UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2007/2008

October/November 2007

MAT 203 – Vector Calculus
[Kalkulus Vektor]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of THIRTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

...2/-

1. **Instruction:** Write the choice of your answer in the exam answer script

(a) If $\underset{\sim}{u} = <2,2,-1>, \underset{\sim}{v} = <-3,4,0>$ find the cosine of the angle θ, between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$.

A. $-\frac{2}{3}$ B. $\frac{2}{3}$ C. $-\frac{2}{15}$ D. $\frac{2}{15}$

E. none of the above

(b) Find a vector parallel to the line of intersection of the planes

$$2x+3y-4z=4 \text{ and } 3x+2y+2z=5$$

A. $<16,14,5>$ B. $<16,-14,5>$ C. $<14,-16,-5>$

D. $<3,2,-1>$ E. none of the above

(c) Find the equation of tangent plane to the graph

$$f(x,y)=2x^3y-3xy^3-xy+4y$$

at the point $(-2,1,-4)$.

A. $8x+20y-z+8=0$ B. $20x+8y-z+8=0$

C. $-8x+20y+z-8=0$ D. $20x-8y+z+8=0$

E. $8x+20y+z+8=0$

(d) Let $f(x,y)=x^3y+x^4y^3$. Find the maximum directional derivative f at $(-1,1)$

A. $\sqrt{3}$ B. 1 C. $\sqrt{5}$ D. $\frac{1}{\sqrt{5}}$

E. none of the above

(e) Let f be a function of two variables with $f(2,3)=8, \nabla f(2,3)=<4,5>$. Estimate $f(1{\cdot}9,3{\cdot}1)$.

A. $7{\cdot}9$ B. $8{\cdot}1$ C. $7{\cdot}1$ D. $8{\cdot}9$

E. none of the above

...3/-

1. ***Arahan:*** *Tulis pilihan jawapan anda dalam skrip buku jawapan*

(a) Jika $\underset{\sim}{u} = <2,2,-1>, \underset{\sim}{v} = <-3,4,0>$ *cari nilai kosinus sudut* θ*, diantara vektor* $\underset{\sim}{u}$ *dan* $\underset{\sim}{v}$.

A. $-\frac{2}{3}$ *B.* $\frac{2}{3}$ *C.* $-\frac{2}{15}$ *D.* $\frac{2}{15}$

E. bukan satu pun di atas

(b) Cari suatu vektor selari dengan garis persilangan satah

$$2x+3y-4z=4 \text{ dan } 3x+2y+2z=5$$

A. $<16,14,5>$ *B.* $<16,-14,5>$ *C.* $<14,-16,-5>$

D. $<3,2,-1>$ *E. bukan satu pun di atas*

(c) Cari persamaan satah tangen kepada graf

$$f(x,y)=2x^3y-3xy^3-xy+4y$$

pada titik $(-2,1,-4)$.

A. $8x+20y-z+8=0$ *B.* $20x+8y-z+8=0$

C. $-8x+20y+z-8=0$ *D.* $20x-8y+z+8=0$

E. $8x+20y+z+8=0$

(d) Biar $f(x,y)=x^3y+x^4y^3$*. Cari terbitan berarah maksimum untuk* f *pada* $(-1,1)$

A. $\sqrt{3}$ *B. 1* *C.* $\sqrt{5}$ *D.* $\frac{1}{\sqrt{5}}$

E. bukan satu pun di atas

(e) Biar f *suatu fungsi dua pembolehubah* $f(2,3)=8, \nabla f(2,3)=<4,5>$*. Anggarkan* $f(1{\cdot}9,3{\cdot}1)$.

A. $7{\cdot}9$ *B.* $8{\cdot}1$ *C.* $7{\cdot}1$ *D.* $8{\cdot}9$

E. bukan satu pun di atas

...4/-

(f) The point $(1,3)$ is a critical point of the function $f(x,y)=6xy-3x^3-y^2x$. Classify the critical point $(1,3)$

A. Local Minimum B. Local maximum

C. Global minimum D. Saddle point

E. none of the above

(g) How many critical points does the function $f(x,y)=8xy-2x^2-y^4$ have?

