A Comparison of the Performances of Various Single Variable Charts

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ABSTRACT

Control charts are used for process monitoring and improvement in industries. Two charts are usually used in the monitoring of both the mean and variance separately. In the past 20 years, numerous control charting approaches that enable a joint monitoring of both the mean and variance on a single chart have been suggested. A joint monitoring of both the mean and variance is more meaningful in a real situation as both the mean and variance may shift simultaneously. Although numerous single variable control charts are available in the literature, not much research is made to compare these charts, in terms of their detection power. This paper compares the performances of several single variable charts, such as the semicircle, MaxEWMA and single MA charts, in terms of their average run length (ARL) results, via a Monte Carlo simulation. The Statistical Analysis System (SAS) software is employed in the simulation study. This comparison serves as a guide to practitioners by helping them to select a suitable single variable chart for process monitoring.

1. INTRODUCTION

Control charts are used for the purpose of detecting assignable causes that affect process stability. Two control charts, one for monitoring the process mean, such as the $\overline{X}$ chart and the other for monitoring the process variance, such as a *R* chart or a *S* chart, are usually run simultaneously. Most charts for variable data found in the literature monitor the process mean and variance separately. As shown by Reynolds and Stoumbos (2004) and mentioned again by Costa and Rahim (2006) that running two charts, one for the mean and the other for the variance, may not always be reliable in identifying the nature of the change. Recently, control charts that can simultaneously monitor both the process mean and the procecss variance have been proposed. These charts are called single variables control charts, and are classified as the Shewhart-type charts, CUSUM-type charts and EWMA-type charts.

For the Shewhart-type single variable chart, White and Schroeder (1987) first introdused the use of one control chart to monitor both process mean and variance on the same chart. This chart was designed using resistant measure and a modified box plot display. Chao and Cheng (1996) proposed a single control chart, called the semicircle (SC) control chart. This chart uses a semicircle to plot a single plotting statistic to indicate the position of the mean and standard deviation, by plotting the standard deviation on the *y*-axis and the mean on the *x*-axis. When a point plots ouside of the semicircle indicating an out-of-control signal, the chart shows whether the mean, the variance or both parameters have shifted. The disadvantage of this chart is that it loses track of the time sequence of the plotted points. Chen and Cheng (1998) proposed a single Shewhart-type control chart, called the Max chart. This Max chart plots the maximum absolute value of the standardized mean and standard deviation. This chart performs like the combined Shewhart charts for the mean and standard deviation, i.e., the combined $\overline{X} - S$ charts. Spiring and Cheng (1998) developed a single variable chart that monitors both the process mean and standard deviation. This chart also plots two variables at the same time and has the advantage of performing equally well for both large and small subgroup sizes. Gan et al. (2004) proposed a single control chart based on the interval approach that combines both $\overline{X}$ and S charts into one scheme. Wu and Tian (2006) suggested a single weighted loss function chart (WL chart) for a simultaneous monitoring of the process mean and variance. İt was shown that the WL chart is significantly more effective than the unadjusted loss function chart and joint $\overline{X} - S$ charts, as well as the other charts.

For the CUSUM-type single chart, the CUSUM M-chart and CUSUM V-chart for detecting small shifts in the process mean and process variance, respectively, were proposed by Yeh et al. (2004). Because these charts have the same distribution when the process is in-control, they can be effectively combined into a single chart, thus, enabling a simultaneous monitoring of the mean and variance to be made on the same chart. A weighted loss function CUSUM (WLC) scheme with variable sampling interval (VSI) that enables a simultaneous monitoring of both the mean shift and an increasing variance shift by using a CUSUM chart was suggested by Zhang and Wu (2006).

