MODELING OF GROUNDWATER BY USING FINITE DIFFERENCE METHODS AND SIMULATION

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Abstract:
In active landslide, the prediction of acceleration of movement is crucial issue for the design and performance of warning systems. Landslide occurs when a sudden increase beyond the critical level of groundwater. This is especially true in tropical weather during the wet season. The purpose of this study is to use numerical model to simulate groundwater
flow. The goal of this modeling is to predict the value of unknown nodal points in the groundwater piezometric head. The numerical technique used is the finite difference method. The finite difference method one of the oldest, most general applicable and most easily understood methods of obtaining numerical solution to steady and unsteady groundwater problems. After we obtain the algebraic approximation equations for each node in solution boundary domain, we solve them with digital computer program. Our research presents a broad, comprehensive overview of the fundamental concepts and applications of computerized groundwater modeling. The research covers finite difference method and includes simulation runs to demonstrate theoretical points described. Our model is able to predict the value of aquifer parameters in particular slope.

1 Introduction:

Groundwater is water located under the ground surface in soil poor spaces and in the fractures of lithologic formations. In the wet season in countries with tropical rainforests, the rate of accumulation of groundwater is greater than normal. Groundwater levels may accelerate as rainfalls increase both in terms of frequency and in amount Figure (1.1). This leads to sudden and unpredicted landslides. Most landslides are preceded by a saturation of a slope. This is commonly caused by heavy rainfall events, periods of extended precipitation that saturate the upper part of the slope, or increase in groundwater levels.

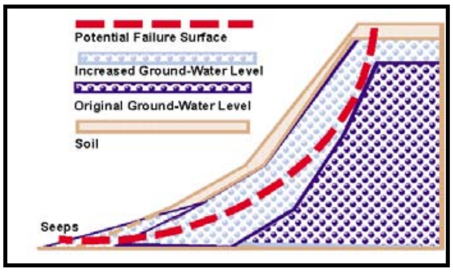


Figure (1.1): The potential for landsliding increases when groundwater levels rise due to rain, or landscape irrigation

Landslides are a recurring hazard in Malaysia. They occur in a majority of states in Malaysia. Additionally, landslides and the many other ground failures result in many direct and indirect expenses to society. Some of these direct costs include lost of life and the actual physical damage which runs the gamut from cleanup and repair to replacement. Indirect costs are harder to measure and include business disruption, loss of tax revenues, reduced property values, loss of productivity, losses in tourism, and losses from litigation. Therefore, we need to study the effects of landslides. , And, as we mentioned earlier, all previous studies about landslide have considered the increase in groundwater level caused by slope collapse. In our research we study a model of groundwater to measure the groundwater in a particular slope in order to predict the amount of water and if it has reached a supercritical level. Groundwater models may be used to predict the effects of hydrology on the behavior of the aquifer. These are often called groundwater simulation models. As the calculations are based on mathematical equations, often with approximate numerical solutions, these models are also called mathematical/numerical groundwater models. A mathematical model consists of a set of differential equations that are known to govern the flow of groundwater. Mathematical models of groundwater flow have been in use since the late 1880's.

As (Wang and Anderson, 1982) state in their book, "Introduction To Groundwater Modeling," they consider two types of models: finite difference models and finite element models. The finite difference model described are based on Lapalace's equation and the finite element model described are based on Poisson's equation. Both of them compare and test the numerical solutions by a classical analytical solution of a similar problem. After one year, (Hunt,1983) wrote in his book, "Mathematical Analysis of Groundwater Recourses," He considers the finite difference method as one of the approximate solutions for boundary-value problems because it is an easily understood method that can provide an approximate solution under very general circumstances. On the other hand, Hunt says, analytical solution is usually easier and more economical to use and interpret then numerical solution. Additionally, they are often useful as standards to test the accuracy of numerical models. (Corominas, Moya, Ledesma , Lloret, and Gili,2004) investigate the prediction of ground displacements and velocities from groundwater level changes at the Vallcebre landslide (Estern Pyrenees ,Spain). Their model to predict both landslide displacements and velocities was performed at Vallcebre by solving the momentum equation in which a viscous term (Bingham and Power Law) was added. They found that the landslide is very sensitive to rainfall, cracks, and drainage pathways. These models used in the time-dependent simulations are based on this "dynamic approach" rather than just considering "static" limit equilibrium.