A. 1 B. 2 C. 3 D. 4

E. none of the above

(h) A moving particle has position vector given by
$\underset{\sim}{r}(t)=5\sin t\ \underset{\sim}{i}-12t\ \underset{\sim}{j}+5\cos t\ \underset{\sim}{k}.$
Find the speed of the particle at any t.

A. 12 B. 22 C. 13 D. $\sqrt{194}$

E. none of the above

(i) To find the volume bounded below by $z=1+\sqrt{x^2+y^2}$ and above by the sphere $x^2+y^2+z^2=16$ we work in cylindrical coordinates and compute the integral as
$$\int_0^{2\pi}\int_0^{B}\int_{1+r}^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta$$
where the value of B is

A. 2 B. 3 C. 4 D. 5

E. none of the above

(j) Evaluate the integral
$$\int_0^1\int_x^1 e^{\frac{x}{y}}\,dy\,dx$$

A. e B. $1+e$ C. $e-1$ D $\frac{e-1}{2}$

E none of the above

...5/-

(f) *Titik* $(1,3)$ *ialah suatu titik genting fungsi* $f(x,y)=6xy-3x^3-y^2x$.
Jeniskan titik genting $(1,3)$

A. *minimum setempat* B. *maksimum setempat*

C. *minimum sejagat* D. *titik pelana*

E. *bukan satu pun di atas*

(g) *Berapakah bilangan titik genting fungsi* $f(x,y)=8xy-2x^2-y^4$ *?*

A. *1* B. *2* C. *3* D. *4*

E. *bukan satu pun di atas*

(h) *Vektor kedudukan suatu jirim bergerak diberi sebagai*
$\underset{\sim}{r}(t)=5\sin t\ \underset{\sim}{i}-12t\ \underset{\sim}{j}+5\cos t\ \underset{\sim}{k}.$
Cari kelajuan jirim tersebut pada sebarang t.

A. *12* B. *22* C. *13* D. $\sqrt{194}$

E. *bukan satu pun di atas*

(i) *Untuk mencari isipadu dibatasi sebelah bawah oleh* $z=1+\sqrt{x^2+y^2}$ *dan sebelah atas oleh sfera* $x^2+y^2+z^2=16$ *kita menggunakan koordinat silinder dan menghitung penkamirannya sebagai*

$$\int_0^{2\pi}\int_0^{B}\int_{1+r}^{\sqrt{16-r^2}} rdz\,drd\theta$$

dengan nilai B ialah

A. *2* B. *3* C. *4* D. *5*

E. *bukan satu pun di atas*

(j) *Nilaikan penkamiran*

$$\int_0^1\int_x^1 e^{\frac{x}{y}}dy\,dx$$

A. e B. $1+e$ C. $e-1$ D $\dfrac{e-1}{2}$

E *bukan satu pun di atas*

...6/-

(k) The equation of the line through $(1,0,2)$ and perpendicular to the plane $4x+6y-6z=3$ is $\underset{\sim}{r}(t)=<A,3t,1-3t>$ where A is

A. $1+4t$ B. $1-2t$ C. $\frac{1}{2}+2t$

D. $-\frac{1}{2}+2t$ E. none of the above

(l) Let $z=1+\ln\left(x+2y^2-2z^2\right)$ compute $\frac{\partial z}{\partial x}$ at $x=1, y=1$ and $z=1$

A. -1 B. $\frac{1}{5}$ C. $-\frac{1}{5}$ D. $\frac{4}{5}$

E. none of the above

(m) Use the method of Lagrange multiplier to find the maximum value of x^3+y^3 with the constraint $x^2+y^2=4$

A. 4 B. $\sqrt{\frac{2}{3}}$ C. 16 D. 32

E. none of the above

(n) Compute the integral $\iint_D \left(\frac{x^2}{9}+\frac{y^2}{4}\right)^{\frac{5}{2}} dx\,dy$ where D is a semi ellipse $\frac{x^2}{9}+\frac{y^2}{4}\le 1; y\ge 0$