For the EWMA-type single chart, numerous single EWMA charts for a simultaneous monitoring of the process mean and variance have been proposed. Domangue and Patch (1991) suggested some omnibus EWMA schemes based on the exponentiation of the absolute value of the standardized sample mean of the observations for a joint monitoring of the mean and variance. Gan (2000) proposed a simultaneous EWMA chart that was developed by combining a chart for the mean and a chart for the variance into one chart by plotting the EWMA of log(S^2) against the EWMA of $\overline{X}$. The control limit of this chart is formed by either using a rectangle or an ellipse. Morais and Pacheco (2000) considered a joint monitoring of the process mean and variance using a combined EWMA (CEWMA) scheme, where the average run length, percentage points of the run length and probability of a misleading signal were investigated. By using a two dimensional Markov chain approximation, these three performance measures are obtained. The MaxEWMA chart which combines the EWMA charts for the process mean and process variance into a single chart was developed by Chen et al. (2001). This chart extends and improves upon the earlier work of Chen and Cheng (1998) on the Max chart. Chen et al. (2004) proposed the EWMA-SC chart by applying the EWMA technique to the statistics employed in the semicircle chart. This proposed chart provides a better detection ability with regards to small shifts in the mean and/or variance in comparison to the SC chart. Costa and Rahim (2004) suggested the use of a single non-central chi-square chart to monitor both the process mean and variance simultaneously. Costa and Rahim (2004) also found that the EWMA chart based on the non-central chi-square statistic has a similar performance to the MaxEWMA chart proposed by Chen et al. (2001). A single EWMA chart which is an extension of the EWMA-SC chart studied by Chen et al. (2004) was suggested by Costa and Rahim (2006).

This paper compares the performances of three single control charts, namely the semicircle (SC), MaxEWMA and single moving average (MA) charts, in terms of thier average run lengths (ARLs), via a Monte Carlo simulation. A simulation study conducted using the Statistical Analysis System (SAS) software shows that the MaxEWMA chart gives the best performance, while the SC chart has the poorest performance. This comparison assist practitioners in selecting a suitable single variable chart for process monitoring.

This paper is organized as follows: Section 2 reviews the semicircle (SC) chart. A review of the MaxEWMA chart is made in Section 3, while Section 4 reviews the single moving average (MA) chart. In Section 5, a performance comparison is made to compare the performances of the SC, MaxEWMA and single MA charts, in terms of their average run length (ARL) profiles. Finally, conclusions are drawn in Section 6.

2. SEMICIRCLE (SC) CHART

Chao and Cheng (1996) proposed a semicircle control chart, where a semicircle is used to plot a single plotting statistic to represent the position of the mean and standard deviation, where the standard deviation is plotted on the *y*-axis and the mean on the *x*-axis. When a point plots outside of the simicircle indicating an out-of-control signal, the chart can easily tell whether the mean, the variance or both the parameters have changed. With its straight forward calculations, this chart can be considerd as a new alternative to the combination of the $\overline{X}$ and R charts.

The plotting statistic proposed by Chao and Cheng (1996) is

$$T = \left(\overline{X} - \mu\right) + S^{*2}. \tag{1}$$

The plotting statistic is based on the sample mean, $\overline{X}$, and the root mean square, $S^* = \left(\frac{n-1}{n}\right)^{\frac{1}{2}} S$, where $S = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n-1}}$ is the sample standard deviation. Under the normality assumption, $\left(\frac{n}{\sigma^2}\right)T$ is distributed exactly as a χ_n^2 random variable for any sample size n. The statistic T in Equation (1) defines a circle, but because $S^* \geq 0,$ when plotting $\left(\overline{X}, S_i^*\right)$ for each sample on the $\left(\overline{X}, S^*\right)$ plane, a semicircle will be sufficient.

