UNIVERSITI SAINS MALAYSIA

First Semester Examination
2009/2010 Academic Session

November 2009

EKC 361 – Process Dynamics and Control
[Kawalan Dinamik dan Proses]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FOURTEEN pages of printed material and TWO page of Appendix before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT BELAS muka surat yang bercetak dan DUA muka surat Lampiran sebelum anda memulakan peperiksaan ini.]

Instruction: Answer **FOUR (4)** questions. Answer any **TWO (2)** questions from Section A. Answer any **TWO (2)** questions from Section B. All questions carry the same marks.

*[**Arahan:** Jawab **EMPAT** (4) soalan. Bahagian A pilih **DUA** (2) soalan sahaja. Bahagian B pilih **DUA** (2) soalan sahaja. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

<u>Section A</u> : Answer any <u>TWO</u> questions.
<u>Bahagian A:</u> Jawab mana-mana <u>DUA</u> soalan.

1. [a] The contents of the stirred-tank heating system shown in Figure Q.1.[a] are heated at a constant rate of Q (kJ/h) using a gas-fired heater. The flow rate, *w* (kg/h) and volume V (m^3) are constant, but the heat loss to the surroundings Q_L (kJ/h) varies with the wind velocity, *v* (m/s) according to the expressions:

Kandungan sistem tangki teraduk yang ditunjukkan dalam Rajah S.1.[a] dipanaskan pada kadar malar Q (kJ/j) menggunakan satu pemanas gas api. Kadar aliran w (kg/j) dan isipadu V (m^3) adalah malar, tetapi haba yang hilang ke persekitaran Q_L (kJ/j) berubah mengikut halaju angin v (m/s) berdasarkan ungkapan:

$$Q_L = UA(T - T_a)$$

$$U(t) = \overline{U} + bv(t)$$

where $\overline{U}$, A, b, and T_a are constants. Derive the transfer function between exit temperature T and wind velocity *v*. List any additional assumptions that you make.

di mana $\overline{U}$, A, b, dan T_a adalah pemalar. Terbitkan rangkap pindah antara suhu keluar T dan halaju angin v. Senaraikan sebarang andaian tambahan yang anda buat.

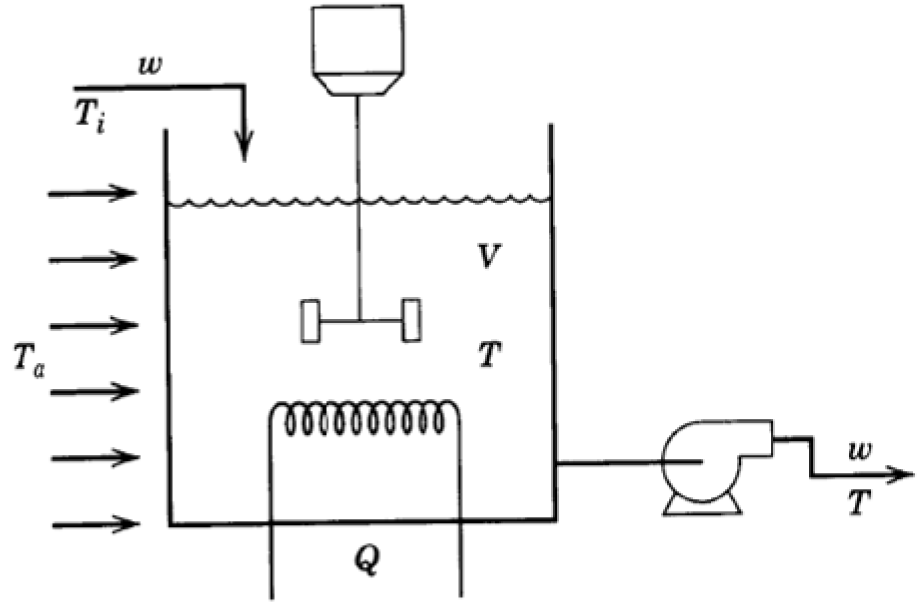

Figure Q.1.[a]: Stirred-tank heating system
Rajah S.1.[a]: Sistem pemanas tangki teraduk

[8 marks/markah]

[b] A feedback control loop is represented by the block diagram of Figure Q.1.[b].

Satu gelung kawalan suap balik diwakili oleh gambarajah blok seperti ditunjukkan dalam Rajah S.1.[b].

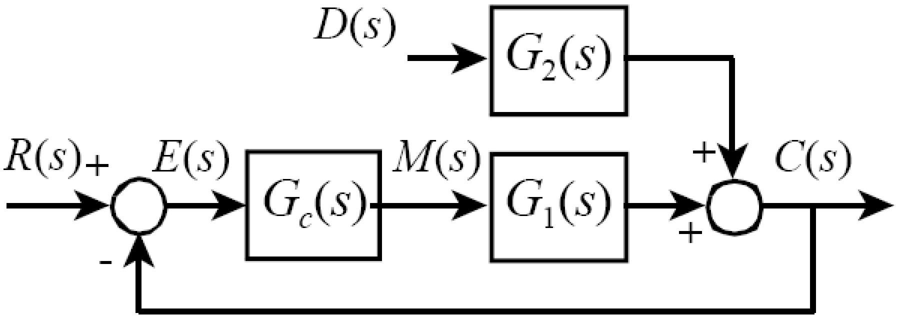

Figure Q.1.[b]: A feedback control loop

Rajah S.1.[b]: Satu gelung kawalan suap balik

The process can be represented by a first-order lag and the controller is proportional-integral (PI):

Proses tersebut boleh diwakili oleh tertib pertama dengan masa lengah dan pengawal merupakan kamiran berkadaran (PI):

$$G_1 = \frac{K}{\tau s + 1}$$

$$G_C = K_C\left(1 + \frac{1}{\tau_I s}\right)$$

Without loss of generality you can set the process time constant τ equal to l, and the process gain K equal to 1.

