
UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2007/2008

October/November 2007

MSS 317 – Coding Theory
[Teori Pengekodan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all six** [6] questions.

Arahan: Jawab **semua enam** [6] soalan.]

...2/-

1. Let C be a linear code over F_3 with generator matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}.$$

- (a) Find a generator matrix for C in the standard form.

[20 marks]

- (b) Find a parity-check matrix for C in the standard form.

[40 marks]

- (c) Use syndrome decoding to decode the received words 2121, 1201 and 2222.

[50 marks]

2. (a) Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

$$\begin{aligned} \text{Let } B_1 &= \{1, 3, 4, 5, 9\}, & B_2 &= \{2, 4, 5, 6, 10\}, & B_3 &= \{3, 5, 6, 7, 11\}, \\ B_4 &= \{1, 4, 6, 7, 8\}, & B_5 &= \{2, 5, 7, 8, 9\}, & B_6 &= \{3, 6, 8, 9, 10\}, \\ B_7 &= \{4, 7, 9, 10, 11\}, & B_8 &= \{1, 5, 8, 10, 11\}, & B_9 &= \{1, 2, 6, 9, 11\}, \\ B_{10} &= \{1, 2, 3, 7, 10\}, & B_{11} &= \{2, 3, 4, 8, 11\} \end{aligned}$$

Is $\{B_i \mid i = 1, 2, \dots, 11\}$ a $(11, 11, 5, 5, 2)$ -BIBD?

[30 marks]

- (b) Show that $A_2(11, 5) \leq 24$.

[30 marks]

- (c) Show that a $(11, 24, 5)$ -code exist

[50 marks]

3. (a) Find all possible 1-dimensional subspaces of F_5^2 .

[30 marks]

- (b) Construct a $[6, 4, 3]$ -Hamming code over F_5

[50 marks]

4. (a) Factorize $x^5 - 1$ into irreducible polynomials over F_2 .

[100 marks]

- (b) Find all cyclic binary codes of length 5.

[20 marks]

...3/-

1. Biar C ialah suatu kod linear atas F_3 dengan matriks penjana

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}.$$

- (a) Cari suatu matriks penjana bagi C dalam bentuk piawai.

[20 markah]

- (b) Cari suatu matriks semakan pariti bagi C dalam bentuk piawai.

[40 markah]

- (c) Guna penyahkodan sinkrom untuk menyahkodkan kata terima 2121, 1201 dan 2222.

[50 markah]

2. (a) Biar $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

$$\text{Biar } B_1 = \{1, 3, 4, 5, 9\}, \quad B_2 = \{2, 4, 5, 6, 10\}, \quad B_3 = \{3, 5, 6, 7, 11\},$$

$$B_4 = \{1, 4, 6, 7, 8\}, \quad B_5 = \{2, 5, 7, 8, 9\}, \quad B_6 = \{3, 6, 8, 9, 10\},$$

$$B_7 = \{4, 7, 9, 10, 11\}, \quad B_8 = \{1, 5, 8, 10, 11\}, \quad B_9 = \{1, 2, 6, 9, 11\},$$

$$B_{10} = \{1, 2, 3, 7, 10\}, \quad B_{11} = \{2, 3, 4, 8, 11\}$$

Adakah $\{B_i \mid i = 1, 2, \dots, 11\}$ suatu BIBD $-(11, 11, 5, 5, 2)$?

[30 markah]

- (b) Tunjukkan $A_2(11, 5) \leq 24$.

[30 markah]

- (c) Tunjukkan kod $-(11, 24, 5)$ wujud

[50 markah]

3. (a) Cari semua subruang berdimensi satu bagi F_5^2 .

[30 markah]

- (b) Bina suatu kod Hamming $-(6, 4, 3)$ atas F_5

[50 markah]

4. (a) Faktorkan $x^5 - 1$ menjadi hasil darab polinomial-polinomial tak terturun atas F_2 .

[100 markah]

- (b) Cari semua kod kitaran dedua yang berpanjang 5.

[20 markah]

...4/-

5. (a) Construct a $(3, 4, 2)$ -code over F_2 .

[40 marks]

(b) Construct a $(9, 4, 6)$ -code over F_2 .

[40 marks]

6. Let F be a finite field such that $\text{char}(F) = p$, p is a prime. Show that

(a) If $p = 2$, then every element $a \in F$, $a = b^2$ for some $b \in F$.

[50 marks]

(b) If $p \neq 2$, then exactly half of the non-zero elements $a \in F$ satisfies $a = b^2$ for some $b \in F$.

[50 marks]

...5/-

5. (a) Bina suatu kod $-(3,4,2)$ atas F_2 .

[40 markah]

(b) Bina suatu kod $-(9,4,6)$ atas F_2 .

[40 markah]

6. Biar F ialah suatu medan terhingga dengan $\text{char}(F) = p$, p ialah nombor perdana. Tunjukkan

(a) Jika $p = 2$, maka bagi setiap unsur $a \in F$, $a = b^2$ bagi suatu $b \in F$.

[50 markah]

(b) Jika $p \neq 2$, maka tepat setengah daripada unsur $a \in F$ mematuhi syarat $a = b^2$ bagi suatu $b \in F$.

[50 markah]

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