A. $\frac{\pi}{6}$ B. π C. $\frac{\pi}{7}$ D. $\frac{12}{7}\pi$

E. none of the above

...7/-

(k) *Persamaan garis melalui* $(1,0,2)$ *dan berserenjang kepada satah* $4x+6y-6z=3$ *ialah* $\underset{\sim}{r}(t)=<A,3t,1-3t>$ *dengan A ialah*

A. $1+4t$ *B.* $1-2t$ *C.* $\frac{1}{2}+2t$

D. $-\frac{1}{2}+2t$ *E.* *bukan satu pun di atas*

(l) *Biar* $z=1+\ln\left(x+2y^2-2z^2\right)$ *hitung* $\frac{\partial z}{\partial x}$ *pada* $x=1, y=1$ *dan* $z=1$

A. -1 *B.* $\frac{1}{5}$ *C.* $-\frac{1}{5}$ *D.* $\frac{4}{5}$

E. *bukan satu pun di atas*

(m) *Guna kaedah pendaraban Lagrange untuk mencari nilai maksimum* x^3+y^3 *dengan kekangan* $x^2+y^2=4$

A. *4* *B.* $\sqrt{\frac{2}{3}}$ *C.* *16* *D.* *32*

E. *bukan satu pun di atas*

(n) *Hitung penkamiran* $\iint_D \left(\frac{x^2}{9}+\frac{y^2}{4}\right)^{\frac{5}{2}} dx\,dy$ *dengan D suatu separuh elips* $\frac{x^2}{9}+\frac{y^2}{4}\le 1; y\ge 0$

A. $\frac{\pi}{6}$ *B.* π *C.* $\frac{\pi}{7}$ *D.* $\frac{12}{7}\pi$

E. *bukan satu pun di atas*

...8/-

(o) Find the length of the curve

$$\underset{\sim}{r}(t) = <2\sqrt{2}t, \cos, \sin t>, \quad 0 \le t \le 2\pi$$

A. 2π B. 4π C. $4\sqrt{2}\pi$ D. 6π

E. none of the above

(p) Compute the integral

$$\iiint_B \left(x^2 + y^2 + z^2\right) dx\, dy\, dz$$

where B is a ball $x^2 + y^2 + z^2 \le 4$

A. $\frac{32}{7}\pi\sqrt{2}$ B. $\frac{64}{7}\pi\sqrt{2}$ C. $\frac{128}{3}\pi$ D. $\frac{2}{3}\pi$

E. none of the above

(q) Evaluate the integral

$$\iiint_B z \sin(x+y)\, dx\, dy\, dz$$

where $D = \left\{(x,y,z) \mid 0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2} \le z \le 2\right\}$

A. 3 B. 4 C. 5 D. 6

E. none of the above

(r) Find the surface area of the paraboloid $z = x^2 + y^2$ which lies above $x^2 + y^2 \le 1$

A. $\frac{\pi}{2}\left(5\sqrt{5} - 1\right)$ B. $\frac{\pi}{4}\left(5\sqrt{5} - 1\right)$ C. $\frac{\pi}{6}\left(\sqrt{5} - 1\right)$

D. $\frac{\pi}{6}\left(5\sqrt{5} - 1\right)$ E. none of the above

...9/-

(o) *Cari pangjang lengkung*

$$\underset{\sim}{r}(t) = <2\sqrt{2}t, \cos, \sin t>, \quad 0 \le t \le 2\pi$$

A. 2π B. 4π C. $4\sqrt{2}\pi$ D. 6π

E. *bukan satu pun di atas*

(p) *Hitungkan penkamiran*

$$\iiint_B \left(x^2 + y^2 + z^2\right) dx\, dy\, dz$$

dengan B suatu bebola $x^2 + y^2 + z^2 \le 4$

A. $\frac{32}{7}\pi\sqrt{2}$ B. $\frac{64}{7}\pi\sqrt{2}$ C. $\frac{128}{3}\pi$ D. $\frac{2}{3}\pi$

E. *bukan satu pun di atas*

(q) *Nilaikan penkamiran*

$$\iiint_B z \sin(x+y)\, dx\, dy\, dz$$

dengan $D = \left\{(x,y,z) \mid 0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2} \le z \le 2\right\}$

