rule.


Figure 3
(25 marks)
(c) Develop an expression for the output $y[n]$ as a function of the input $x[n]$ for the multi-rate structure shown in Figure 4.


Figure 4
(25 marks)
4. (a) Explain why the histogram of a discrete image is not flat after histogram equalization.
(40 marks)
(b) The probability density distribution $p_{r}$ of an image $f$ with gray value $r$ ranging from 0 to 9 is tabulated in Figure 5(a) while Figure 5(b) shows the specified distribution $p_{z}$.

| Gray scale, $r$ | Number of <br> pixel, $n_{r}$ |
| :---: | :---: |
| 0 | 2 |
| 1 | 3 |
| 2 | 4 |
| 3 | 12 |
| 4 | 18 |
| 5 | 8 |
| 6 | 5 |
| 7 | 6 |
| 8 | 3 |
| 9 | 3 |

Figure 5(a)

| Gray scale, $z$ | Number of <br> pixel, $n_{z}$ |
| :---: | :---: |
| 0 | 4 |
| 1 | 6 |
| 2 | 9 |
| 3 | 9 |
| 4 | 10 |
| 5 | 10 |
| 6 | 8 |
| 7 | 6 |
| 8 | 1 |
| 9 | 1 |

Figure 5(b)
(i) Plot the histogram distribution of $f$,
(20 marks)
(ii) Repeat 4(b)(i) but after histogram equalization,
(20 marks)
(iii) Perform histogram specification on $f$ using $p_{z}$ and, hence, tabulate the new gray scale which maps $r \rightarrow z$.
(20 marks)
5. (a) The blurred image is shown in Figure 6


Figure 6
Clearly explain three principal techniques to estimate the degradation function of Figure 6.
(b) A degradation function of a certain image capturing device can be modeled as the convolution of the captured image with the spatial, circularly symmetric function such as

$$
\left.h \mathbf{\zeta}=\boldsymbol{4}^{2}-\sigma^{2}\right\rangle \sigma^{4} \bar{e}^{-r^{2} / 2 \sigma^{2}}
$$

where $r^{2}=x^{2}+y^{2}$. Show that the degradation in the frequency domain is given by

$$
H\left(l, v=-\sqrt{2 \pi} \sigma \mathbf{l}^{2}+v^{2} e^{-2 \pi^{2} \sigma^{2} \boldsymbol{l}^{2}+v^{2}}\right.
$$

## Given

$$
\begin{aligned}
& \mathfrak{J} \nabla^{2} f\left(x, y^{-}=-\ell^{2}+v^{2} F\left(\downarrow, v^{2}\right.\right. \\
& \left.\mathfrak{F} A e^{-\boldsymbol{r}^{2}+y^{2}},\right]=A \sqrt{2 \pi} \sigma^{-2 \pi^{2} \sigma^{2} \mathbf{l}^{2}+v^{2}}
\end{aligned}
$$

(60 marks)
6. (a) Write an expression for $\psi 3,3$ in terms of the Haar scaling function. Hence draw wavelet $\psi 3,3$ for the Haar wavelet function.
(40 marks)
(b) Consider the $2 \times 2$ image shown in Figure 6(b).

$$
f(x, y)=\left\lceil\begin{array}{ll}
3 & 1 \\
6 & 2
\end{array}\right\rceil
$$

Figure 6(b)
(i) Draw the require filter bank to implement the two-dimensional FWT with respect to Haar wavelets of Figure 6(b). Label all inputs and outputs with the proper arrays.
(ii) Use the result from 6(b)(i) to draw the require filter bank to implement the two-dimensional inverse FWT. Label all inputs and outputs with the proper arrays.
(30 marks)

## Given:

The wavelet functions are defined as:

$$
\begin{aligned}
& \psi_{j, k}<2^{\frac{j}{2}} \psi \boldsymbol{p}^{j} x-k \\
& \psi<\sum_{n}=h_{\psi}\left(i \sqrt{2} \varphi 2^{x-n}\right.
\end{aligned}
$$

The Haar scaling function is defined as :

$$
\varphi(x)=\left\{\begin{array}{ll}
1 & ; \quad 0 \leq x<1 \\
0 & ;
\end{array}\right. \text { elsewher }
$$

The scaling function coefficients for the Haar function are given by:

$$
h_{\varphi}(\boldsymbol{a})=\left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\} \quad \text { for } n=0,1
$$

The scaling function coefficients for the Haar wavelet are given by:

$$
h_{\psi}\left(\frac{1}{=}=\left\{\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right\} \quad \text { for } n=0,1\right.
$$