The subject of this research is using the numerical methods to solve mathematical model that simulate groundwater flow and contaminant transport. We consider the finite difference method to be the numerical technique to solve our model. The finite difference method is an easily understood method that can provide approximate solutions under general circumstances. A mathematical groundwater model for steady flow conditions consists of a governing equation and boundary conditions which simulate the flow of groundwater in a particular problem domain. To solve our model we have to calculate the value of head at each point in system. The numerical techniques that we consider are finite difference methods which provide a rationale for operating on the differential equations. Using a computer, one can solve large number of algebraic equations by iterative solution as in our model. We solved 29 algebraic equations by using Gauss Seidel iterative methods.

2 The Finite – Difference Method

The finite difference method is considered as the most applicable and easily understood methods of obtaining numerical solutions to steady and unsteady groundwater flow problems. The general method consists of superimposing a finite – deference grid of nodes upon the solution domain. Actually, each node is given a global identification number and its surrounding of each of these nodes, where the dependent variable is approximated with a finite – degree polynomial whose coefficients are written in forma of the unknown values of the dependent variable at the surrounding nodes. So this polynomial is used to obtain an algebraic approximation for the partial differential equation for each internal nodes beside an algebraic approximation for boundary condition at each node site upon or near the solution domain boundary.

After we obtain the algebraic equations for each node we can solve the equation simultaneously to obtain the unknown value of the dependent variable at all nodes.

3 Solution of the Finite-Difference Equations

There are two method to solved the system of simultaneous equation that is generated by writing equation at each interior node and equation at each boundary node we can solved by direct elimination method such as Gaussian elimination or iterative method such as the Gauss –Seidel iteration. We can say the direct method more efficient under certain circumstances. However, some of applications consider here may need simultaneous solution of one or two thousand equations with sparse matrix and relatively large diagonal terms in the coefficient matrix. For these conditions ,the iterative methods are easier to cod for computer , where it's take less computer storage and less computational time for that we will consider the iterative methods.

4 Analysis and Discussion

4.1 The model of groundwater

The groundwater models are representations of reality and, if properly constructed, can be valuable predictive tools for the management of groundwater as one of water resources and a tool to predict the effects of groundwater on the movement of landslide as our case study. Of course, the validity of the predictions is predicated on how well the model approximates filed conditions.

The development and building of models for groundwater levels have to follow a specific number of well defined steps:

4.1.1- Identify the particular problem

For our research we want to know how much water can be in a particular area in the slope or the groundwater level at the slope. Therefore, one of the main factors of occur the landslide is the level of and the forces associated with ground- water. If we can predict the groundwater when it's reaching the critical level in the slope this will be useful as an early warning message to the people who could be affected by the landslide.

4.1.2- Formulate the boundary value problem

The problem of formulation assists us in organizing our thinking about a particular problem. We will give each node a global identification number, as well as in the neighborhood of each of these nodes. The dependent variable h (piezometric head) is approximated with a finite degree polynomial whose coefficients are written in terms of unknown values of the dependent variable at the surrounding nodes.

We will use this polynomial to obtain two algebraic equations

i. An algebraic approximation for the partial differential equation at each interior node.

ii. An algebraic approximation for a boundary condition at each node that lies upon or beside the solution domain boundary.

