UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2009/2010

November 2009

EEE 512 – ADVANCED DIGITAL SIGNAL AND IMAGE PROCESSING

Duration: 3 hours

INSTRUCTION TO CANDIDATE:

Please check that this examination paper contains **NINE (9)** pages of printed material before you begin the examination.

This paper contains **SIX (6)** questions.

Instructions: Answer **FIVE (5)** questions.

Answer to any question must start on a new page.

Distribution of marks for each question is given accordingly.

All questions must be answered in English.

1. (a) Define a stable and causal system? (10 marks)

(b) Find the DFT coefficients for the sequence; (20 marks)

$$x_p(n) = 10\sin\left(2\pi\frac{n}{3}\right)$$

(c) The difference equation of a recursive filter is given by

$$y(n)=0.7y(n-1)-0.3y(n-2)+6x(n-1)$$

(i) Obtain its pulse transfer function G(z).

(ii) What is the static gain of the filter?

(iii) Locate poles and zeros on a z-plane diagram and comment on its stability.

(iv) Given that the sample frequency is 1 kHz, find the gain and phase shift of the filter operating on a sinusoidal component of 250 Hz.

(40 marks)

(d) Obtain the difference equation for the IIR filter with transfer function:

$$H(z) = \frac{z^2 - 0.2z - 0.08}{z^2 + 0.5}$$

(30 marks)

2. (a) What are the differences between FIR and IIR filters? (10 marks)

(b) An FIR filter is described by the I/O equation: $y(n) = x(n) - x(n-4)$. Determine its transfer function and sketch the impulse and frequency responses of this filter.

(35 marks)

(c) A symmetric FIR lowpass filter of length N = 3 is designed via the Parks-McClellan algorithm. The passband edge is $\omega_p = \pi/3$ and the stopband edge is $\omega_s = \pi/2$.

$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

(i) Plot the magnitude of the frequency response of this filter over $-\pi < \omega < \pi$. Show as much detail as possible. (Recall that the frequency response is the DTFT of the impulse response of the filter.) The following values of the sine function may be useful:

$$\sin\left(\frac{\pi}{6}\right)=\frac{1}{2};\quad \sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2};\quad \sin\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}};\quad \sin\left(\frac{\pi}{2}\right)=1;\quad \sin\left(\frac{3\pi}{4}\right)=\frac{1}{\sqrt{2}};\quad \sin(\pi)=0$$

(ii) Plot the phase of the frequency response of this filter over $-\pi < \omega < \pi$.

- Is the phase linear over the passband?
- What is the delay of this filter (in discrete time units)?

(iii) The frequency response at $\omega = 0$ is one. What is the maximum deviation from one over the passband? That is, what is the passband ripple ∂_1?

(iv) What is the maximum absolute deviation from zero over the stopband? That is, what is the stopband ripple ∂_2?

(55 marks)

3. For the problem below, you might find the following values of the tangent function useful:

$$\tan\left\{\frac{\pi}{4}\right\} = 1; \qquad \tan\left\{\frac{\pi}{3}\right\} = \sqrt{3}; \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

If the values $1/\sqrt{3}$ or $1/\sqrt{2}$ show up in the problem, just carry them along. A digital IIR filter is designed from an analog Butterworth filter via the Bilinear Transform method through the transformation $s = (z-1)/(z+1)$. A canonical analog Butterworth filter with a 3 dB cut-off frequency of *Ω= 1 rads/sec* has three poles at the following locations in the s-plane:

$$s_1 = e^{j\frac{2\pi}{3}}; \quad s_2 = -1; \quad s_3 = e^{-j\frac{2\pi}{3}}$$

The analog Butterworth filter has a gain of one (unity) at DC.

(a) Determine the locations of the three poles of the resulting digital IIR filter.

(10 marks)

(b) Plot a pole-zero diagram for the resulting digital filter. Be sure to show where the zeros are located, as well as the poles.

(10 marks)

(c) Is the resulting digital filter stable or unstable? Explain your answer.

(10 marks)

(d) Plot the magnitude of the frequency response of the digital IIR filter over $-\pi < \omega < \pi$. show as much detail as possible. You must clearly indicate what the magnitude of the frequency response is at the following three digital frequencies: $\omega = 0$, $\omega = \pi/2$, and $\omega = \pi$.

(20 marks)

(e) Find the output of the digital filter to the following input which is a sum of sine waves "turned on" for all *n*.

$$x[n] = 1 + (j)^n + (-1)^n$$

(20 marks)

(f) For the digital filter, what frequency is the 3 dB point? That is, at what frequency does the magnitude drop from one at $\omega = 0$ to a value equal to $1/\sqrt{2}$?

(10 marks)

(g) Write the difference equation for the resulting digital filter.