Anda boleh mengambil pemalar masa proses dan gandaan proses bersamaan 1.

[i] Write the closed-loop transfer function and the characteristic equation of the loop. Is there any offset?

Tuliskan rangkap pindah gelung tertutup dan persamaan ciri bagi gelung tersebut. Adakah terdapat sebarang ofset?

[ii] Determine the response of the closed loop to a step change in set point for $\tau_I = \tau$ as the controller gain varies from zero to infinity.

Tentukan sambutan gelung tertutup tersebut terhadap satu perubahan langkah titik set bagi $\tau_I = \tau$ *apabila gandaan pengawal berubah daripada kosong kepada infiniti.*

[10 marks/markah]

[c] A gas detector is used to determine the concentration of a flammable gas in a gas stream. Normally the gas concentration is 1 % by volume, well below the alarm limit of 4 % and the lower flammability limit of 5 %. If the gas concentration is above the lower flammability limit, it is flammable. A particular gas detector demonstrates first-order behavior with a time constant of 5 s. At a particular time, the gas stream is flowing at 1 m^3/s through a duct with a cross-sectional area of 1 m^2. If the gas concentration suddenly increases from 1 to 7 % by volume, how many cubic meters of flammable gas pass the sensor before the alarm is sounded.

Satu pengesan gas digunakan untuk menentukan kandungan gas mudah bakar dalam satu alur gas. Biasanya kandungan gas tersebut ialah 1 % berdasarkan isipadu, jauh di bawah had penggera iaitu 4 % dan had bawah kemudahbakaran iaitu 5 %. Jika kandungan gas berada di atas had bawah kemudahbakaran tersebut, ia adalah mudah bakar. Sejenis pengesan gas menunjukkan kelakuan tertib pertama dengan pemalar masa 5 s. Pada satu-satu masa, alur gas tersebut mengalir pada 1 m^3/s *melalui satu salur dengan luas keratan rentas 1* m^2*. Jika kandungan gas tiba-tiba bertambah daripada 1 kepada 7 % berdasarkan isipadu, berapa meter padu gas mudah bakar perlu melalui penderia sebelum penggera berbunyi.*

[7 marks/markah]

2. [a] A surge tank system is to be installed as part of a pilot plant facility. The initial proposal calls for the configuration shown in Figure Q.2.[a]. Each tank is 1.5 m high and 0.9 m in diameter. The design flow rate is $q_i = 0.38$ m^3/min. It has been suggested that an improved design will result if the two-tank system is replaced by a single tank that is 1.2 m in diameter and has the same total volume (i.e.,$V = V_1 + V_2$).

Satu tangki pusuan akan dipasang sebagai sebahagian kemudahan satu loji pandu. Tatarajah awal yang dicadangkan ialah seperti yang ditunjukkan dalam Rajah S.2.[a]. Setiap tangki mempunyai tinggi 1.5 m dan diameter 0.9 m. Kadar aliran yang direkabentuk ialah $q_i = 0.38$ m^3/min*. Sistem dua tangki dengan diameter 1.2 m dan jumlah isipadu yang sama (iaitu* $V = V_1 + V_2$*) telah dicadangkan untuk menggantikan satu tangki bagi memperbaiki rekabentuk.*

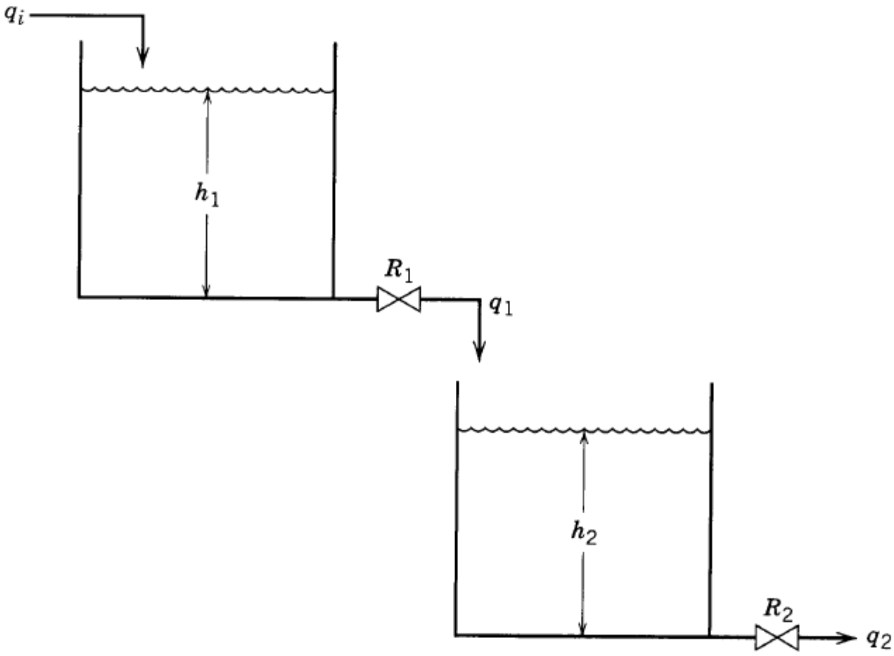

Figure Q.2.[a]
Rajah S.2.[a]

[i] Which surge system (original or modified) can handle larger step disturbances in q_i? Justify your answer.