If we consider a five node grid shown in Figure (1. 2) the node has the constant spacing of Δ in each coordinate direction, where the integer's h_0 through h_1, h_2, h_3 *and* h_4 are used as local node identification number. The dependent variable h (piezometric head) will be represented in the neighborhood of these nodes with polynomial where we use Taylor series about node 0:

$$h(x,y) = h_\circ + \frac{\partial h_\circ}{\partial x}x + \frac{\partial h_\circ}{\partial y}y + \frac{1}{2!}\frac{\partial^2 h_\circ}{\partial x^2}x^2 + \frac{1}{2!}\frac{\partial^2 h_\circ}{\partial x \partial y}xy + \frac{1}{2!}\frac{\partial^2 h_\circ}{\partial y^2}y^2 + \frac{1}{3!}\frac{\partial^3 h_\circ}{\partial x^3}x^3 + \ldots\ldots \quad (1.1)$$

By setting (x,y) equal to the coordinates of nods 1 through 4 we obtain.The four equations:

$$h_1 = h_\circ + \frac{\partial h_\circ}{\partial x}\Delta + \frac{1}{2!}\frac{\partial^2 h_\circ}{\partial x^2}\Delta^2 + \frac{1}{3!}\frac{\partial^3 h_\circ}{\partial x^3}\Delta^3 + 0(\Delta^4) \quad (1.2)$$

$$h_2 = h_\circ + \frac{\partial h_\circ}{\partial y}\Delta + \frac{1}{2!}\frac{\partial^2 h_\circ}{\partial y^2}\Delta^2 + \frac{1}{3!}\frac{\partial^3 h_\circ}{\partial y^3}\Delta^3 + 0(\Delta^4) \quad (1.3)$$

$$h_3 = h_\circ - \frac{\partial h_\circ}{\partial x}\Delta + \frac{1}{2!}\frac{\partial^2 h_\circ}{\partial x^2}\Delta^2 - \frac{1}{3!}\frac{\partial^3 h_\circ}{\partial x^3}\Delta^3 + 0(\Delta^4) \quad (1.4)$$

$$h_4 = h_\circ - \frac{\partial h_\circ}{\partial y}\Delta + \frac{1}{2!}\frac{\partial^2 h_\circ}{\partial y^2}\Delta^2 - \frac{1}{3!}\frac{\partial^3 h_\circ}{\partial y^3}\Delta^3 + 0(\Delta^4) \quad (1.5)$$

By subtracting Equations (1.2)and (1.4) and Equations (1.3) and (1.5)

$$\frac{\partial h_\circ}{\partial x} = \frac{h_1 - h_3}{2\Delta} + 0(\Delta^2) \quad (1.6)$$

$$\frac{\partial h_\circ}{\partial y} = \frac{h_2 - h_4}{2\Delta} + 0(\Delta^2) \quad (1.7)$$

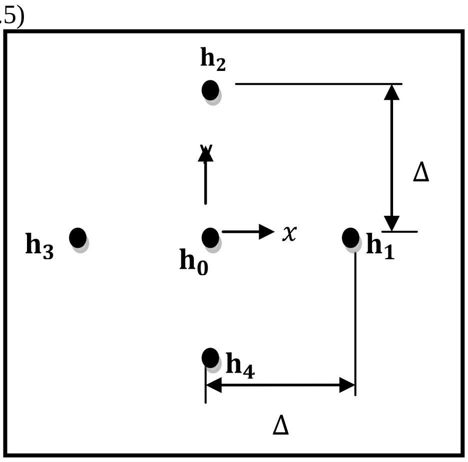


Figure (1.2): A typical five –node grid with a node spacing of Δ

The equation for a steady two dimensional flow is.

$$\bar{\nabla}.(T\bar{\nabla}h) = \left[\frac{\grave{k}}{\grave{B}}\right](h - \grave{h}) + \frac{Q_0}{A} - R \quad (1.8)$$

Where

$\bar{\nabla}$:Vector operator del (L-1)

T: Transmnissivity (L2 T-1)

h: piezometric head

$\grave{k}$: aquifer permeability (L T-1)

$\grave{B}$: aquifar thickness (L)

Q_0: discharge form a well at node 0.

R: rainfall recharge.

