UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2009/2010 Academic Session

April/May 2010

EKC 314 – Transport Phenomena
[Fenomena Pengangkutan]

Duration : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains TEN printed pages and FOUR printed page of Appendix before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak dan EMPAT muka surat Lampiran sebelum anda memulakan peperiksaan ini.]

Instruction: Answer **FIVE (5)** questions. Answer **ALL (3)** questions from Section A. Answer any **TWO (2)** questions from Section B. All questions carry the same marks.

*[**Arahan:** Jawab **LIMA (5)** soalan. Jawab **SEMUA (3)** soalan dari Bahagian A. Jawab mana-mana **DUA (2)** soalan dari Bahagian B. Semua soalan membawa jumlah markah yang sama.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah digunapakai].

<u>Section A</u> : Answer <u>ALL</u> questions.
<u>Bahagian A</u> : Jawab <u>SEMUA</u> soalan.

1. Heat is transferred from a duct to a fluid surrounding it by means of heat conduction through a fin heat exchanger. A diagram of a rectangular fin used for this purpose with a constant cross sectional area is shown in Figure Q.1.

Haba dipindahkan daripada satu saluran ke bendalir di sekitarnya secara pengaliran haba melalui penukar haba sirip. Gambarajah sirip segiempat yang digunakan dengan luas keratan rentas sekata ditunjukkan dalam Rajah S.1.

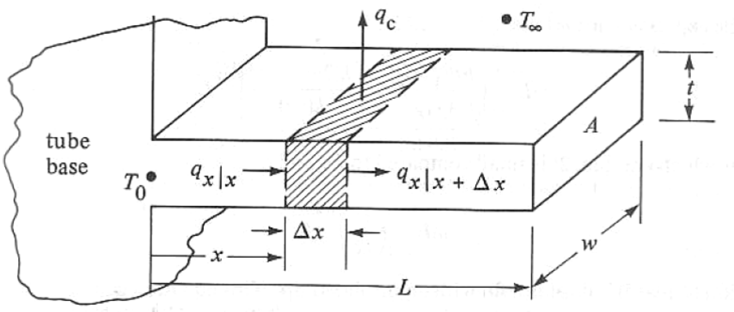

Figure Q.1.
Rajah S.1.

[a] Obtain an equation correlating the temperature profile along the fin assuming that the end or edges of the fin is insulated.

Dapatkan satu persamaan profil suhu sepanjang sirip tersebut dengan menganggap bahawa penghujung atau pinggir sirip tertebat.

[14 marks/markah]

[b] Define the term fin efficiency (η_f) for the fin.
Takrifkan istilah kecekapan sirip (η_f) untuk sirip tersebut.

[2 marks/markah]

[c] Show that the fin efficiency (η_f) is given by the equation below

Tunjukkan bahawa kecekapan sirip (η_f) adalah diberikan oleh persamaan di bawah

$$\eta_f = \frac{\tan h\, mL}{mL} \quad \text{whereby} \quad m = \left(\frac{hP}{kA}\right)^{\frac{1}{2}}$$

whereby, *k* is the thermal conductivity, *h* is the convective heat transfer coefficient, *P* is the perimeter of the cross sectional area, *A* is the cross sectional area and *L* is the length of the fin.

dengan k *adalah keberaliran haba,* h *adalah pekali pemindahan haba berolak,* P *adalah perimeter keratan rentas,* A *adalah luas keratan rentas dan* L *adalah panjang sirip.*

[4 marks/markah]

2. [a] Determine the mean molecular velocity, $\bar{u}$ in (m/s) and the mean free path, λ, in (m) for nitrogen at 1 atm and 273.2 K. A reasonable value for diameter between the two molecules d, is 2.5 Å.

Tentukan halaju molekul min, $\bar{u}$ *dalam (m/s) dan laluan bebas min,* λ*, dalam (m) untuk nitrogen pada 1 atm dan 273.2 K. Suatu nilai yang bersesuaian untuk diameter di antara dua molekul,* d *ialah 2.5 A.*

What is the ratio of the mean free path to the molecular diameter under these conditions?

Apakah nisbah laluan bebas kepada diameter molekul di bawah keadaan-keadaan tersebut?