İn order to construct the semicircle chart, the following formula can be used in determining the radius r, where α is the size of the Type-I error (Chao and Cheng, 1996):

$$P\left(T < r^2\right) = P\left(\frac{n}{\sigma^2}T < \frac{n}{\sigma^2}r^2\right)$$
$$= 1 - \alpha,$$

Since $\frac{n}{\sigma^2}T \sim \chi_n^2,$ we have $\frac{n}{\sigma^2}r^2 \sim \chi^2_{n,\,(1-\alpha)},$ i.e.,

$$r = \left(\frac{\chi^2_{n,(1-\alpha)}}{n} \right)^{1/2} \sigma. \quad (2)$$

Here, $\chi^2_{n,\,(1-\alpha)}$ is a 100(1−α)% percentile of the χ^2_n distribution. Note that if parameters are unknown, $\overline{\overline{X}}$ is used to estimate μ and $\overline{S}^*$ to estimate σ.
Then we have (Chao and Cheng, 1996)

$$\hat{r} = q\overline{S}^* \quad (3)$$

where $q = \left(\frac{\chi^2_{n,\,(1-\alpha)}}{n} \right)^{1/2}$.

3. MaxEWMA CHART

An exponentially weighted moving average (EWMA) chart is a control chart for variable data. İt plots weighted moving averages. A weighting factor is chosen by the user to determine how older data points affect the mean value compared to more recent ones. Because the EWMA chart uses information from all samples, it detects smaller process shifts quicker than the Shewhart control chart. The MaxEWMA chart is constructed as follows (Chen et al., 2001):

Assume that a sequence of individual measurements, X_{ij}, in sample *i*, follow a *N*(μ,σ) distribution, for *i* = 1, 2, ... and *j* = 1, 2, ..., n_i. Let μ_0 be the nominal process mean and σ_0 be a known value of the process standard deviation. Assume that the process parameters μ and σ can be expressed as $\mu = \mu_0 + a\sigma_0$ and $\sigma = b\sigma_0$, where *a* and *b* (>0) are constants. The process is in-control when *a* = 0 and *b* = 1; otherwise the process has changed.

Let $\overline{X}_i = \frac{\sum_{j=1}^{n_i} X_{ij}}{n_i}$ be the i^{th} sample mean and $S_i^2 = \frac{\sum_{j=1}^{n_i} \left(X_{ij} - \overline{X}_i \right)^2}{(n_i - 1)}$ be the i^{th} sample variance. Define (Chen et al., 2001)

$$U_i = \frac{\left(\overline{X}_i - \mu_0 \right)}{\sigma_0 / \sqrt{n_i}} \quad (4)$$

and

$$V_i = \Phi^{-1}\left\{ H\left(\frac{(n_i - 1)S_i^2}{\sigma_0^2}; n_i - 1 \right) \right\}, \quad (5)$$

where $\Phi(z) = P(Z \le z)$, for *Z* ~ *N*(0, 1), $\Phi^{-1}(\cdot)$ is the inverse function of $\Phi(\cdot)$ and $H(\omega; v) = P(W \le \omega)$, where *W* follows a chi-square distribution with *v* degrees of freedom.

It is known that U_i and V_i are independent when *a* = 0 and *b* = 1 because $\overline{X}_i$ and S_i^2 are independent. İt can be shown that $U_i \sim N(0,1)$ and $V_i \sim N(0,1)$ (Chen et al., 2001). The distributions of U_i and V_i are both independent of the sample size n_i, when a = 0 and b = 1, therefore the variable sample size problem can be handled easily by the MaxEWMA chart. Since both U_i and V_i have the same distribution, a single variable chart to monitor both the process mean and process variability can be constructed (Chen et al., 2001).

First, define

$$Y_i = \lambda U_i + (1-\lambda) Y_{i-1}, \quad 0 < \lambda \le 1, \quad \text{for } i = 1, 2, \ldots \tag{6a}$$

and

$$Z_i = \lambda V_i + (1-\lambda) Z_{i-1}, \quad 0 < \lambda \le 1, \quad \text{for } i = 1, 2, \ldots, \tag{6b}$$

with Y_0 and Z_0 as the starting values, respectively. Then, the above two EWMA statistics are combined into a single chart by defining a new statistic M_i given by

$$M_i = \max\{|Y_i|, |Z_i|\}\,. \tag{7}$$

The statistic M_i will be large when the process mean has shifted away from μ and/or when the process variability has increased or decreased. On the other hand, the statistic M_i will be small when the process mean and process variabiity stay close to their respective targets.