By integration the equation (1. 8)

$$\int_T T \frac{dh}{dn} ds = \int_A [\left[\frac{\grave{k}}{\grave{B}}\right](h - \grave{h}) + \frac{Q_0}{A} - R] \, dA \tag{1.9}$$

An approximate calculation of the integrals in terms of variables at nodes 0 though 4 gives

$$\sum_{i=1}^{4} \frac{1}{2}(T_0 - T_i)\frac{h_i - h_0}{\Delta}\Delta = \left[\left[\frac{\grave{k}}{\grave{B}}\right](h - \grave{h}) + \frac{Q_0}{A} - R\right]\Delta^2 + Q_0 \tag{1.10}$$

$$\frac{1}{2}(T_0 - T_1)(h_1 - h_0) + \frac{1}{2}(T_0 - T_2)(h_2 - h_0) + \frac{1}{2}(T_0 - T_3)(h_3 - h_0) + \frac{1}{2}(T_0 - T_4)(h_4 - h_0) = \left[\left[\frac{\grave{k}}{\grave{B}}\right](h_0 - \grave{h}) + \frac{Q_0}{A} - R\right]\Delta^2 + Q_0 \tag{1.11}$$

By simplifying this equation and putting in more convenient forms by collection terms which are coefficients of the unknown values of h:

$$A_1 h_1 + A_2 h_2 + A_3 h_3 + A_4 h_4 - A_0 h_0 = -B_0 \tag{1.12a}$$

$$A_i = \frac{1}{2}(T_0 - T_i) \tag{1.12b}$$

$$A_0 = A_1 + A_2 + A_3 + \left[\frac{\grave{k}}{\grave{B}}\right]\Delta^2 \tag{1.12 c}$$

$$B_0 = \left[\left[\frac{\grave{k}}{\grave{B}}\right](\grave{h}_0) + \frac{Q_0}{A} - R\right]\Delta^2 + Q_0 \tag{1.12d}$$

We conclude from Equation (1. 12) the approximate algebraic for the partial differential equation at each interior node. With this equation we will solve it in our code because it gives us the values of dependent variable h at every interior node that lies inside the solution domain boundary. After we obtain the algebraic approximation for each interior node .We have to obtain the algebraic approximation for each interior node which lies upon or beside the solution domain.

Along the boundaries, the condition to obtain an algebraic approximation is

$$\alpha \frac{dh}{dn} + \beta h = F \tag{1.13}$$

Where α : Aquifer bulk coefficient of compressibility.

β: Bulk coefficient of compressibility for water .

F: a prescribed function equal to the elevation of the water surface above the coordinate origin.

In general α, β and F maybe discontinuous functions. We can put $\beta = 1, \alpha = 0$ and F=f or$\beta = 1,\ \alpha = 1$ and F=0.

We will approximated the equation (1.13) in the neighborhood of the three node grid show in figure(1. 3a) by using the following first –degree polynomial:

$$h(x,y) = h_0 + \frac{h_0 - h_2}{\Delta}x + \frac{h_0 - h_2}{\Delta}y + 0(\Delta^2) \qquad (5.14)$$

The coefficients in the previous equation(1.14) have been obtained by evaluating the polynomial at nodes 0,1,2 by assuming that point p is the nearest point on the boundary to node 0, $\widehat{e_n} = (N_x, N_y)$be the outward normal to the boundary at point p and δ be the distance between zero and p as it shown in figure (5.3b).We note that the radial coordinate δ is positive or negative.

Depending upon whether node 0 lies within or outside of the solution domain, respectively, For that equation(5. 14)gives

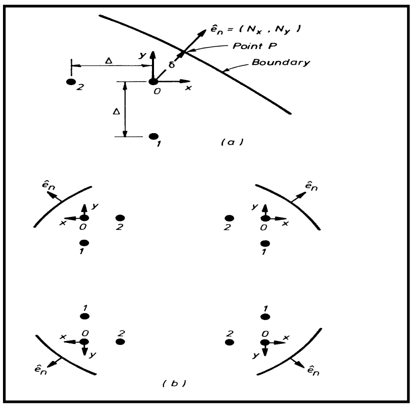


Figure (1.3) a & b: A three node grid along a boundary

$$\left[\frac{dh}{dn}\right]_p = (\overline{\nabla} h.\widehat{e_n})_p = \frac{h_0 - h_2}{\Delta}N_x + \frac{h_0 - h_1}{\Delta}N_y \qquad (1.15)$$

$$(h)_p = h_0 + \frac{h_0 - h_2}{\Delta}N_x\delta + \frac{h_0 - h_1}{\Delta}N_y\,\delta \qquad (1.16)$$