A. 3 B. 4 C. 5 D. 6

E. *bukan satu pun di atas*

(r) *Cari luas permukaan paraboloid* $z = x^2 + y^2$ *yang terletak ke atas* $x^2 + y^2 \le 1$

A. $\frac{\pi}{2}\left(5\sqrt{5} - 1\right)$ B. $\frac{\pi}{4}\left(5\sqrt{5} - 1\right)$ C. $\frac{\pi}{6}\left(\sqrt{5} - 1\right)$

D. $\frac{\pi}{6}\left(5\sqrt{5} - 1\right)$ E. *bukan satu pun di atas*

...10/-

(s) Let f be a smooth function in two variables. If $\nabla f(a,b) = <0,0>$ and $f_{xx}(a,b) = 2$, $f_{xy}(a,b) = -1$ and $f_{yy}(a,b) = 2$, then (a,b) is

A. a local maximum point B. a local minimum

C. a saddle point D. a global minimum point

E. none of the above

(t) Let $c > 0$ be a constant and f be a smooth function of one variable. Let $s = x + ct$ and let $w = f(s)$, then $\frac{\partial^2 w}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} =$

A. 200 B. $2f''(s)$ C. $2(f'(s))^2$

D. 0 E. none of the above

[40 marks]

2. (a) Let $f(x,y) = x^2 + y^2 + 4$ and $D = \{(x,y) \mid 9x^2 + 4y^2 \leq 36\}$

(i) Find the point on the surface nearest to the point $(0,1,1)$.

(ii) Find the absolute maximum and absolute minimum of f in D.

(iii) Find the volume below the surface $f(x,y)$ and above the disc $x^2 + y^2 \leq 36$.

(b) Suppose $z = f(x,y)$ is differentiable at $(1,2)$, $f(1,2) = -2$, $\frac{\partial z}{\partial x}(1,2) = -4$ and $\frac{\partial z}{\partial y}(1,2) = -3$

(i) Find the equation of the tangent plane to the graph f at the point $(1,2,-2)$. Write also your solution in the form of $ax + by + cz + d = 0$.

(ii) Approximate the value of $f(1{\cdot}1, 1{\cdot}9)$.

(c) Use the method of Lagrange multiplier to find the maximum and minimum of

$$f(x,y) = 6x - 5y - 3$$

subject to the constraint

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

[30 marks]

...11/-

(s) *Biar f suatu fungsi licin dalam due pembolehubah. Jika If* $\nabla f(a,b) = <0,0>$ *dan* $f_{xx}(a,b) = 2, f_{xy}(a,b) = -1$ *dan* $f_{yy}(a,b) = 2$, *maka* (a,b) *ialah*

A. *suatu titik maksimum setempat* B. *suatu titik minimum setempat*

C. *suatu titik pelana* D. *suatu titik minimum sejagat*

E. *bukan satu pun di atas*

(t) *Biar* $c > 0$ *suatu niali malar dan f sebagai fungsi licin satu pembolehubah. Biar* $s = x + ct$ *dan* $w = f(s)$, *maka* $\frac{\partial^2 w}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} =$

A. *200* B. $2f''(s)$ C. $2(f'(s))^2$

D. *0* E. *bukan satu pun di atas*

[40 markah]

2. (a) *Biar* $f(x,y) = x^2 + y^2 + 4$ *dan* $D = \{(x,y) \mid 9x^2 + 4y^2 \le 36\}$

(i) *Cari titik pada permukaan yang terdekat dengan titik* $(0,1,1)$.

(ii) *Cari maksimum dan minimum mutlak f dalam D.*

(iii) *Cari isipadu ke bawah permukaan* $f(x,y)$ *dan ke atas cakera* $x^2 + y^2 \le 36$.

(b) *Andaikan* $z = f(x,y)$ *adalah terbezakan pada* $(1,2)$, $f(1,2) = -2$, $\frac{\partial z}{\partial x}(1,2) = -4$ *dan* $\frac{\partial z}{\partial y}(1,2) = -3$

(i) *Cari persamaan satah tangen kepada geraf f pada titik* $(1,2,-2)$. *Tulisikan juga penyelesaian anda dalam bentuk* $ax + by + cz + d = 0$.

