

UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
2009/2010 Academic Session

April/May 2010

**ESA 251/3 – Control System Theory**  
*Teori Sistem Kawalan*

Duration : 3 hours  
[Masa : 3 jam]

**INSTRUCTION TO CANDIDATES**  
**ARAHAN KEPADA CALON**

Please ensure that this paper contains **THIRTEEN (13)** printed pages and **FIVE (5)** questions before you begin examination.

*Sila pastikan bahawa kertas soalan ini mengandungi **TIGABELAS (13)** mukasurat bercetak dan **LIMA (5)** soalan sebelum anda memulakan peperiksaan.*

Part A: Answer **ALL** questions. Part B: Answer **TWO (2)** questions.

*Bahagian A: Jawab **SEMUA** soalan. Bahagian B: Jawab **DUA (2)** soalan.*

Appendix/Lampiran:

1. Appendix A/*Lampiran A* [1 page/mukasurat]
2. Second order time domain specification  
*Spesifikasi domain masa sistem tertib kedua* [1 page/mukasurat]

Student may answer the questions either in English or Bahasa Malaysia.

*Pelajar boleh menjawab soalan dalam Bahasa Inggeris atau Bahasa Malaysia.*

Each questions must begin from a new page.

*Setiap soalan mestilah dimulakan pada mukasurat yang baru.*

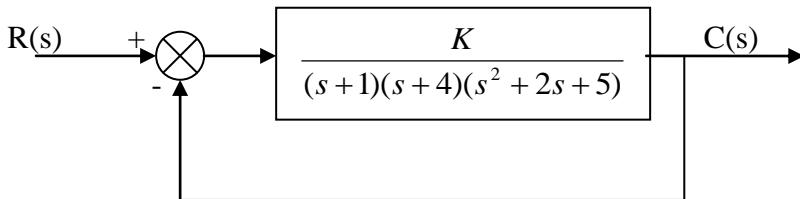
In the event of any discrepancies, the English version shall be used.

*Sekiranya terdapat sebarang percanggahan pada kertas soalan, versi Bahasa Inggeris hendaklah diguna pakai.*

**PART A/BAHAGIAN A**

1. **Figure 1** shows a block diagram of a mechanical system:

*Rajah blok bagi sebuah sistem mekanikal ditunjukkan seperti dalam **Rajah 1**:*



**Figure 1 / Rajah 1**

Where  $K \geq 0$  ( $K$  varies from 0 to infinity)

*Di mana  $K \geq 0$  ( $K$  berubah daripada 0 sehingga infinity)*

- (a) Using the rules for plotting the root locus, sketch the root locus plot on the graph paper provided.

*Dengan menggunakan peraturan binaan londar punca, lakarkan plot londar punca di atas kertas graf yang disediakan.*

**(12 marks/markah)**

- (b) Determine:

*Tentukan:*

- (i) The root locus angle of departure from the complex poles.

*Sudut berlepas londar punca dari kutub kompleks.*

**(3 marks/markah)**

- (ii) The root locus point of intersection on the imaginary axis

*Titik persilangan londar punca di paksi khayal*

**(3 marks/markah)**

- (iii) The value of gain K on the imaginary axis

*Nilai gandaan K di paksi khayal*

**(3 marks/markah)**

- (iv) The range of gain K for the closed-loop to be stable

*Julat gandaan K supaya sistem gelung tertutup adalah stabil*

**(3 marks/markah)**

- (c) Show that the break-away point of the root locus at  $s = -3$ .

*Tunjukkan titik pecah londar punca berada pada  $s = -3$ .*

**(1 marks/markah)**

2. A unity feedback system has an open-loop transfer function given as follow:

*Rangkap pindah gelung buka bagi satu sistem suap balik unit seperti berikut:*

$$G(s) = \frac{K(s+1)}{s(s+10)(s+5)^2}$$

- (a) Write the analytical expression for

*Tuliskan persamaan analitik bagi*

- (i) The magnitude frequency response  $|G(jw)|$  in unit of dB.

