Performance Measure of Some Subspace-based Methods for Closed-loop System Identification

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***Abstract* — This paper presents some of the methods under subspace-based family to perform closed-loop system identification. Three methods have been observed; those are the ORT method, MOESP method and CCA method. The performance of these models is evaluated by identifying the same experimental systems. Three performance evaluation tests according to mean square errors, variance accounted for and best fit are used to verify the accuracy of the models to identify the given systems. The noise model is also introduced in order to see whether the developed model can improve the overall identification performance or not. Based on the simulation results, some justifications are made as to conclude the efficacy of the observed models to identify a closed-loop data system.**

Keywords- subspace method; closed-loop system; system identification

I. INTRODUCTION

Subspace identification methods (SIMs) has proven to be such a valuable tool in identification area since past few years. This interest is due to the ability of the subspace approach in providing accurate state-space models for multivariable linear systems directly from input-output data [1,2,3,4,5]. Recently, system identification is focused on closed-loop system applications since there are many cases where open-loop experiments are impossible due to safety and stability consideration [2,6,7]. Closed-loop experiments are also necessary if the open-loop plant is unstable, or the feedback is an inherent mechanism of the systems. At the early development of subspace method, this approach seems to provide bias when implemented on a system that works under closed-loop operation. This is due to the correlation between disturbances and the control signal, induced by loop, in which the ordinary subspace methods failed to solve. However, with special treatment, now the subspace methods are also able to identify the closed-loop system (See for some published examples in [6,7,8,9,10]).

There are three common approaches to closed-loop identification: (i) direct approach, (ii) indirect approach and (iii) joint input-output approach. These approaches have their own advantages and disadvantages (Details explanation can be referred in [2,11]). The direct approach usually provides biased estimates unless the noise effect is not so significant and can be neglected. However, this is not always true in practical applications. On the other hand, the indirect approach requires the information about the controller transfer function is known. The advantage of the joint input-output approach is that the knowledge of the controller is not required. However, the major drawback is that the identified model has an order which is equal to the sum of the plant and controller order. Therefore, model reduction step is required in the procedure.

Instrumental variable (IV) techniques are usually adopted in subspace approaches to identify plant models of systems operating in closed-loop. The IV are mainly used as an instrument to remove the effect of the noise term, since the geometrical properties of the ordinary subspace equation are lost in the presence of noise term [6,12,13].

In this paper, three subspace approaches are observed in order to perform closed-loop system identification. These three approaches are the ORT (Orthogonal Decomposition) method, MOESP (Multivariable Output Error State Space) method and CCA (Canonical Correlation Analysis) method. Some of the published paper in relation to these three approaches can be referred for example in [7,9,14,15,16,17].

The objective of this paper is to analyse the performance of the models in identifying same experimental data system. The accuracy of those three models will be discussed based on results from the evaluation tests. Three tests are used; the mean square errors (MSE) test, the variance accounted for (VAF) test and the best fit (BF) test. In addition, the noise model is also introduced as to see whether the developed models can improve the overall identification performance or not. The significant contribution of this paper is the analyses that have been done for three different approaches of the same subspace family, in terms of method differences and performance demonstrated during the simulation and evaluation procedure. In addition, the introduction of noise model and pre-filtering process can be considered as an improvised approach for the existing observed methods. Having few differences in terms of computation of system matrices for estimation, some justifications can be made as to investigate any significant improvement with respect to performance of those models.

This paper is organized as follows. Closed-loop framework will be shown in Section II. Model description

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of closed-loop system is elaborated in detail during this section. In Section III, problem formulation to identify closed-loop system will be explained and notations used in subspace identification process will be shown. Detail description on subspace methods used in this performance measure is discussed in Section IV. As to compare the performance on those three approaches in obtaining the state space model, simulation example is demonstrated in Section V. Results from evaluation tests are also tabulated in a table. Discussion based on the observation over the results obtained will be clearly justified. Finally, Section VI concludes the paper.

II. A Closed-Loop Framework

The experimental setup for the closed-loop system is shown in Fig. 1. The identification setup considered in this paper is organized as follows: $u \in \mathbb{R}^{n_u \text{x}1}$, $y \in \mathbb{R}^{n_y \text{x}1}$, $r \in \mathbb{R}^{n_y \text{x}1}$ and $r_s \in \mathbb{R}^{n_u \text{x}1}$ are the input, output, reference and set point signals generated by the closed-loop configuration in Figure 1. The controller output is denoted by $u_c \in \mathbb{R}^{n_u \text{x}1}$ while $e \in \mathbb{R}^{n_y \text{x}1}$ is unobserved, zero mean, white noise vector sequence. It is assumed, without loss of generality, that the set point, $r_s = 0$ while an excitation signal is added to the controller output, via r.