Given that;
Diberi;

Boltzmann constant, $k = \frac{R}{N}$

Gas constant, R = 8.314 J mol^{-1} K^{-1}
= 82.058 cm^3 (atm) mol^{-1} K^{-1}

Avogadro's number, N = 6.02214 x 10^{23}

[4 marks/markah]

[b] A fluid with density ρ is flowing upwards in the direction opposing the gravity in between two coaxial cylinders of radii ξR and R for the inner cylinder and outer cylinder respectively. The radius from the centre to the fully-developed flow in between the annulus is given by γR. Assuming that the flow is at a steady-state condition;

Suatu bendalir dengan ketumpatan, ρ *mengalir ke arah atas melawan graviti antara dua silinder berpaksi tegak dengan masing-masing jejari-jejari* ξR *dan R bagi silinder dalam dan silinder luar. Jejari dari titik tengah silinder ke pembentukan penuh aliran antara anulus, diberi oleh* γR*. Andaikan yang aliran adalah pada keadaan mantap;*

[i] Draw a schematic diagram of the boundary condition, stating the velocity distribution of the flow.

Lukiskan suatu gambarajah skima penghasilan keadaan sempadan dengan menyatakan taburan halaju aliran.

[4 marks/markah]

[ii] Prove that the velocity distribution of the flow upwards the annulus is given by;

Buktikan bahawa halaju agihan bagi aliran ke atas melalui anulus di beri oleh;

$$v_z = \frac{(P_0 - P_L)R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2 - \frac{1-\xi^2}{\ln 1/\xi}\ln\left(\frac{R}{r}\right)\right]$$

[8 marks/markah]

[iii] According to the drawn boundary condition in 2.[b].[i]. above, when is the maximum velocity of the flow occurs and find $v_{z,max}$.

Berdasarkan gambarajah yang dilukis di dalam 2.[b].[i]. di atas, bilakah berlakunya halaju maksimum dan cari $v_{z,max}$.

[4 marks/markah]

3. A solution containing a component A is transported through a cylindrical pipe of diameter *d* and length *l*, constructed with a material B. The component A however is reactive with the component B leading to a corrosive reaction,

Suatu larutan yang mengandungi komponen A dipindahkan melalui sebatang paip silinder dengan garispusat d *dan panjang* l. *Paip tersebut dibina dengan bahan B. Walau bagaimanapun komponen A bertindakbalas dengan komponen B yang membawa kepada suatu tindakbalas kakisan,*

A+B→ C where the reaction rate r_A is first order and is given by
A+B→ C di mana kadar tindakbalas r_A ialah tertib pertama dan diberi oleh

$r_A = -k_1C_A$ moles/m^2 of surface/s
$r_A = -k_1C_A$ mol/m^2 permukaan/s

[a] Write down the general differential equation for the variation of concentration of A with the length at steady state.

Tuliskan persamaan pembezaan umum bagi perubahan kepekatan A dengan panjang pada keadaan mantap.

[7 marks/markah]

[b] Assume that there is no variation of axial velocity of the solution with the radius at steady flow conditions but diffusional effects along the length of the pipe exist. Show that the concentration of A (C_A) at a distance z from the pipe entry and the overall rate of corrosion of pipe measured in terms of N_A can be expressed as:

Andaikan yang halaju paksi larutan tidak berubah dengan jejari pada keadaan aliran mantap tetapi kesan resapan sepanjang paip wujud. Tunjukkan bahawa kepekatan A (C_A) pada jarak z dari titik masuk paip dan kadar keseluruhan kakisan paip yang diukur dalam N_A boleh masing-masing diungkapkan sebagai:

[i] $C_A = K_1 \sinh \alpha z + K_2 \cosh \alpha z$

[6 marks/markah]

and
dan

[ii] $N_A = D_{AB} . \alpha [K_1 . \cosh \alpha l + K_2 \sinh \alpha l]$

[7 marks/markah]

respectively, where:
di mana,

$\alpha^2 = 4k_1/(D_{AB}\, d)$

Derive the above equations starting from the three dimensional diffusion and continuity equations.

Terbitkan persamaan-persamaan di atas bermula dengan persamaan-persamaan resapan dan keselanjaran tiga dimensi.

Outline how the parameters K_1 and K_2 could be estimated.
Rangkakan bagaimana parameter-parameter K_1 *dan* K_2 *boleh dianggarkan.*

<u>Section B:</u> Answer any <u>TWO</u> questions.
<u>Bahagian B:</u> Jawab mana-mana <u>DUA</u> soalan.

4. A hot stream of liquid flowing under laminar condition at temperature of T_1 was cooled while flowing through a circular pipe shown in Figure Q.4. Heat is conducted away from the fluid radially through the cool pipe wall at a constant heat flux (q_o) as well as through the liquid flow in the z direction.