*Sambutan frekuensi gandaan  $|G(jw)|$  dalam unit dB.*

**(2 marks/markah)**

- (ii) The phase frequency response  $\angle G(jw)$  in unit of degree.

*Sambutan frekuensi fasa  $\angle G(jw)$  dalam unit darjah.*

**(2 marks/markah)**

- (b) If  $K=1000$ , sketch the bode magnitude and phase plots using straight-line approximation. Determine:

*Sekiranya  $K=100$ , lakarkan plot bode magnitud dan fasa menggunakan kaedah penghampiran garis lurus. Tentukan:*

**(10 marks/markah)**

- (i) Gain margin, G.M

*Jidar gandaan, G.M*

**(2 marks/markah)**

- (ii) Phase margin,  $\phi.M$

*Jidar fasa,  $\phi.M$*

**(2 marks/markah)**

- (iii) The range of K for stability of system from the bode plots

*Julat K supaya sistem stabil dari plot Bode.*

**(2 marks/markah)**

- (c) Determine the new value of gain  $K_1$  for the system to be stable critical. Draw the bode magnitude plot at the new value of  $K_1$  and find the phase margin.

*Tentukan nilai baru gandaan  $K_1$  supaya sistem menjadi stabil kritikal. Lukiskan magnitud bode bagi nilai  $K_1$  dan tentukan jidar fasanya.*

**(5 marks/markah)**

**PART B/BAHAGIAN B**

3. (a) An aircraft's attitude varies in roll, pitch and yaw as defined in **Figure 3(a)**. The system measures the actual roll angle with gyro and compares the actual roll angle with the desired roll angle. The ailerons respond to the roll-angle error by undergoing an angular deflection. The aircraft responds to this angular deflection, producing a roll angle rate.

*Sikap sesebuah kapal terbang berubah secara roll, pitch dan yaw seperti yang ditunjukkan dalam **Rajah 3(a)**. Sistem tersebut mengukur sudut roll sebenar dengan menggunakan giroskop dan membandingkan sudut roll sebenar dengan sudut rujukan roll. Kemudian, aileron akan bertindak balas terhadap ralat sudut roll dengan mengubah sudut pesongan. Kapal terbang akan bertindak berdasarkan sudut pesongan dan menghasilkan kadar sudut roll.*

- (i) Draw a functional block diagram for a closed-loop system that stabilizes the roll.

*Lukiskan gambarajah blok untuk sistem gelung tertutup yang menstabilkan roll.*

**(3 marks/markah)**

- (ii) Identify all the components of the block diagram including Input, output transducers, the controller and the plant.

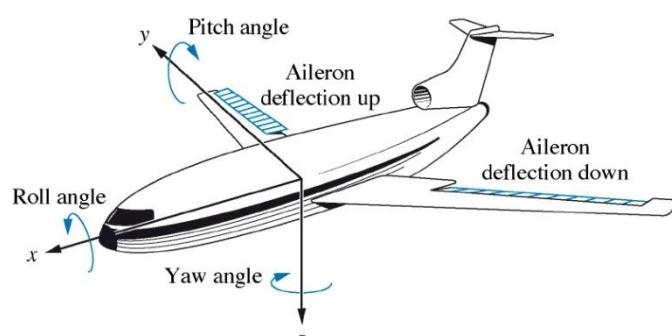
*Tentukan semua komponen gambar rajah blok termasuk masukkan, keluaran tranducer, pengawal dan pelan.*

**(3 marks/markah)**

- (iii) State the type of the system.

*Tentukan jenis sistem tersebut.*

**(1 marks/markah)**



**Figure 3(a) / Rajah 3(a)**

- (b) **Figure 3(b)** shows a translational mechanical network. Based on the model.

**Rajah 3(b)** menunjukkan sebuah sistem translasi mekanikal. Berdasarkan model tersebut.

- (i) Write the equation of motion.