Sistem pusuan manakah (yang asal atau yang diubahsuai) boleh mengendali gangguan langkah dalam q_i yang lebih besar? Jelaskan jawapan anda.

[ii] Which system provides the best damping of step disturbances in q_i? Justify your answer.

Sistem manakah yang memberikan redaman terbaik bagi gangguan langkah dalam q_i? Jelaskan jawapan anda.

In your analysis you may assume that:

- The valves on the exit lines act as linear resistances.
- The valves are adjusted so that each tank is half full at the nominal design condition of $q_i = 0.38$ m^3/min.

Dalam analisis, anda boleh menganggap:

- *Injap pada aliran keluar bertindak sebagai rintangan lelurus.*
- *Injap diubah supaya setiap tangki berada separuh penuh daripada keadaan rekabentuk namaan $q_i = 0.38$ m^3/min.*

[15 marks/markah]

[b] A step change from 15 to 31 psi in actual pressure results in the measured response from a pressure-indicating element shown in Figure Q.2.[b].

Satu perubahan langkah terhadap tekanan sebenar dari 5 ke 31 psi menghasilkan sambutan daripada satu elemen penunjuk tekanan seperti dalam Rajah S.2.[b].

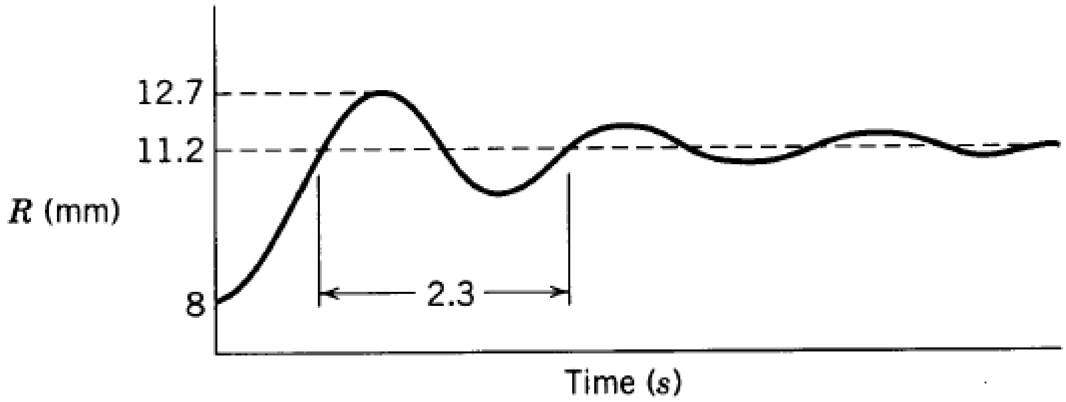

Figure Q.2.[b]
Rajah S.2.[b]

[i] Assuming second-order dynamics, calculate all important parameters and write an approximate transfer function in the form

Anggap dinamik tertib kedua, kirakan semua parameter penting dan tuliskan rangkap pindah dalam bentuk

$$\frac{R'(s)}{P'(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

where R′ is the instrument output deviation (mm), P′ is the actual pressure deviation (psi).

di mana R′ ialah sisihan keluaran bagi alatan (mm), P′ ialah sisihan tekanan sebenar (psi).

[ii] Write an equivalent differential equation model in terms of actual (not deviation) variables.

Tuliskan model persamaan kebezaan berdasarkan pembolehubah sebenar (bukan sisihan).

Given:
Diberi:

Time to first peak: $t_p = \frac{\pi\tau}{\sqrt{1-\zeta^2}}$

Masa puncak pertama:

Overshoot: $\mathrm{OS} = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$

Terlajak:

Decay ratio: $\mathrm{DR} = \exp\left(-\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right) = (\mathrm{OS})^2$

Nisbah susut:

Period: $\mathrm{P} = \frac{2\pi\tau}{\sqrt{1-\zeta^2}}$

Tempoh: *[10 marks/markah]*

3. [a] A mixing process consists of a single stirred-tank instrumented as shown in Figure Q.3.[a]. The concentration of a single species A in the feed stream varies. The controller attempts to compensate for this by varying the flow rate of pure A through the control valve.

Satu proses pencampuran terdiri daripada satu tangki teraduk dengan alatan seperti yang ditunjukkan dalam Rajah S.3.[a]. Kepekatan spesies A dalam aliran masuk adalah berubah. Pengawal cuba untuk memampas perubahan ini dengan mengubah kadar aliran A tulen dengan menggunakan injap kawalan.