(20 marks)

4. (a) Using neatly drawn diagrams, explain clearly the processes involved in Hormomorphic filtering. Hence, discuss one useful application of this filtering technique.

(40 marks)

(b) The unrotated Laplacian of an image $f(x, y)$ can be expressed as,

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Meanwhile the rotated version of the Laplacian is given by

$$\nabla^2 f = \frac{\partial^2 f}{\partial x'^2} + \frac{\partial^2 f}{\partial y'^2}$$

where,

$$x = x^{'}\cos\theta - y^{'}\sin\theta \text{ and } x = x^{'}\sin\theta + y^{'}\cos\theta$$

and θ is the axis of rotation.

Show that the Laplacian operation defined above is isotropic, i.e. invariant to rotation.

(60 marks)

Given:

If $w = f(x, y)$, $x = x(u, v)$ and $y = y(u, v)$ then

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial u}$$
$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial v}$$

5. (a) Using the Kronecker product recursion defined below, prove a $2^n \times 2^n$ Hadamard transform is orthogonal.

(40 marks)

(b) Consider the 4×4 image shown in Figure 5(b).

$$f(x, y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 5(b)

(i) construct a 4×4 Walsh-Hadamard matrix,

(5 marks)

(ii) perform Walsh-Hadamard transformation on $f(x,y)$,

(15 marks)

(iii) reconstruct $f(x,y)$ using the first three basis functions,

(15 marks)

(iv) repeat 5(b)(iii) using the expansion of vector outer product,

(15 marks)

(v) hence, calculate the sum square error.

(10 marks)

<u>Given:</u>

Hadamard core matrix is given by

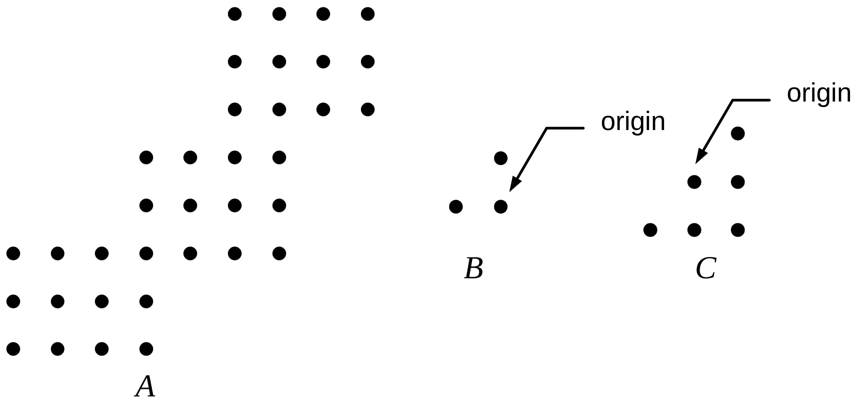
$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

and, the Kronecker product recursion is given by

$$H_n = H_{n-1} \otimes H_1 = H_1 \otimes H_{n-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} H_{n-1} & H_{n-1} \\ H_{n-1} & -H_{n-1} \end{pmatrix}$$

and, for $n = 1,2,3,....$

6. (a) A digital image *A,* structuring elements *B* and *C* are shown in Figures 6(a), (b) and (c), respectively.



(a) (b) (c)

Figure 6

Sketch the output images resulting from erosion and dilation respectively using *B* and *C*.

(40 marks)

(b) A computer-based image inspection system was proposed to inspect a 4×3 rectangular block in an image *A* of size 8×8. However, due imperfection of image capturing devices, noise in the form of small hole and protrusion appear in the block as shown in Figure 6(b).

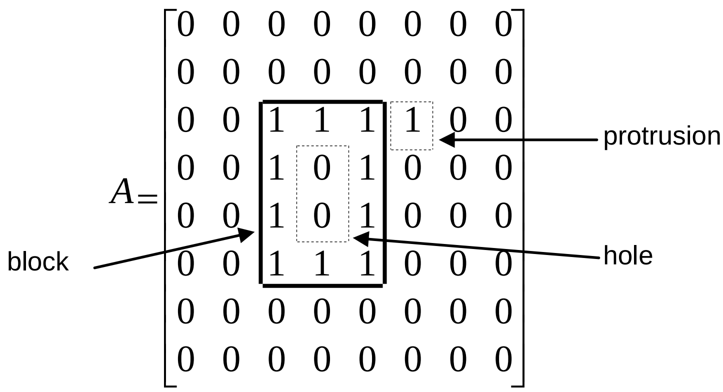


Figure 6(b)

Device a strategy to automatically inspect and locate these types of defects in *A*. Show clearly the results of the proposed strategy using image in Figure 6(b). Hence, discuss one main drawback of the proposed strategy.

(60 marks)

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