Since M_i is non-negative, only a UCL is needed. The UCL is given by

$$\text{UCL} = E(M_i) + K\sqrt{Var(M_i)}, \tag{8}$$

where $E(M_i)$ is the mean of M_i and $Var(M_i)$ is the variance of M_i, when a = 0 and b = 1. Here, K is a multiplier, which together with λ, controls the performance of the new chart. Because this chart is based on M_i, the maximum of $|Y_i|$ and $|Z_i|$, it is called the MaxEWMA chart.

4. SINGLE MOVING AVERAGE (MA) CHART

Khoo and Yap (2005) suggested the use of a joint moving average control chart for a simultaneous monitoring of the process mean and variance. Besides being efficient in detecting increases and decreases in the process mean and/or variability, the joint MA chart is also able to indicate the source and direction of a shift.

Let X_{ij} for i =1, 2, ..., and j = 1, 2, ..., n_i, be observations from subgroups of size n_i, with i representing the subgroup number. It is assumed that $X_{ij} \sim N(\mu + a\sigma, b^2\sigma^2)$, where a = 0 and b = 1 indicate that the process is in-control; otherwise, the process has shifted.

Let $\overline{X}_i = \dfrac{(X_{i1} + X_{i2} + \cdots + X_{in_i})}{n_i}$ be the i^{th} sample mean and let $S_i^2 = \dfrac{\sum_{j=1}^{n_i} (X_{ij} - \overline{X}_i)^2}{n_i - 1}$ be the i^{th} sample variance. Khoo and Yap (2005) define the following statistics:

$$U_i = \frac{(\overline{X}_i - \mu)}{\sigma/\sqrt{n_i}} \sim N(0,1), \quad \text{for } i = 1, 2, \ldots \tag{9}$$

and

$$V_i = \Phi^{-1}\left\{H\left(\frac{(n_i-1)S_i^2}{\sigma^2}; n_i-1\right)\right\} \sim N(0,1), \quad \text{for } i = 1, 2, \ldots, \tag{10}$$

where $\Phi^{-1}(\,\cdot\,)$ and $H(\,\cdot\,)$ denote the inverse standard normal distribution function and the chi-square distribution function with $n_i - 1$ degrees of freedom, respectively. Because the sample mean, $\overline{X}_i$ and sample variance, S_i^2, are independent, U_i and V_i are also independent. The sample grand average, $\overline{\overline{X}}$ and R/d_2 or $\overline{S}/c_4$ are substituted for μ and σ, respectively, if the target values of these parameters are unknown, where d_2 and c_4 are the control chart constants.

The plotting statistic, K_i, of the chart can then be defined as (Khoo and Yap, 2005)

$$K_i = \max\{|L_i|, |M_i|\}, \tag{11}$$

where

$$L_i = \frac{U_i + U_{i-1} + \cdots + U_{i-w+1}}{w}$$

and

$$M_i = \frac{V_i + V_{i-1} + \cdots + V_{i-w+1}}{w},$$

for $i \geq w$, while for $i < w$, say $w = 3$,

$$L_1 = \frac{U_1}{1},\ L_2 = \frac{U_1 + U_2}{2},\ L_3 = \frac{U_1 + U_2 + U_3}{3}$$

and

$$M_1 = \frac{V_1}{1}, M_2 = \frac{V_1 + V_2}{2}, M_3 = \frac{V_1 + V_2 + V_3}{3}.$$

Note that w is the span of the moving average statistic. The statistic K_i will be large when the process mean has shifted away from its target value and/or when the process variance has increased or decreased. Only the upper control limit, UCL is applied on the joint MA chart as K_i is non-negative.