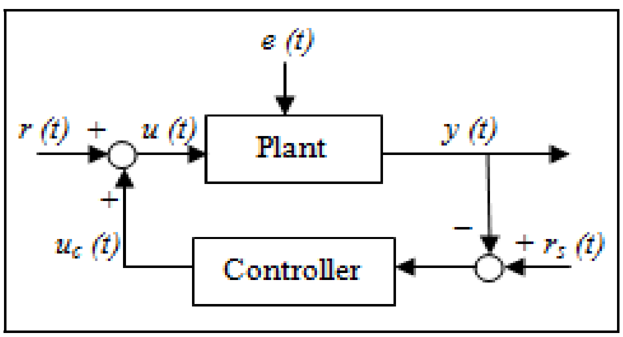

Figure 1. Closed-loop system setup

The signals are connected through the following state space representations.

- The plant equations:

$$x(t+1) = Ax(t) + Bu(t) + Ke(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) + e(t) \tag{2}$$

- The controller equations are defined as

$$x_c(t+1) = A_c x_c(t) - B_c y(t) \tag{3}$$

$$u_c(t) = C_c x_c(t) - D_c y(t) \tag{4}$$

where $u(t) = r(t) + u_c(t)$. We assume that the closed-loop identification problem is well posed, in a sense that the output $y(t)$ is uniquely determined by the states of the plant, by the controller and by the disturbances and reference input. This generic condition is satisfied when $(I_{n_y} + DD_c)$ is non-singular [6,7].

III. Problem And Notations

A. Problem formulation

The problem treated in this paper can now be stated as

Given:

- Input, output and reference data: $u(t)$, $y(t)$ and $r(t)$, $t = 0,1,...,N+2k-2$ are given with number of data, $N \to \infty$ and $k > n$ where k is number of block rows of Hankel matrix and n is order of the system.

Find:

- The system matrices A, B, C, D up to within a similarity transformation.

B. Notations

Some system related matrices need to be defined first, before presenting the solution to this problem. The usual block Hankel matrix of past input is defined as

$$U_p = \begin{bmatrix} u(0) & u(1) & \cdots & u(N-1) \\ u(1) & u(2) & \cdots & u(N) \\ \vdots & \vdots & \ddots & \vdots \\ u(k-1) & u(k) & \cdots & u(N+k-2) \end{bmatrix} \tag{5}$$

and future input as

$$U_f = \begin{bmatrix} u(k) & u(k+1) & \cdots & u(k+N-1) \\ u(k+1) & u(k+2) & \cdots & u(k+N) \\ \vdots & \vdots & \ddots & \vdots \\ u(2k-1) & u(2k) & \cdots & u(N+2k-2) \end{bmatrix} \tag{6}$$

where $U_p, U_f \in \mathbb{R}^{km\text{x}N}$. Similarly, we define past and future output $Y_p, Y_f \in \mathbb{R}^{kp\text{x}N}$ respectively. Therefore, the constructed input and output are given as

$$U = \begin{bmatrix} U_p \\ U_f \end{bmatrix}, \qquad Y = \begin{bmatrix} Y_p \\ Y_f \end{bmatrix} \tag{7}$$

The extended observability matrix of order i is defined as

$$\Gamma_i = \begin{bmatrix} C^T & (CA)^T & \cdots & (CA^{i-1})^T \end{bmatrix}^T \tag{8}$$

and the block lower triangular Toeplitz matrix as

$$
\mathrm{H}_i = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-2}B & CA^{i-3}B & \cdots & D \end{bmatrix} \tag{9}
$$

IV. Subspace Identification Methods

The subspace identification based on ORT, MOESP and CCA methods are observed in this paper. The algorithms are developed for both deterministic and stochastic components. Details regarding this algorithm can be referred in [2,5,15]. The main idea for models based on ORT and MOESP methods is to reconstruct the past input and past output data as instrumental variables. Then, the past input-output and future input-output data are projected onto the space of exogenous inputs by using the Linear Quadratic (LQ) decomposition in order to obtain their deterministic components.