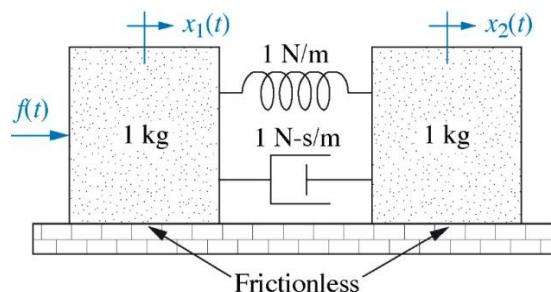
Tuliskan persamaan gerakan

(6 marks/markah)

- (ii) Find the transfer function relating the output  $X_2(s)$  to the input  $F(s)$ .

Dapatkan rangkap pindah yang menghubungkan keluaran  $X_2(s)$  kepada masukkan  $F(s)$

(6 marks/markah)



**Figure 3(b) / Rajah 3(b)**

- (c) A missile in flight, as shown in **Figure 3(c)**, is subject to several forces: thrust, lift, drag and gravity. The missile flies at an angle of attack,  $\alpha$ , from its longitudinal axis, creating lift. For steering, the body angle from vertical,  $\phi$  is controlled by rotating the engine at the tail. The transfer function relating the body angle,  $\phi$  to the angular displacement of the engine is of the form:

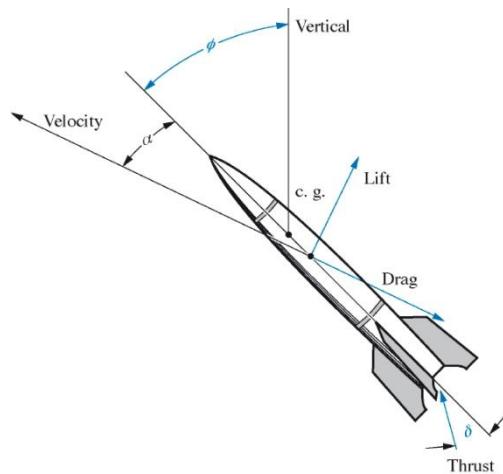
Satu peluru berpandu telah dilancarkan seperti ditunjukkan dalam **Rajah 3(c)** mengalami beberapa daya: daya tujahan, daya angkat, daya geseran dan graviti. Peluru berpandu tersebut meluncur pada sudut serangan,  $\alpha$  daripada paksi longitud yang menyebabkan ia terangkat. Untuk mengemudi, sudut badan merujuk kepada garisan menegak  $\phi$  dikawal dengan memusingkan enjin di bahagian belakang. Rangkap pindah yang menghubungkan sudut badan  $\phi$  terhadap sudut sasaran adalah seperti yang diberikan:

$$\frac{\phi(s)}{\delta(s)} = \frac{s^2 + 3s + 7}{(s+1)(s^2 + 5s + 4)}$$

- (i) Represent the missile steering control in state space.

Dapatkan kawalan pengemudian peluru berpandu tersebut dalam 'state space'.

**(6 marks/markah)**



**Figure 3(c)/Rajah 3(c)**

4. **Figure 4(a)** shows a mechanical system block diagram which is subjected to a unit-step input. The system output time response is shown in **Figure 4(b)**. Determine:

*Rajah 4(a) menunjukkan rajah blok sistem mekanikal yang diberikan masukan unit langkah. Sambutan masa keluaran sistem diberikan oleh Rajah 4(b). Tentukan:*

- (i) The values of  $K_1$  and  $K_2$  of the closed-loop system.