By substituting equations (1.15),(1.16) in to equation (1.13) gives the result

$$\alpha(\frac{h_0 - h_2}{\Delta})N_x + \alpha(\frac{h_0 - h_1}{\Delta})N_y + \beta(\frac{h_0 - h_2}{\Delta})N_x\delta + \beta(\frac{h_0 - h_1}{\Delta})N_y\delta + \beta h_\circ = F_p \qquad (1.17)$$

$$-h_1(\alpha + \beta\delta)\frac{N_y}{\Delta} - h_2(\alpha + \beta\delta)\frac{N_x}{\Delta} + \beta h_0 + h_0(\alpha + \beta\delta)\frac{N_y}{\Delta} - h_0(\alpha + \beta\delta)\frac{N_y}{\Delta} = F_p \qquad (1.18)$$

$$A_0 h_0 - A_1 h_1 - A_2 h_2 = F_p \qquad (1.19)$$

In which F_p denotes the value of F at point P and coefficients, A_1, A_2, A_0 are given by

$$A_1 = (\alpha + \beta\delta)\frac{N_y}{\Delta} \qquad (1.20a)$$

$$A_2 = (\alpha + \beta\delta)\frac{N_x}{\Delta} \qquad (1.20b)$$

$$A_0 = A_1 + A_2 + \beta \qquad (1.20c)$$

Equation (1. 19) is the algebraic approximation to equation (1.13) that is written for each node that lies either on or adjacent to the solution domain boundary . Therefore, we can state that we obtain the two algebraic approximations. The first one is for the interior nodes and the second one is for the nodes which lie upon or adjacent to the solution domain boundary.

$$A_1h_1 + A_2h_2 + A_3h_3 + A_4h_4 - A_0h_0 = B_0 \qquad (interior\ nods) \qquad (1.12)$$

$$A_0h_0 - A_1h_1 - A_2h_2 = F_p \qquad (boundry\ nodes) \qquad (1.19)$$

These two equations are the formulation of the boundary value problem and we will solve these two equations in step four by using the finite difference method after we blog our data.

4.1.3- Collect field data

Prior to starting step three, we need to know that step two and step three are serial because a correctly posed boundary value problem cannot be constructed without first knowing these basics, such as the location and the type of aquifer boundaries, the sources of recharge, etc. Additionally, collecting large amounts of field data before the problem is defined mathematically can lead to large expenditures of time and money in collecting data that is unnecessary.

Figure(5.4): General view of Jalan Tun Sadron slope

This step is usually the most expensive and time consuming part of any investigation in that we have to use the skills and experiences of a large number of people in various fields. Actually, we need different experts for different aspects. We obtain the recharge rates from rainfall for the slope which is in our study. The location of this slope is in Jalan Tun Sadron Pulau Pinang see figure (1.4).

We determined the accumulation of rainfall for the period from 30-Agu-2008 until 13-Sep-2008. We will use the two weeks to demonstrate how much the acumulation of rainfall changes when the weather is taken into consideration for this area of Malaysia which is located in the tropical rainforest. Figure (1.5) illustrates the water level Vs time.

We have selected the 30th of August, 2008, because there is an increasing accumulation of rainfall. The recharge of rainfall is significant data for our study because it is one of the coefficients that will be used in our algebraic equation (1.12) and R will create a specified maximum for h of the accumulation of rainfall R for 30th August 2008 is 0.8×10^{-6} m.