In comparison between ORT and MOESP methods, differences in their developed algorithms can be stated as follows:

- Even though both methods use the same LQ decomposition, the way of utilizing the L factors is different. The ORT [2] utilizes the L_{42} to obtain

  $$\lim_{N\to\infty} \frac{1}{\sqrt{N}} \left[L_{42} \right] = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \Gamma_i X_f \left[Q_2^T \right]$$

  whereas the MOESP [5] method utilizes [$L_{42} L_{43}$] to obtain

  $$\lim_{N\to\infty} \frac{1}{\sqrt{N}} \left[L_{42} \quad L_{43} \right] = \lim_{N\to\infty} \frac{1}{\sqrt{N}} \Gamma_i X_f \left[Q_2^T \quad Q_3^T \right]$$

- Due to the above mentioned factorization, the way of obtaining the extended observability matrix is also different. Hence, the computation of A and C matrix is also treated differently.
- In stochastic algorithm formulation the difference can be observed during the computation of residuals, $\left(\rho_w \ \& \ \rho_v\right)$. The ORT computes the residuals using the following equation

  $$\begin{bmatrix} \widehat{X}_{i+1} \\ \hat{Y}_{i|i} \end{bmatrix} = \begin{bmatrix} A_s \\ C_s \end{bmatrix} \widehat{X}_i + \begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix} \tag{10}$$

  whereas the MOESP computes the residuals using the following equation

  $$\begin{bmatrix} X_i \\ L_u \end{bmatrix} = \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \begin{bmatrix} X_i \\ L_y \end{bmatrix} + \begin{bmatrix} \rho_w \\ \rho_v \end{bmatrix} \tag{11}$$

  L_u is row in L corresponding to the $U_{i,1,N}$ and L_y is row corresponding to $Y_{i,1,N}$.

Both subspace methods are then applied to deterministic components like the direct approach in order to derive state-space models of the plant. In the other hand, the stochastic part will derive Kalman gain to develop a pre-filter computed as follows.

$$\tilde{L}(z^{-1}) = \widehat{C}(zI - \widehat{A})^{-1} K + I \tag{12}$$

The noise model is obtained from developed pre-filter and reconstructed input. The input-output data will be filtered by the noise model and the new instruments are generated based on the estimated system matrices from the deterministic part of both methods. By having the new instruments and the filtered data, the IV estimate of the process is finally determined.

For model based on CCA approach, a stochastic component is directly used to determine an estimation of system matrices of the process. In terms of LQ decomposition, this approach is slightly different from the previous two methods. Here, the instrumental variable is produced in the dimension of the joint process of past input output data. $W_p = [U_p; Y_p]$. Hence, the 3x3 matrix is used instead of 4x4 as in ORT and MOESP methods. The remarkable difference is observed in the use of the normalized SVD of the conditional covariance matrix. Details regarding this algorithm can be referred in [2].

In summary the identification procedure that runs using these methods goes as follows.

Let $\{u, y\}$ the input-output data of the system (1)-(2) and $\{\tilde{u}, \tilde{y}\}$ the reconstructed input-output sequence from the following simulation of the closed-loop system

$$x(t+1) = \hat{A}x(t) + \hat{B}\tilde{u}(t) \tag{13}$$

$$\tilde{y}(t) = \hat{C}x(t) + \hat{D}\tilde{u}(t) \tag{14}$$

$$x_c(t+1) = A_c x_c(t) - B_c \tilde{y}(t) \tag{15}$$

$$u_c(t) = C_c x_c(t) - D_c \tilde{y}(t) \tag{16}$$

Step 1: Construct data matrices of
U_p, U_f, Y_p, Y_f for ORT and MOESP methods
U_f, Y_f, W_p for CCA method

Step 2: Perform LQ factorization.

Step 3: Perform SVD to the working matrix
ORT: $[L_{42}]$
MOESP: [$L_{42} L_{43}$]
CCA: $Lfi * [L_{32} * L_{22}{}'] * Lpi'$
(Lfi and Lpi denote some other matrix multiplication as can be referred in [2])

Step 4: Solve the stochastic components and compute the residuals and the estimate of covariance matrices based on Equation (10) for ORT and Equation (11) for MOESP and CCA.

Step 5: Determine the system matrices ($\hat{A}, \hat{B}, \hat{C}, \hat{D}, \hat{K}$).
Step 6: Compute noise model based on Equation (12).
Step 7: Generate new instruments and filter the I/O data with noise model.
Step 8: Using information in Step 7, repeat Step 1 – 4.
Step 9: Generate the predicted output.

V. SIMULATION EXAMPLE

A. *Example 1- Heating System*

The first example is a heating system data taken from [18]. The physical apparatus consists of a 300 Watt Halogen lamp suspended several centimeters above a thin steel plate. The purpose is to model the change in temperature of the metal plate for changes in lamp power. The system input was the driving voltage for the lamp. The output consists of the measurements taken by a thermocouple mounted on the back of the plate.