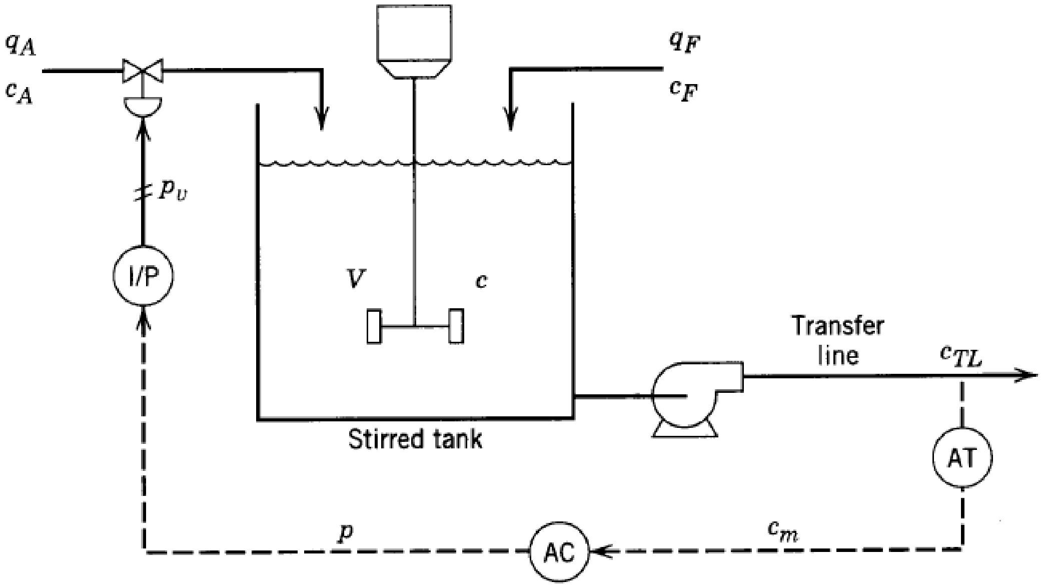

Figure Q.3.[a]: A single stirred-tank for a mixing process
Rajah S.3.[a]: Satu tangki teraduk bagi proses pencampuran

[i] Draw a block diagram for the controlled process.
Lukiskan gambarajah blok bagi proses terkawal tersebut.

[ii] Derive a transfer function for each block in your block diagram.
Terbitkan rangkap pindah bagi setiap blok dalam gambarajah blok anda.

Given:
Diberi:

For Process:

- The volume is constant (5 m^3).
- The feed flow rate is constant ($\overline{q}_F$ = 7 m^3/min).
- The flow rate of the A stream varies but is small compared to $\overline{q}_F$ ($\overline{q}_A$=0.5 m^3/min).
- $\overline{c}_F$ = 50 kg/m^3 and $\overline{c}_A$=800 kg/m^3.
- All densities are constant and equal.

Bagi proses:

- *Isipadu adalah malar (5 m^3).*
- *Kadar aliran masuk adalah malar ($\overline{q}_F$ = 7 m^3/min).*
- *Perubahan kadar aliran bagi aliran A adalah kecil dibandingkan dengan $\overline{q}_F$ ($\overline{q}_A$=0.5 m^3/min).*
- *$\overline{c}_F$ = 50 kg/m^3 dan $\overline{c}_A$=800 kg/m^3.*
- *Semua ketumpatan adalah malar dan sama.*

For Transfer Line

- The transfer line is 20 m long and has 0.5 m inside diameter.
- Pump volume can be neglected.

Bagi garis pemindah

- *Panjang garis pemindah adalah 20 m dan diameter dalamnya ialah 0.5m.*
- *Isipadu pam boleh diabaikan.*

For Composition Transmitter Data
Data bagi kandungan terhantar

c (kg/m^3)	c_m (mA)
0	4
200	20

For PID Controller

- Derivative on measurement only
- Direct or reverse acting, as required
- Current (mA) input and output signals

Bagi pengawal PID

- *Hasil kebezaan hanya pada pengukuran sahaja*
- *Tindakan terus atau songsang, yang mana diperlukan*
- *Isyarat arus masukan dan keluaran (mA)*

For I/P Transducer Data
Data bagi I/P Tranduser

p (mA)	p_v (psig)
4	3
20	15

For Control Valve
Bagi Injap Kawalan

An equal percentage valve is used, which has the following relation:
Injap peratusan sama digunakan yang mempunyai hubungan berikut:

$$q_A = 0.17 + 0.03(20)^{\frac{pv-3}{12}}$$

For a step change in input pressure, the valve requires approximately 1 min to move to its new position.

Bagi suatu perubahan langkah pada tekanan masukan, injap memerlukan lebih kurang 1 minit untuk bergerak kepada kedudukan barunya.

[18 marks/markah]

[b] Assume that the following equation describes a certain process
Anggap persamaan berikut mewakili satu proses tertentu

$$\frac{Y(s)}{X(s)} = \frac{3e^{-0.5s}}{5s + 0.2}$$

[i] Obtain the steady-state gain, time constant and dead time of this process.

Carikan gandaan keadaan mantap, pemalar masa dan masa lengah bagi proses tersebut.

[ii] The initial condition of the variable y is y(0) = 2. For a forcing function as shown in Figure Q.3.[b], what is the final value of y(t)?

Keadaan awal bagi pembolehubah y iaitu y(0) = 2. Bagi satu fungsi paksa seperti yang ditunjukkan dalam Rajah S.3.[b], apakah nilai akhir bagi y(t)?

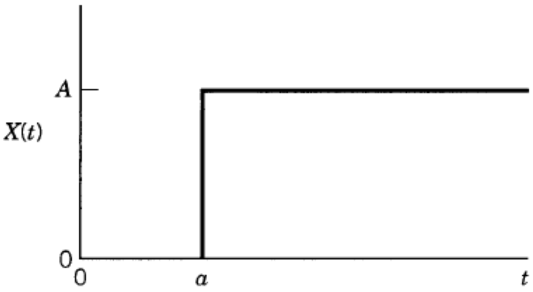

Figure Q.3.[b]
Rajah S.3.[b]

[7 marks/markah]

<u>Section B</u> : Answer any <u>TWO</u> questions.
<u>Bahagian B</u>: Jawab mana-mana <u>DUA</u> soalan.