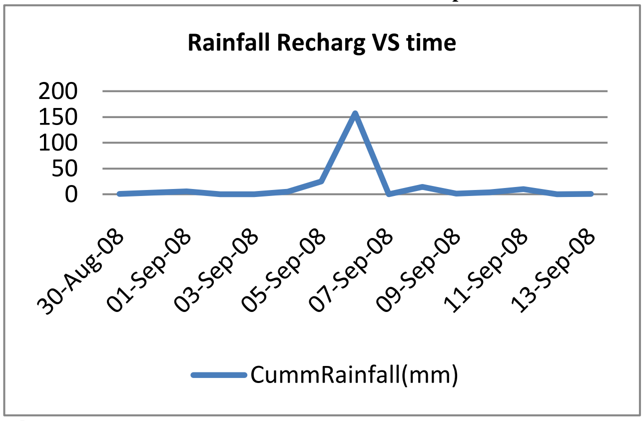


Figure (1.5): Rainfall data

We now need to determine the piezometric head for each interior point or for each point that lies on or next to the boundary domain. Since we don’t have the kind of tools needed, which are difficult to obtain and which are too costly, we will use the same piezometric head that Hunt (1983) employed as well as our recharge of rainfall. Let us assume the solution domain boundary as shown in figure (1. 6) and NB the number of boundary points, and N the number of nodes in the interior grid.

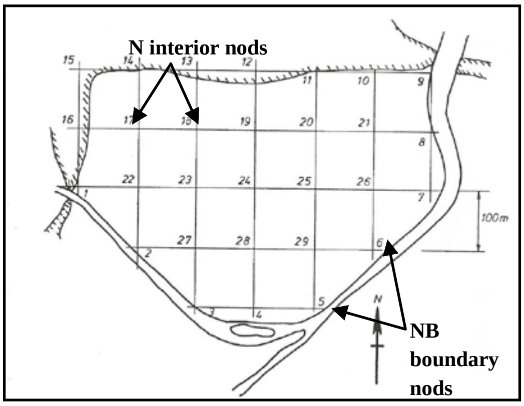


Figure (1.6): A finite difference grid superimposed upon a solution domain.

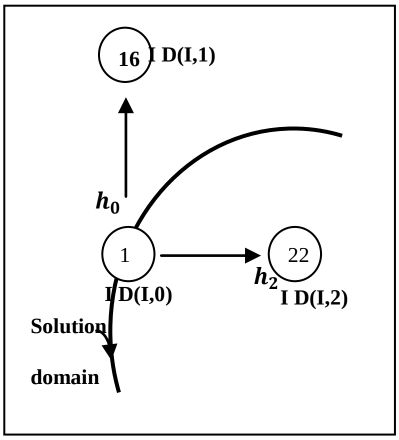
Probably the main difficulty is to relate the global and the local numbering schemes in a way which has enough flexibility to solve problems on solution domain with curved boundaries.We can overcome this problem with the flowing scheme:

1- Each points that lies on or next to the boundary is assigned a global number between 1 and NB. Also, each interior point is assigned a global number between NB+1 and N.

2 - Each of these points is assigned a series of either four or two integers that give the global number of surrounding nods. For the interior nodes N these integers will be called in our cod I D(I,J) where I goes from I through 4. For the boundary nodes NB I D(I,J)where J goes from 1 through 2. These integers and global number of the nods that correspond with local numbering.

For example, to make this clear we take

1- Boundary node 1, I D(1,j) where J=1and 2, I D (1, 1)=16, I D(1,2)=22. As figure (1.7) and this describe our input data where we take the NB=16



4.1.4- Solve the boundary value problem that was formulated in step two

In step four we calculate answers to the equations that were posed in step two and the data gathered in step three is incorporated directly into the formulation and solution of the problem. In our study to solve the system of simultaneous equations that are generated by writing equation (1. 19) at each interior node and equation (1.12) at each boundary node, we can solve them through various methods. We must first choose the finite difference method because it is an easily understood method that can provide approximate solutions under very general circumstances. In short, iterative methods consist of guessing and adjustments. There are three iterative techniques: Jacobi iteration, Gauss Seidel iteration, and Successive over Relaxation (SOR). Jacobi iteration is the least efficient and is seldom used. Gauss Seidel iteration can be considered to be a special case of Successive over Relaxation.