The data obtained from this plant is used to evaluate the performance of three observed methods. Length of data sequence used to estimate the process model is $N = 800$. A signal to noise ratio is

$$\text{SNR} = 10\log(\frac{P_{y_d}}{P_e}) = 37.9968 \text{ dB} \tag{17}$$

where P_{y_d} denotes the signal power and P_e is the noise power.

The input and output of heating data system is shown as in Fig. 2. The graph of superimpose between simulated true system data and simulated output obtained from ORT, MOESP and CCA models are shown in Fig.3.

B. *Example 2- Electrical Servo Motor*

Second example is an electrical servo motor system which has been used as in [6]. The plant corresponds to a discrete-time model of a laboratory plant set-up of two circular plates rotated by an electrical servo motor with flexible shafts. The plant has a state-space description as in (1)-(2) with

$$A = \begin{bmatrix} 4.40 & 1 & 0 & 0 & 0 \\ -8.09 & 0 & 1 & 0 & 0 \\ 7.83 & 0 & 0 & 1 & 0 \\ -4.00 & 0 & 0 & 0 & 1 \\ 0.86 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0.00098 \\ 0.01299 \\ 0.01859 \\ 0.00330 \\ -0.00002 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$D = 0$ and e_k is a Gaussian white noise sequence. The plant has one integrator, and therefore is only marginally stable. The controller has a state-space description as in (3)-(4) with

$$A_c = \begin{bmatrix} 2.65 & -3.11 & 1.75 & -0.39 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_c = \begin{bmatrix} -0.4135 & 0.8629 & -0.7625 & 0.2521 \end{bmatrix}$$

$$D_c = 0.61$$

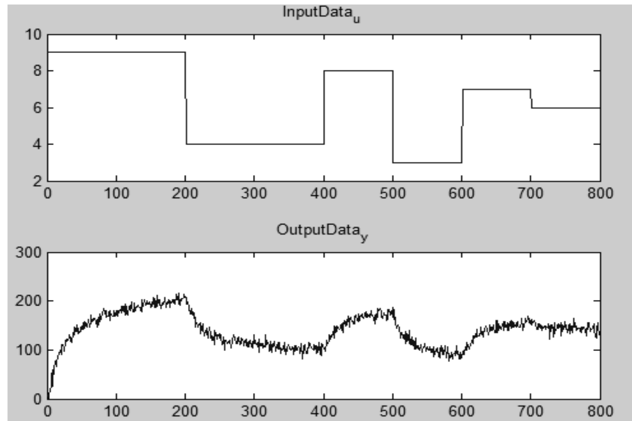

Figure 2. Input and output data of example 1 with noise

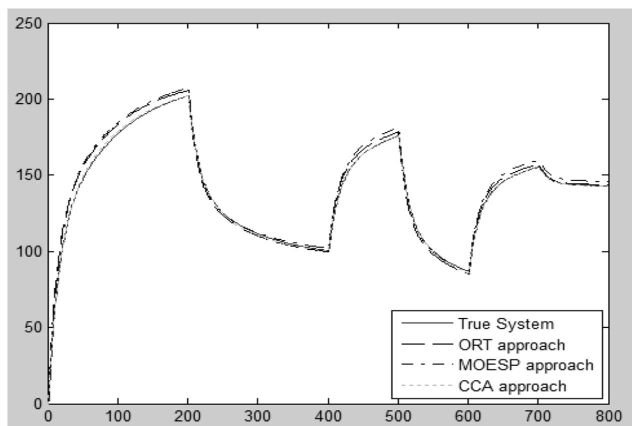

Figure 3. Superimpose between simulated true data system of example 1 and simulated output obtained from estimated system matrices of ORT, MOESP and CCA methods.

The excitation signal r_k is a Gaussian white noise sequence with variance 1. The data obtained from this plant is used to evaluate the performance of three observed methods. Length of data sequence used to estimate the model is $N = 1000$.

The signal to noise ratio of this data is 14.8179 dB. The input and output of electrical servo motor data is shown as in Fig. 4. The graph of superimpose between simulated true system data and simulated output obtained from ORT, MOESP and CCA models are shown in Fig.5.