4. Table Q.4. shows the experimental process reaction curve of an open-loop system with a PI controller. Using these values:

   Jadual S.4. menunjukkan lengkungan tindakbalas proses dari eksperimen bagi sistem gelung terbuka kawalan PI. Dengan menggunakan nilai-nilai tersebut:

   [a] Approximate the open-loop response with that of a first-order system plus dead time where you need to find the values of K, τ and t_d.

   Anggarkan sambutan gelung terbuka dengan sistem tertib pertama dan masa lengah di mana anda perlu mencari nilai-nilai K, τ *dan* t_d.

   [9 marks/markah]

   [b] Select the controller setting for both disturbance and set-point tracking using the Smith and Corripio ITAE performance index.
   Pilih pengesetan pengawal untuk gangguan dan pengesanan titik-set menggunakan indeks pencapaian ITAE Smith dan Corripio.

   [8 marks/markah]

   [c] Use the numerical analysis to determine the controller setting using the Ziegler-Nichols open-loop tuning rule.

   Gunakan analisa berangka untuk menentukan pengesetan pengawalan menggunakan peraturan penalaan gelung terbuka Ziegler-Nichols.

   [8 marks/markah]

Table Q.4.
Jadual S.4.

Time (Min) *Masa (Min)*	Manipulated Input *Masukan Manipulatif*	Measurement of output *Ukuran Keluaran*
-2	100	200
-1	100	200
0	150	200.1
0.2	150	201.1
0.4	150	204.0
0.6	150	227.0
0.8	150	251.0
1.0	150	280.0
1.2	150	302.5
1.4	150	318.0
1.6	150	329.5
1.8	150	336.0
2.0	150	339.0
2.2	150	340.5
2.4	150	341.0

5. A three-tank mixing process is shown in Figure Q.5. The goal is to have the outlet concentration as close to its set point.

Proses pengadukan tiga tangki ditunjukkan dalam Rajah S.5. Pengadukan itu bertujuan untuk mencapai kepekatan keluaran menghampiri titik set.

Assumptions:

- All tanks are well mixed
- Dynamics of the valve and sensor are negligible
- No transportation delays (dead times) exist.
- A linear relationship exists between the valve opening and the flow of component **A**.
- Densities of components are equal.

Andaian:

- *Semua tangki teraduk dengan sempurna.*
- *Dinamik injap dan penderia boleh diabaikan.*
- *Tiada masa pengangkutan lengah (masa lengah) yang wujud.*
- *Wujud penghubungan linear antara bukaan injap dan aliran komponen A.*
- *Ketumpatan komponen-komponen adalah sama.*

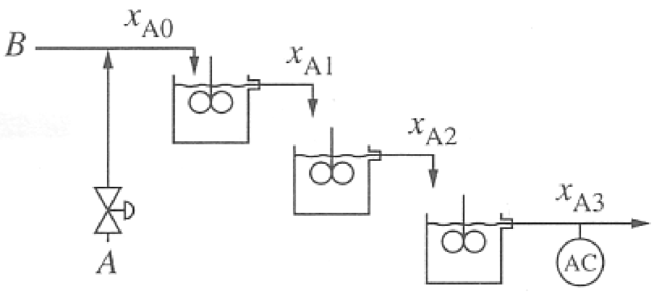

Figure Q.5.
Rajah S.5.

Data:

V = volume of each tank = 35 m^3
F_B = flow rate of stream B = 6.9 m^3 /min
x_{Ai} = concentration of A in all tanks and outlet flow = 3% A
F_A = flow rate of stream A = 0.14 m^3/min
$(x_A)_B$ = concentration of stream B = 1% A
$(x_A)_A$ = concentration of stream A = 100% A
v = valve position = 50% open

$$K_v = 0.0028 \frac{m^3/\min}{\%\ open}$$

Maklumat:
V = isipadu setiap tangki = 35 m^3
F_B = kadar aliran B = 6.9 m^3 /min
x_{Ai} = kepekatan A dalam semua tangki dan aliran keluar = 3% A
F_A = kadar aliran A= 0.14 m^3/min
$(x_A)_B$ = kepekatan aliran B = 1% A
$(x_A)_A$ = kepekatan aliran A = 100% A
v = kedudukan injap = 50% buka

$$K_v = 0.0028 \frac{m^3/\min}{\%\ bukaan}$$

Thus, the product flow rate is essentially the flow of stream B; that is, $F_B >> F_A$
Jadi, kadar aliran produk adalah aliran arus B, iaitu $F_B >> F_A$

[a] Determine the process gain and time constant of this process.
Tentukan gandaan proses dan pemalar masa proses tersebut.
[6 marks/markah]

[b] The three-tank mixing process has been reduced to a single-tank with proportional control where the composition sensor at the exit. The process transfer function, which includes an ideal sensor and fast final element dynamics is given as

Proses pengadukan tiga tangki telah dikurangkan ke satu tangki dengan kawalan perkadaran di mana penderia komposisi pada keluarnya. Rangkap pindah proses di mana dinamik penderia unggul dan elemen akhir pantas diberi:

$$G_c(s) = K_c \qquad ; \quad G_p(s)G_v(s)G_s(s) = \frac{0.039}{5s+1}$$

Show and determine the stability by Bode method.
Tunjuk dan tentukan kestabilan dengan kaedah Bode.