Figure (1.7): A typical three node boundary node

The change between two successive and Gauss Seidel iterations is called" residual c. In the method of SOR, the Gauss Seidel residual multiplied by relaxation factor $\omega \geq 1$. For the SOR method $0 < \omega < 1$.For the Gauss Seidel method $\omega = 1$.

However, the applications considered here require the simultaneous solution of twenty nine equations with a sparse matrix and relatively large diagonal terms in the coefficient matrix. With these conditions, our choice is the Gauss Seidel iterative method since it is easier to code for a computer ,it requires considerable less computer storage, and it uses less computational time. The Gauss Seidel method will be the last step for the code. After calculating the coefficient of two previous equations to obtain the matrix, we will solve it by Gauss Seidel iteration to estimate for (piezometric head) h.**Then** we will run the program by using our data with recharge of rain full R=0.0000008m/s=2104mm/month, the output as below in the Figure(1.8)

5 Analysis for output:

From the Figure (1.8) above we note that aquifer has been assumed homogeneous, that h will be the change in water level created by the uniformly distributed recharge rate R and that the problem is inverse problem in which the unknown is the value for R that will create a specified maximum for h. However, this inverse problem is easily to solved because the problem is liner in both hand R. Thus, multiplying all of the equations by the same constant λ, shows that if a recharge rate of R creates a water level change of h. Then a recharge rate λR will create a water level change of λh. The output for the computer program gives the water table rise for a recharge rate of R (Hunt data) **R = $\mathbf{0.1 * 10^{-6}}$ m/s = 263 mm/month**. Since the maximum computed rise is 0.315m at the node 12 .

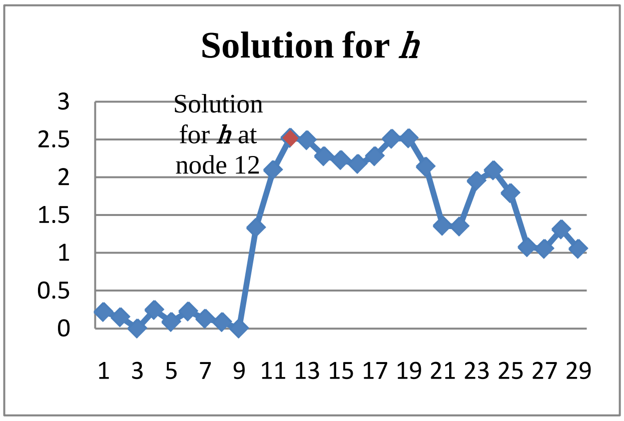


Figure (1.8): the values of the h at each node in solution boundary domain

The value for R that will give a maximum water level rise of 1 meter is

$R = \frac{1m}{0.315} * 263$ mm / month=835 mm/month .

The output for the computer program gives the water table rise for a recharge rate of R (our data) **R $= \mathbf{0.8 * 10^{-6}}$ m/s = 2104 mm/month**. Since the maximum computed rise is 2.519m at the node 12 as is shown in Figure(1.8).

We calculate the value for R that will give a maximum water level rise of 1 meter is

$R = \frac{1m}{2.519} * 2104$ mm/month = 835.252 mm/month.

This of course, is the maximum rate at which recharge water could actually be allowed to enter the saturated portion of the aquifer. For that we conclude the increase of recharge of the rainfall as show Figure (1.9) especially for the countries with tropical rainforests as our case this leads to increasing in the water table then unpredicted landslide.

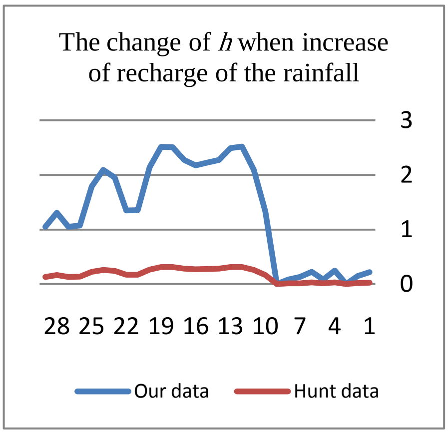


Figure (1.9): The change of h when increase of recharge of the rainfall

6 Conclusion:

Groundwater flow models have a very long history and come in many diverse forms. Early flow models were based primarily on the finite difference method of the approximation of governing field equations. Finite difference models, both simple in their concept and computationally efficient, found broad acceptance by the general groundwater community.