The density function of K_i, for the in-control case is (Khoo and Yap, 2005)

$$f(k) = 4\sqrt{w}\phi\left(k\sqrt{w}\right)\left\{2\Phi\left(k\sqrt{w}\right) - 1\right\}, \quad \text{for } k \geq 0. \tag{12}$$

Here, $\phi(\,\cdot\,)$ and $\Phi(\,\cdot\,)$ are the density and distribution functions of a standard normal random variable, respectively. Suppose that the desired Type-I error set by management based on some predetemined factors is α, then UCL is obtained by solving the following definite integral:

$$\int_{UCL}^{\infty} f(k)\,dk = \alpha \tag{13}$$

For a MA control chart, the control limits for periods $i < w$ are wider than their steady-state value. Besides having the desirable properties of the moving average chart, the variable sample size problem can also be handled automatically with the application of the joint MA chart. Therefore, the joint MA chart can be considered as an attractive alternative to the combined $\overline{X} - R$ or $\overline{X} - S$ charts (Khoo and Yap, 2005).

5. PERFORMANCE COMPARISON

A simulation study is conducted using SAS version 9 to study the performances of the SC, MaxEWMA and single MA charts. The sample size of $n = 5$ is considered. The shifts in the mean and variance considered are $\mu_1 = \mu_0 + a\sigma_0$ and $\sigma_1 = b\sigma_0$, respectively, where $a \in \{0, 0.25, 0.5, 1, 2\}$ and $b \in \{0.25, 0.5, 1, 1.5, 2\}$. Note that when the process is in-control, $a = 0$ and $b = 1$. The in-control ARL (ARL_0) is fixed as 185. For the SC chart, the radius of the chart for $n = 5$ is $r = 1.8174$. For the MaxEWMA chart, $\lambda \in \{0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.8, 1\}$ are considered and their corresponding K values are determined. The values of the moving span $w \in \{2, 3, 4, 5\}$ are employed and their corresponding UCLs are determined for the single MA chart.

The ARL profiles for the SC, single MA and MaxEWMA charts are given in Tables 1, 2 and 3, respectively. Generally, the results show that the MaxEWMA chart is superior to the other two charts. The SC chart is found to have the poorest performance. Note that for an arbitrary combination of (a, b), where $a > 0$ and $b \neq 1$, the MaxEWMA chart has the lowest out-of-control ARL, followed by the single MA chart. On the contrary, the SC chart always has the highest out-of-control ARL among the three charts.

6. CONCLUSIONS

In this paper, the performances of three single variable control charts are compared based on their ARLs. Overall, the MaxEWMA chart provides a better performance than the other two charts, while the SC chart gives the poorest performance.

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Table 1. ARL profiles for the SC chart with $ARL_0 = 185$ and $n = 5$

	a					
	b	0.00	0.25	0.50	1.00	2.00
$r = 1.8174$	0.25	1.3	1.5	1.0	1.0	1.0
	0.50	8.7	1.1	1.0	1.0	1.0
	1.00	185.0	123.7	52.0	8.3	1.2
	1.50	5.0	4.7	4.0	2.4	1.2
	2.00	1.9	1.8	1.7	1.5	1.1

Table 2. ARL profiles for the single MA chart with $ARL_0 = 185$ and $n = 5$

	a					
(*w*, UCL)	*b*	0.00	0.25	0.50	1.00	2.00
$w = 2$ UCL= 2.1233	0.25	1.3	1.3	1.3	1.1	1.0
	0.50	11.5	11.5	11.0	1.8	1.0
	1.00	185.0	64.7	14.3	2.2	1.0
	1.50	5.0	4.5	3.4	1.8	1.1
	2.00	1.7	1.7	1.6	1.4	1.1
$w = 3$ UCL= 1.7358	0.25	1.0	1.0	1.0	1.0	1.0
	0.50	5.2	5.2	4.5	1.1	1.0
	1.00	185.0	46.7	9.0	1.6	1.0
	1.50	3.8	3.5	2.7	1.5	1.0
	2.00	1.5	1.4	1.4	1.2	1.0
$w = 4$ UCL= 1.5066	0.25	1.0	1.0	1.0	1.0	1.0
	0.50	3.3	3.3	2.6	1.0	1.0
	1.00	185.0	36.9	6.4	1.4	1.0
	1.50	3.1	2.8	2.2	1.3	1.0
	2.00	1.3	1.3	1.2	1.1	1.0
$w = 5$ UCL= 1.3540	0.25	1.0	1.0	1.0	1.0	1.0
	0.50	2.4	2.4	1.9	1.0	1.0
	1.00	185.0	29.8	5.0	1.3	1.0
	1.50	2.6	2.4	1.8	1.2	1.0
	2.00	1.2	1.2	1.2	1.1	1.0