[9 marks/markah]

[c] The problem of single tank system is considered again here with the valve and sensor exhibit certain dynamics as below:

Permasalahan sistem satu tangki dibincangkan semula dengan injap dan penderia memiliki dinamik tertentu seperti di bawah:

$$G_c(s) = K_c; \quad G_p(s) = \frac{0.0039}{5s+1}; \quad G_v(s) = \frac{1}{0.033s+1}; \quad G_s(s) = \frac{1}{0.25s+1}$$

Show the Bode plot for the system. Also, analyse if K_c of 1 , 500 and 6000 are used.
Tunjukkan plot Bode bagi sistem ini. Buat analisa juga jika K_c adalah 1,500 dan 6000.

[10 marks/markah]

6. A stirred-tank chemical reactor is shown in Figure Q.6. with the following reaction.
Satu reaktor kimia teraduk ditunjukkan dalam Rajah S.6. dengan tindakbalas berikut.

$$A \rightarrow B \qquad \Delta H_{\text{rxn}} = 0$$

$$r_A = -k_o e^{-E/RT} \frac{C_A}{1+kC_x}$$

The available sensors and control valves are shown in the Figure Q.6., and no changes to these are allowed. The goal is to control the reactor concentration of A by single-loop or cascade control, whichever is better. For each of the following disturbances, design the best control system and explain your design:

Penderia-penderia dan injap-injap kawalan yang sedia ada ditunjukkan dalam Rajah S.6., dan sebarang perubahan aturan tidak dibenarkan. Matlamat kawalan adalah mengawal kepekatan A dalam reaktor dengan gelung tunggal atau kawalan lata, mana-mana yang lebih baik. Untuk setiap gangguan-gangguan berikut, rekabentukkan sistem kawalan terbaik dan terangkan rekabentuk anda:

[a] the heating medium pressure (P1)
tekanan bagi medium pemanasan (P1)

[10 marks/markah]

[b] the solvent feed temperature (T_s)
suhu suapan bagi pelarut (T_s)

[8 marks/markah]

[c] the reactant feed pressure (P_2)
tekanan bagi suapan bahan tindakbalas (P_2)

[7 marks/markah]

(Prepare a block diagram for each case showing the appropriate single-loop or cascade control system).
(Sediakan gambarajah blok bagi setiap kes dengan gelung tunggal dan sistem kawalan lata yang sesuai).

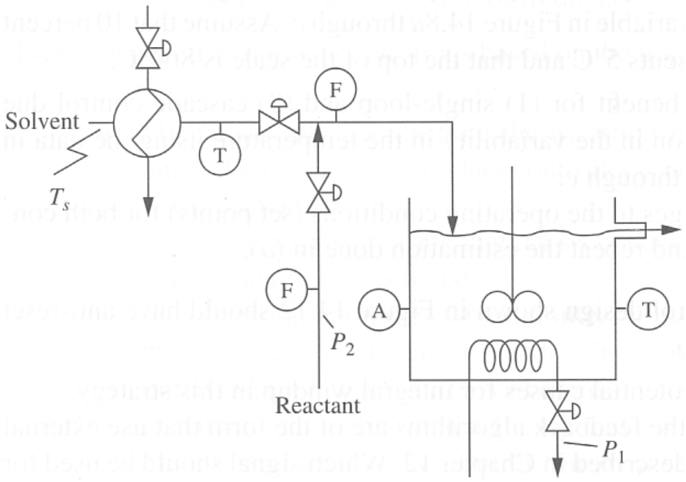

Figure Q.6.
Rajah S.6.