*Nilai  $K_1$  dan  $K_2$  bagi sistem tertutup.*

**(5 marks/markah)**

- (ii) The transfer function  $\frac{C(s)}{R(s)}$

*Rangkap pindah  $\frac{C(s)}{R(s)}$*

**(5 marks/markah)**

- (iii) The output time response,  $C(t)$

*Sambutan masa keluaran,  $C(t)$*

**(5 marks/markah)**

- (iv) The static error constant,  $K_p$

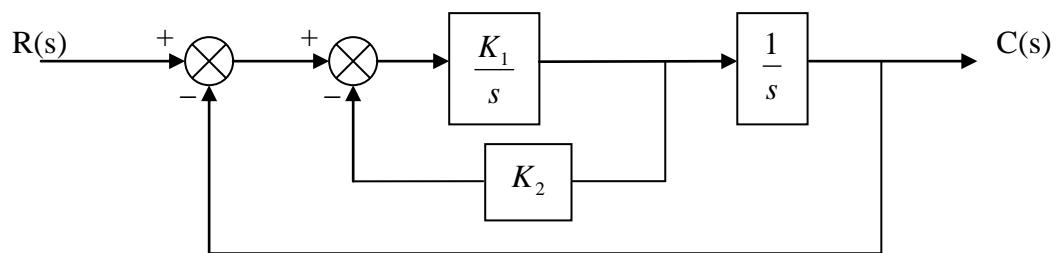
*Pemalar ralat statik,  $K_p$*

**(5 marks/markah)**

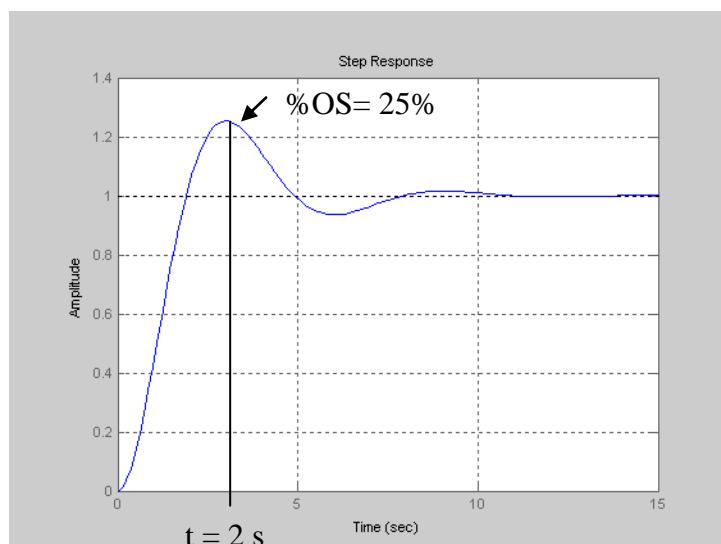
- (v) The steady state error,  $e_{ss}$

*Alat keadaan mantap,  $e_{ss}$*

**(5 marks/markah)**



**Figure 4(a)/Rajah 4(a)**



**Figure 4(b)/Rajah 4(b)**

5. (a) Simplify the block diagram shown in **Figure 5** by using block reduction

technique and obtain the closed-loop transfer function  $\frac{C(s)}{R(s)}$ .

*Ringkaskan gambarajah blok yang ditunjukkan pada **Rajah 5** dengan menggunakan teknik pengurangan blok dan dapatkan rangkap pindah gelang tertutup  $\frac{C(s)}{R(s)}$ .*

(12 marks/markah)