(ii) *Anggar nilai* $f(1{\cdot}1, 1{\cdot}9)$.

(c) *Guna kaedah pendaraban Lagrange untuk mencari minimum dan maksimum*

$$f(x,y) = 6x - 5y - 3$$

dengan kekangan

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

[30 markah]

...12/-

3. (a) Let $F = < y^2, 2xy - z\sin(yz), -y\sin(yz) + 6z^2 >$

(i) Find the potential function of F.

(ii) Find the work done by F in moving a particle along a path C parameterized by

$$\underset{\sim}{r}(t) = < t^4, te^{t^2} - e^{t^2}\sin\left(\frac{\pi}{2}t\right), 1 - \cos\left(\frac{\pi}{2}t\right) >$$

(b) Let $F(x, y, z) = < z, y, x >$. Consider the parameterized surface S given by $f(u,r) = (u\cos v, u\sin v)$ where $0 \le u \le 1 \cdot 0, \quad 0 \le v \le \frac{\pi}{2}$.

(i) Find curl F.

(ii) Rewrite (but do not evaluate), the surface integral

$$\iint_S \text{curl } \underset{\sim}{F} \cdot \underset{\sim}{n}\, dS$$

as a double integral over the region

$$D = \left\{ (u,v) \middle| 0 \le u1 \cdot 0, 0 \le v\frac{\pi}{2} \right\}$$

(iii) Use Stokes Theorem to evaluate the path integral

$$\oint_{\partial s} \underset{\sim}{F} \cdot d\underset{\sim}{s}$$

(c) Evaluate $\iint_S \underset{\sim}{F} \bullet d\underset{\sim}{S}$ where $\underset{\sim}{F} = -y\underset{\sim}{j} + z\underset{\sim}{k}$ and S is the surface given by the paraboloid $y = x^2 + z^2, 0 \le y \le 1$ and the disc $x^2 + z^2 \le 1$ at $y = 1$.

[30 marks]

...13/-

3. *(a) Biarkan* $F = < y^2, 2xy - z\sin(yz), -y\sin(yz) + 6z^2 >$

   (i) *Cari fungsi potensi untuk F.*

   (ii) *Cari kerja yang dilaksana F untuk menggerakkan suatu jisim sepanjang laluan C di parameter oleh*

$$\underset{\sim}{r}(t) = < t^4, te^{t^2} - e^{t^2}\sin\left(\frac{\pi}{2}t\right), 1 - \cos\left(\frac{\pi}{2}t\right) >$$

(b) Biar $F(x, y, z) = < z, y, x >$ *Pertimbangkan permukaan S di beri sebagai*

$$f(u,r) = (u\cos v, u\sin v) \text{ dengan } 0 \le u \le 1\cdot 0, \quad 0 \le v \le \frac{\pi}{2}.$$

(i) *Cari keikalan F.*

(ii) *Tulis semula (tetapi jangn menilai) kamiran permukaan*

$$\iint_s \text{curl}\, \underset{\sim}{F} \cdot \underset{\sim}{n}\, dS$$

Sebagai kamiran berganda pada rantau

$$D = \left\{ (u,v) \middle| 0 \le u1\cdot 0, 0 \le v\frac{\pi}{2} \right\}$$

(iii) *Gunakan Teorem Stoke untuk menilai penkamiran garis*

$$\oint_{\partial s} \underset{\sim}{F} \cdot d\underset{\sim}{r}$$

(c) Nilaikan $\iint_s \underset{\sim}{F} \bullet d\underset{\sim}{s}$ *dengan* $\underset{\sim}{F} = -y\underset{\sim}{j} + z\underset{\sim}{k}$ *dan S adalah permulaan diberi sebagai paraboloid* $y = x^2 + z^2, 0 \le y \le 1$ *dan cakera* $x^2 + z^2 \le 1$ *at* $y = 1$.

[30 markah]

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