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Appendix

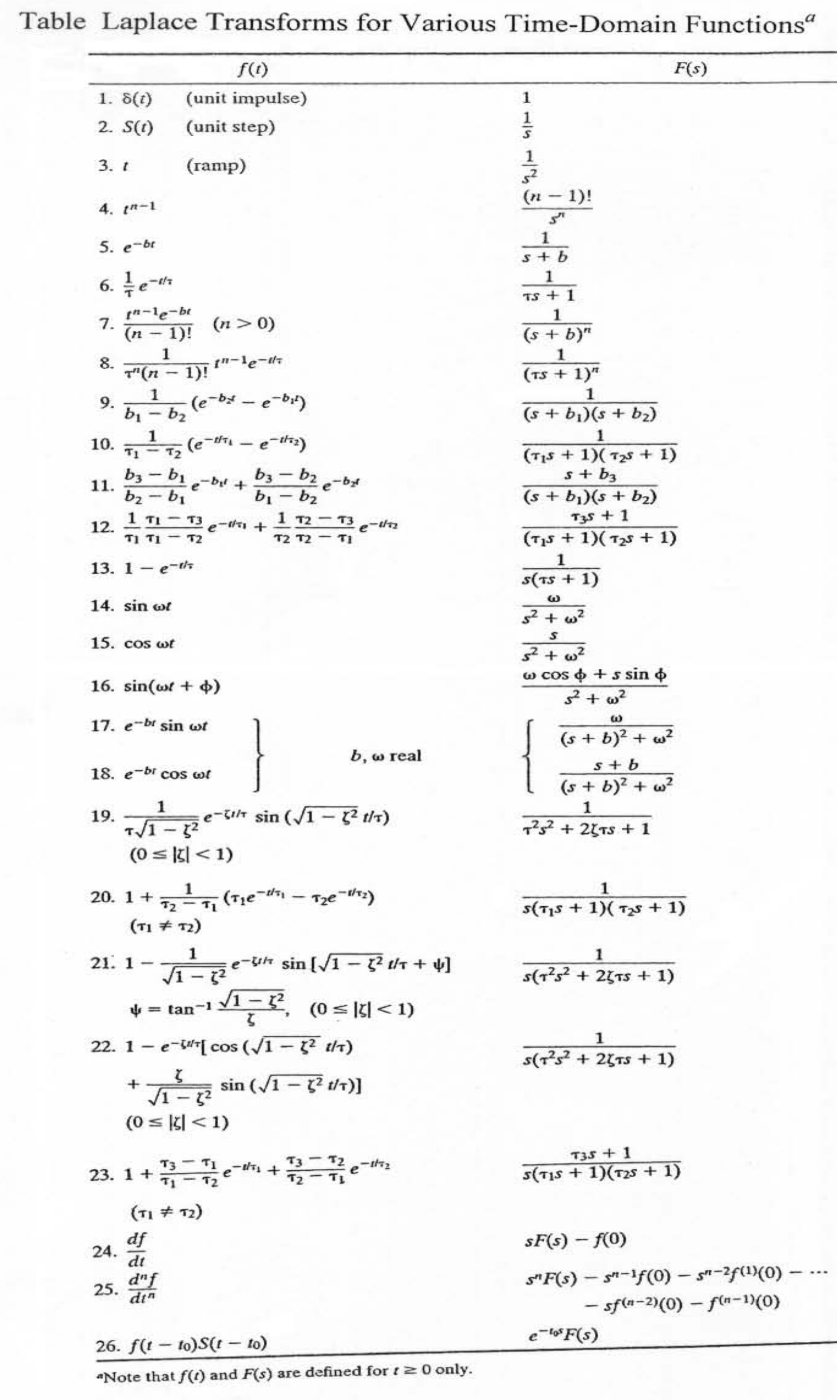
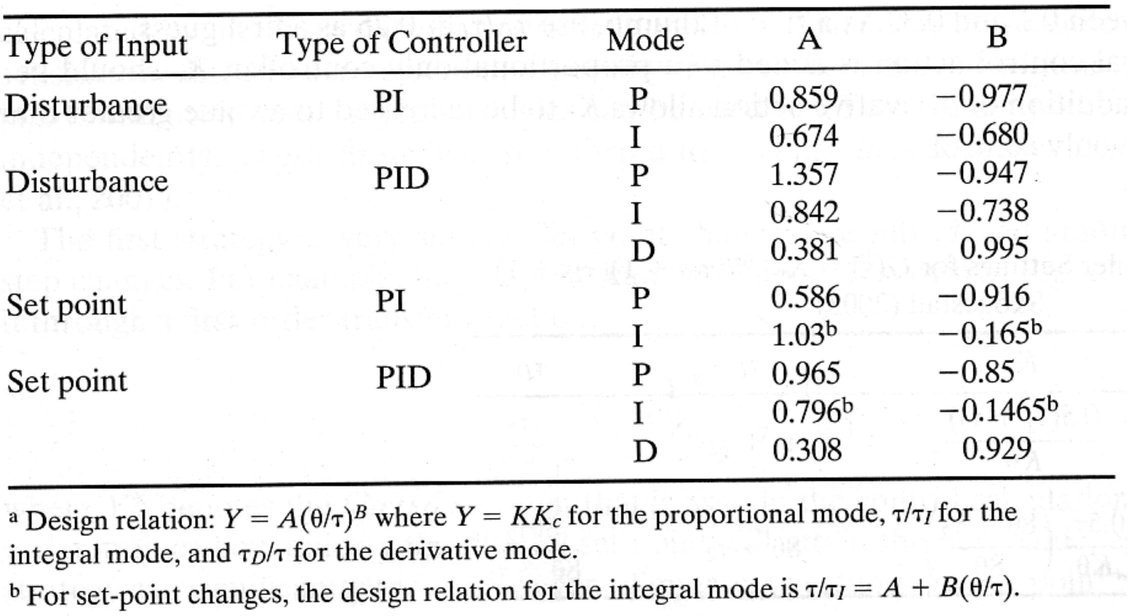


Table Laplace Transforms for Various Time-Domain Functions[a]