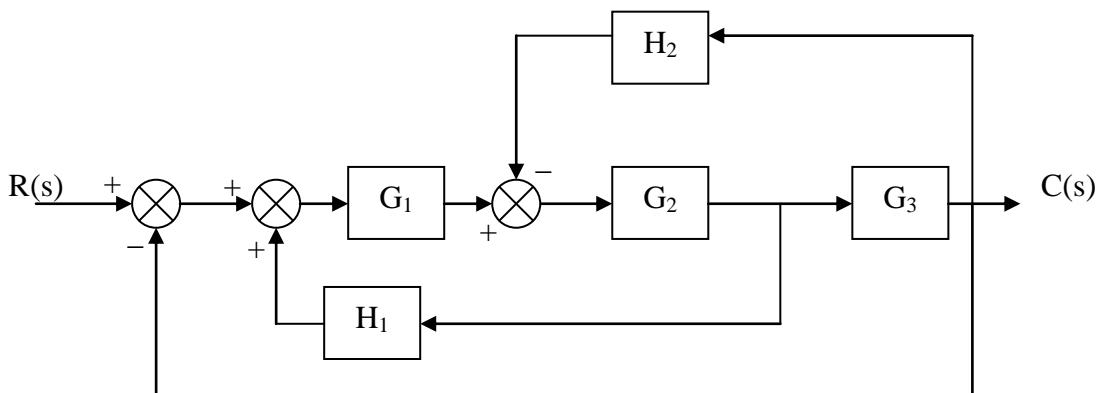
- (b) Use Mason's rule to find the transfer function  $\frac{C(s)}{R(s)}$  for the block diagram in

**Figure 5(a)**

*Dengan menggunakan peraturan Mason, dapatkan rangkap pindah  $\frac{C(s)}{R(s)}$*

*bagi blok diagram dalam **Rajah 5(a)***

(13 marks/markah)



**Figure 5/[Rajah 5]**

## APPENDIX A

## LAPLACE TRANSFORMS

Laplace transform	Time function	Description of time function
1		A unit impulse
$\frac{1}{s}$		A unit step function
$\frac{e^{-st}}{s}$		A delayed unit step function
$\frac{1 - e^{-st}}{s}$		A rectangular pulse of duration $T$
$\frac{1}{s^2}$	$t$	A unit slope ramp function
$\frac{1}{s^3}$	$\frac{t^2}{2}$	
$\frac{1}{s+a}$	$e^{-at}$	Exponential decay
$\frac{1}{(s+a)^2}$	$te^{-at}$	
$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	
$\frac{a}{s(s+a)}$	$1 - e^{-at}$	Exponential growth
$\frac{a}{s^2(s+a)}$	$t - \frac{(1 - e^{-at})}{a}$	
$\frac{a^2}{s(s+a)^2}$	$1 - e^{-at} - ate^{-at}$	
$\frac{s}{(s+a)^2}$	$(1 - at)e^{-at}$	
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at} - e^{-bt}}{b - a}$	
$\frac{ab}{s(s+a)(s+b)}$	$1 - \frac{b}{b-a}e^{-at} + \frac{a}{b-a}e^{-bt}$	
$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-a)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$	
$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	Sine wave
$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	Cosine wave
$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	Damped sine wave
$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	Damped cosine wave
$\frac{\omega^2}{s(s^2 + \omega^2)}$	$1 - \cos \omega t$	
$\frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$	$\frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \sin [\omega \sqrt{1-\zeta^2} t]$	
$\frac{\omega^2}{s(s^2 + 2\zeta\omega s + \omega^2)}$ with $\zeta < 1$	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \sin [\omega \sqrt{1-\zeta^2} t + \phi]$	

**SECOND ORDER TIME DOMAIN SPECIFICATION**  
**[SPESIFIKASI DOMAIN MASA SISTEM TERTIB KEDUA]**

$$\% \text{ overshoot} / \% \text{ Lajakan Maksimum}, \% C_p = 100e^{\left[ -\frac{\zeta\pi}{\sqrt{1-\zeta^2}} \right]}$$

$$\text{Peak time/ Masa puncak, } t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$\text{Rise time/ Masa menaik, } t_r = \frac{\pi - \cos^{-1} \zeta}{\omega_n \sqrt{1-\zeta^2}}$$

$$\text{Settling time /Masa pengenapan, } t_s = \frac{5}{\zeta\omega_n} \text{ (criteria /kriteria 2%)}$$

$$\text{Steady state error/ Ralat keadaan mantap, } e_{ss} = \lim_{s \rightarrow 0} \frac{sR(s)}{1+G(s)H(s)}$$

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