Each model should be constructed to answer specific questions. Indeed, the detail and accuracy of a model depends on the question it is designed to answer. Our model is designed to predict the amount of water in a particular solution boundary domain in a slope that possibly can collapse. The answers generated using mathematical models are dependent on the quality and the quantity of the field data available that is used to define the input parameters and boundary conditions. Collection of this field data is the most difficult, the most expensive, and the most time consuming part of modeling, as mentioned in chapter two. The success of any modeling largely depends upon the availability and reliability of measured or recorded data required for the study. Usually, in most modeling projects, the time and effort spent on the pre-processing and post-processing of data far exceeds the time spent on other project activities. The reliability of predictions from groundwater models depends on how well the model approximates the field situation. Inevitably, simple assumptions must be made in order to construct a model because the field situation is too complex to be simulated exactly. To deal with more realistic situations like ours, we have to solve the mathematical models approximately using a numerical technique. From many studies in groundwater modeling we found the most easily understood and most applicable method in obtaining numerical solutions to steady groundwater flow to be the finite difference method. Therefore, the finite difference method is our choice to obtain the algebraic approximation equations for the solution boundary domain we have established. When we have to obtain two kinds of algebraic approximation equations, the first one for the nodes lies upon or beside the boundary while the second one is the partial differential equation for the interior nodes. After obtaining the algebraic equations for each node we can solve the equations simultaneously to obtain the unknown value of the dependent variables at all nodes.

Figure (1.9): The change of

h when increase of recharge

The dramatic advances in the efficiency of digital computers during this past decade have provided hydrologists with a powerful tool for numerical modeling of groundwater systems. We used the computer program in Matlab 7.0 to solve a large number of algebraic equations by iterative techniques as in the Gauss Seidel method. The result of the program is the groundwater peizomtric head. This helps us to predict the amount of water in ground surfs or in the aquifer. It also gives us a clear idea of the effect of a recharge of rainfall and how much it will create a specific maximum for h(piezometric head). Additionally, we obtain the value of R (rainfall recharge) that will give a maximum water level rise of 1 meter. The model can be used to predict the acceleration of movement for landslide.

8 References

Baharum, A. (1997). "Prototype *Modeling of Landslide Prevention*", Operational Research Society Annual Conference, U K. 3-5 September 1997.

Biavati, G, J. W. Godt, et al. (2006). "*Drainage effects on the transient, near -surface hydrologic response of a steep hillslope to rainfall: implications for slope stability, Edmonds, Washington, USA* ", Natural Hazards and Earth System Sciences.

Corominas. J, Moya. J, et al. (2005). "*Prediction of ground displacements and velocities from groundwater level changes at the Vallcebre landslide (Eastern Pyrenees, Spain)*." Landslides , Springer 2(2): 83-96.

Hunt, B. (1983). "*Mathematical Analysis of Groundwater Recourses*". Butterworth, Butterworth &CO.

Koay, P. S, L. Habibah, Baharum. A. et al. (2008). "*The preliminary study on landslide prediction model in Malaysia*", ICL conference, Tokyo, Japan.

Ghosh, N. C. and Sharma, K. D. (2006). "*Groundwater Modeling and Management*". New Delhi, Capital Publishing Company

Pinder, G. F. (2002). "*Groundwater modeling using geographical information systems*", Wiley, New York, USA.

Wang, H. F. and M. P. Anderson (1995). "*Introducing to groundwater modeling. Finite difference and finite element methods*." Academic Press, 6277 Sea Harbor Dr. Orlando Florida 32887 USA, 1995, 237.

Websites

Wikipedia :http://en.wikipedia.org/wiki/Groundwater_models

landslide groundwater: http://www.wgwa.org/articles/landslidesandgroundwater.pdf