$f(t)$	$F(s)$
1. $\delta(t)$ (unit impulse)	1
2. $S(t)$ (unit step)	$\frac{1}{s}$
3. t (ramp)	$\frac{1}{s^2}$
4. t^{n-1}	$\frac{(n-1)!}{s^n}$
5. e^{-bt}	$\frac{1}{s+b}$
6. $\frac{1}{\tau}e^{-t/\tau}$	$\frac{1}{\tau s+1}$
7. $\frac{t^{n-1}e^{-bt}}{(n-1)!}$ $(n>0)$	$\frac{1}{(s+b)^n}$
8. $\frac{1}{\tau^n(n-1)!}t^{n-1}e^{-t/\tau}$	$\frac{1}{(\tau s+1)^n}$
9. $\frac{1}{b_1-b_2}(e^{-b_2t}-e^{-b_1t})$	$\frac{1}{(s+b_1)(s+b_2)}$
10. $\frac{1}{\tau_1-\tau_2}(e^{-t/\tau_1}-e^{-t/\tau_2})$	$\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$
11. $\frac{b_3-b_1}{b_2-b_1}e^{-b_1t}+\frac{b_3-b_2}{b_1-b_2}e^{-b_2t}$	$\frac{s+b_3}{(s+b_1)(s+b_2)}$
12. $\frac{1}{\tau_1}\frac{\tau_1-\tau_3}{\tau_1-\tau_2}e^{-t/\tau_1}+\frac{1}{\tau_2}\frac{\tau_2-\tau_3}{\tau_2-\tau_1}e^{-t/\tau_2}$	$\frac{\tau_3 s+1}{(\tau_1 s+1)(\tau_2 s+1)}$
13. $1-e^{-t/\tau}$	$\frac{1}{s(\tau s+1)}$
14. $\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
15. $\cos\omega t$	$\frac{s}{s^2+\omega^2}$
16. $\sin(\omega t+\phi)$	$\frac{\omega\cos\phi+s\sin\phi}{s^2+\omega^2}$
17. $e^{-bt}\sin\omega t$ $\quad b, \omega$ real	$\frac{\omega}{(s+b)^2+\omega^2}$
18. $e^{-bt}\cos\omega t$	$\frac{s+b}{(s+b)^2+\omega^2}$
19. $\frac{1}{\tau\sqrt{1-\zeta^2}}e^{-\zeta t/\tau}\sin(\sqrt{1-\zeta^2}\,t/\tau)$ $(0\le\lvert\zeta\rvert<1)$	$\frac{1}{\tau^2s^2+2\zeta\tau s+1}$
20. $1+\frac{1}{\tau_2-\tau_1}(\tau_1e^{-t/\tau_1}-\tau_2e^{-t/\tau_2})$ $(\tau_1\ne\tau_2)$	$\frac{1}{s(\tau_1 s+1)(\tau_2 s+1)}$
21. $1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta t/\tau}\sin[\sqrt{1-\zeta^2}\,t/\tau+\psi]$ $\psi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$, $(0\le\lvert\zeta\rvert<1)$	$\frac{1}{s(\tau^2s^2+2\zeta\tau s+1)}$
22. $1-e^{-\zeta t/\tau}[\cos(\sqrt{1-\zeta^2}\,t/\tau)$ $+\frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\sqrt{1-\zeta^2}\,t/\tau)]$ $(0\le\lvert\zeta\rvert<1)$	$\frac{1}{s(\tau^2s^2+2\zeta\tau s+1)}$
23. $1+\frac{\tau_3-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1}+\frac{\tau_3-\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}$ $(\tau_1\ne\tau_2)$	$\frac{\tau_3 s+1}{s(\tau_1 s+1)(\tau_2 s+1)}$
24. $\frac{df}{dt}$	$sF(s)-f(0)$
25. $\frac{d^nf}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f^{(1)}(0)-\cdots$ $-sf^{(n-2)}(0)-f^{(n-1)}(0)$
26. $f(t-t_0)S(t-t_0)$	$e^{-t_0s}F(s)$

[a] Note that $f(t)$ and $F(s)$ are defined for $t\ge0$ only.

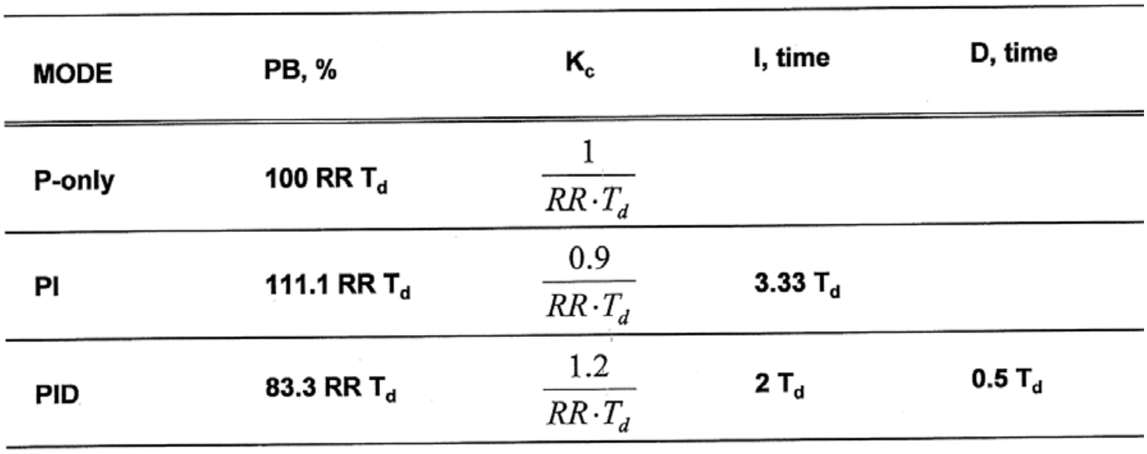


Type of Input	Type of Controller	Mode	A	B
Disturbance	PI	P	0.859	−0.977
		I	0.674	−0.680
Disturbance	PID	P	1.357	−0.947
		I	0.842	−0.738
		D	0.381	0.995
Set point	PI	P	0.586	−0.916
		I	1.03[b]	−0.165[b]
Set point	PID	P	0.965	−0.85
		I	0.796[b]	−0.1465[b]
		D	0.308	0.929

[a] Design relation: $Y = A(\theta/\tau)^B$ where $Y = KK_c$ for the proportional mode, τ/τ_I for the integral mode, and τ_D/τ for the derivative mode.

[b] For set-point changes, the design relation for the integral mode is $\tau/\tau_I = A + B(\theta/\tau)$.

ITAE performance index (Smith and Corripio, 1997)



MODE	**PB, %**	**K_c**	**I, time**	**D, time**
P-only	**100 RR T_d**	$\frac{1}{RR \cdot T_d}$		
PI	**111.1 RR T_d**	$\frac{0.9}{RR \cdot T_d}$	**3.33 T_d**	
PID	**83.3 RR T_d**	$\frac{1.2}{RR \cdot T_d}$	**2 T_d**	**0.5 T_d**

Ziegler-Nichols open loop tuning rule