Table 3. ARL profiles for the MaxEWMA chart with ARL_0 =185 and n =5

	a					
	b	0.00	0.25	0.50	1.00	2.00
λ=0.05 *K*=3.3833	0.25	2.0	2.0	2.0	1.9	1.0
	0.50	3.5	3.5	3.4	2.2	1.0
	1.00	185.5	12.1	5.1	2.3	1.2
	1.50	4.1	3.9	3.3	2.2	1.2
	2.00	2.1	2.1	2.0	1.7	1.2
λ=0.10 *K*=3.4028	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	3.3	3.3	3.1	2.0	1.0
	1.00	185.0	12.3	4.8	2.1	1.1
	1.50	3.7	3.5	3.0	2.0	1.2
	2.00	1.9	1.9	1.8	1.6	1.2
λ=0.15 *K*=3.2254	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	3.3	3.3	3.1	2.0	1.0
	1.00	185.0	14.7	4.0	2.1	1.1
	1.50	3.7	3.5	2.9	1.9	1.2
	2.00	1.9	1.8	1.8	1.5	1.2
λ=0.20 *K*=3.0996	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	3.5	3.5	3.3	2.0	1.0
	1.00	185.1	18.4	5.4	2.1	1.1
	1.50	3.9	3.6	3.0	2.0	1.1
	2.00	1.9	1.9	1.8	1.5	1.2
λ=0.25 *K*=3.0415	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	3.7	3.7	3.5	2.1	1.0
	1.00	185.1	23.1	6.0	2.2	1.1
	1.50	4.1	3.8	3.1	2.0	1.2
	2.00	1.9	1.9	1.8	1.6	1.2
λ=0.30 *K*=3.0278	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	4.1	4.0	3.8	2.1	1.0
	1.00	185.0	28.5	6.6	2.2	1.1
	1.50	4.3	4.0	3.2	2.0	1.2
	2.00	1.9	1.9	1.8	1.6	1.2
λ=0.40 *K*=3.0499	0.25	1.9	1.9	1.9	1.8	1.0
	0.50	4.9	4.9	4.7	2.2	1.0
	1.00	185.0	38.3	8.3	2.3	1.1
	1.50	4.7	4.3	3.4	2.1	1.2
	2.00	2.0	1.9	1.8	1.6	1.2
λ=0.50 *K*=3.0884	0.25	1.9	1.9	1.9	1.8	1.0
	0.50	6.5	6.5	6.2	2.3	1.0
	1.00	185.0	47.7	10.25	2.5	1.1
	1.50	5.0	4.6	3.6	2.1	1.2
	2.00	2.0	2.0	1.9	1.6	1.2
λ=0.80 *K*=3.0019	0.25	2.6	2.6	2.6	2.4	1.0
	0.50	21.4	21.4	21.1	4.2	1.0
	1.00	185.0	76.8	19.9	3.3	1.1
	1.50	6.5	5.9	4.5	2.4	1.2
	2.00	2.2	2.1	2.0	1.7	1.2
λ=1.00 *K*=2.6918	0.25	4.8	4.8	4.8	4.7	1.0
	0.50	50.4	50.4	50.2	12.1	1.0
	1.00	185.0	96.8	30.3	4.4	1.1
	1.50	7.3	6.7	5.1	2.7	1.2
	2.00	2.3	2.2	2.1	1.7	1.2