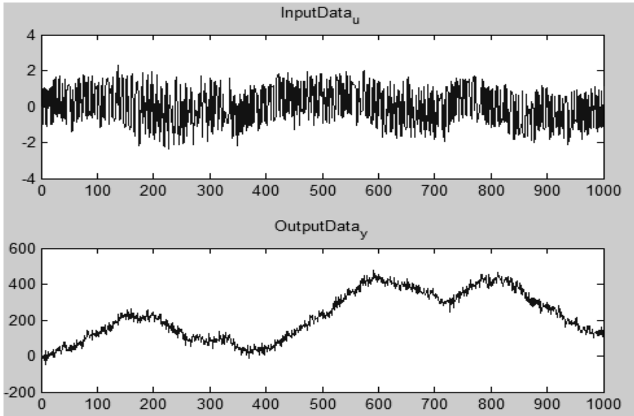

Figure 4. Input and output data of example 2 with noise

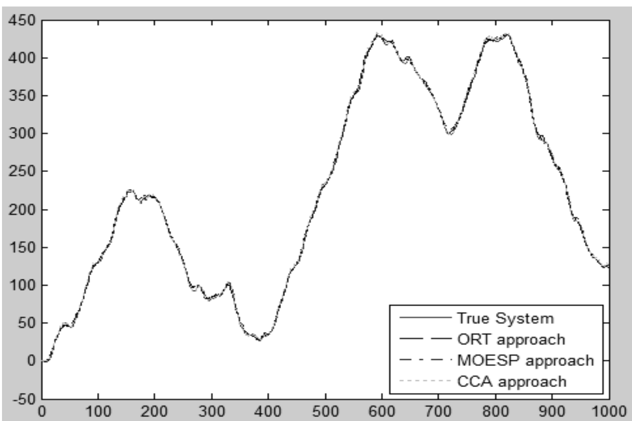

Figure 5. Superimpose between simulated true data system of example 2 and simulated output obtained from estimated system matrices of ORT, MOESP and CCA methods.

The following test will be used for model evaluation

- Mean Squares Error (MSE) Test

$$\text{MSE} = \frac{1}{N}\sum_{i-1}^{N} | \, y(t) - \hat{y}(t) \, |^2 \tag{18}$$

where $y(t)$ =measured output; $\hat{y}(t)$ =estimated output

- Best Fit (BF)

$$\text{BF} = (1 - \frac{| \, y - \hat{y} \, |}{| \, y - \bar{y} \, |})\text{x}100\% \tag{19}$$

where $y(t)$ =measured output; $\hat{y}(t)$ =estimated output; $\bar{y}(t)$ =mean output

- Variance Accounted For (VAF)

$$\text{VAF} = (1 - \frac{\text{VAR}(y(t) - \hat{y}(t))}{\text{VAR}(y(t))})\text{x}100\% \tag{20}$$

where $y(t)$ =measured output; $\hat{y}(t)$ =estimated output

Table I and Table II tabulate the results obtained from the evaluation tests of example 1 and example 2 respectively.

TABLE I. VALIDATION TEST RESULTS OF EXAMPLE 1

Type	Output	MSE	BF	VAF
ORT	***Without noise model***	$1.0500\text{x}10^3$	3.5587	94.2576
	With noise model	15.0948	88.4225	98.9213
MOESP	***Without noise model***	382.2326	41.7407	98.3505
	With noise model	21.2063	86.2775	99.2504
CCA	***Without noise model***	1.1908	96.7483	99.8943
	With noise model	NO	NO	NO

TABLE II. VALIDATION TEST RESULTS OF EXAMPLE 2

Type	Output	MSE	BF	VAF
ORT	***Without noise model***	15.4028	97.0117	99.9257
	With noise model	5.2023	98.2633	99.9714
MOESP	***Without noise model***	60.4090	94.0820	99.8937
	With noise model	9.7487	97.6226	99.9631
CCA	***Without noise model***	6.1440	98.1127	99.9766
	With noise model	NO	NO	NO

Based on Table I and Table II, the following justifications are made:

- In general, these three methods are able to identify the corrupted closed-loop data system successfully.
- The performance of ORT and MOESP approach is improved when the noise model is introduced during the identification process.
- However, the performance given by CCA is good even though without the noise model.
- In consideration on stochastic observation only (without considering the adoption of the noise model), it shows that the CCA method gives the best performance.
- In terms of computational load and computational time, the CCA is also chosen as the best model since the identification process already completed at step 5.
- Based on example 2, the CCA method requires only 100 of block rows of Hankel matrix to obtain the same performance, in which ORT and MOESP require 150 block rows. Again, the CCA is better in terms of computational time.

CONCLUSION

Three methods under the subspace-based family are observed to develop a state-space model based on identification from closed-loop data. Given the heating system and the electrical servo motor system as an example, these three approaches are able to identify the corrupted closed-loop data system successfully. Among all, the CCA method gives the best performance overall. The CCA method can be considered as a good model since it does not need the noise model and require less block rows to obtain the same performance as the other two approaches. It also requires less computational load and time.

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