Suatu aliran cecair panas di bawah aliran lamina pada suhu T_1 disejukkan semasa mengalir melalui paip bulat yang ditunjukkan dalam Rajah S.4. Haba dialirkan daripada cecair secara jejarian melalui dinding paip yang sejuk pada fluks haba malar (q_o) dan melalui aliran cecair pada arah z.

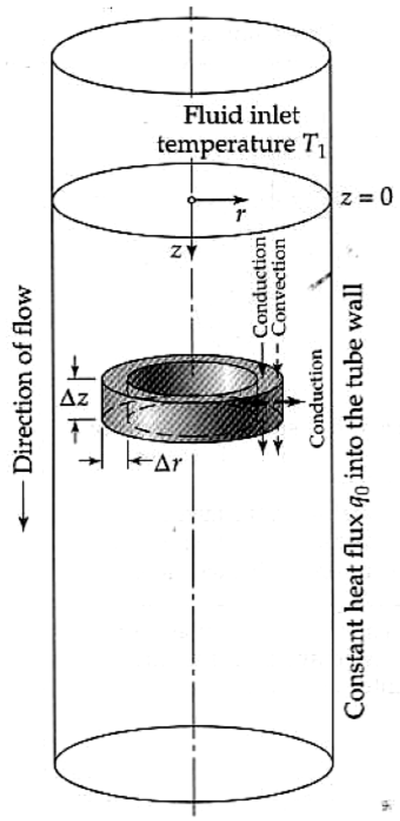

Figure Q.4.
Rajah S.4.

[a] Using the notation in the Figure Q.4., conduct a heat balance on the darken annular region and show that the temperature profile of the stream in the pipe is given by the equation below. Indicate all assumptions made.

Dengan menggunakan tatatanda dalam Rajah S.4., lakukan imbangan haba terhadap anulus yang digelapkan dan tunjukkan bahawa profil suhu aliran dalam paip diberikan oleh persamaan di bawah. Nyatakan semua anggapan yang dibuat.

$$\rho\widehat{C_p}v_z\left[1-\left(\frac{r}{R}\right)^2\right]\frac{\partial T}{\partial z}=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right]$$

[12 marks/markah]

…7/-

[b] In an experiment, due to the turbulent flow of the fluid, it was observed that the velocity and temperature did not change along the radial direction and the temperature gradient, $\partial T / \partial z$, could be assumed to be constant. Under these conditions, show that the heat loss of the fluid flowing in the pipe is given by the equation below where the c_1 and c_2 are constants.

Dalam suatu eksperimen, disebabkan aliran gelora oleh bendalir, dapat diperhatikan bahawa halaju dan suhu tidak berubah pada arah jejarian dan kecerunan suhu, $\partial T/\partial z$, *adalah dianggap malar. Dalam keadaan sedemikian, tunjukkan bahawa kehilangan haba oleh bendalir yang mengalir melalui paip diberikan oleh persamaan di bawah dengan* c_1 *dan* c_2 *adalah pemalar.*

$$q_o = 2\pi Rh\left[\frac{c_1 z^2}{2} + (c_2 - T_o)z\right]$$

[8 marks/markah]

5. [a] Derive the following continuity relationship for single phase fluid flow:

Terbitkan persamaan keselanjaran di bawah untuk aliran fasa tunggal bendalir:

$$\frac{\partial \rho}{\partial t} = -\left(\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)\right) \qquad \text{Q.5.1.}$$

S.5.1

where ρ is the fluid density, t is time and v_x, v_y, and v_z, are the velocities in the x, y and z-directions respectively.

di mana ρ *adalah ketumpatan bendalir,* t *ialah masa dan* v_x, v_y, *dan* v_z, *adalah masing-masing halaju-halaju pada arah* x, y *dan* z.

[6 marks/markah]

[b] To what form does the above equation reduce for an incompressible (constant density) fluid?

Kepada bentuk apakah persamaan di atas akan diringkaskan bagi bendalir tidak-boleh-mampat (ketumpatan malar).

[2 marks/markah]

[c] In the near wall region, and where pressure gradients and gravitational forces are negligible, the Reynolds averaged form of the Navier-Stokes Equation reduces, for a 1-dimensional flow to:

Pada kawasan berdekatan dengan dinding, di mana kecerunan tekanan dan daya graviti diabaikan, purata-Reynolds daripada Persamaan Navier-Stokes diringkaskan kepada aliran 1-dimensi;

$$(\mu + \mu_t)\frac{\partial^2 v_x}{\partial y^2} \approx 0 \qquad \text{Q.5.2.}$$

S.5.2.

where μ and μ_t are the molecular and turbulent viscosities, $\overline{v_x}$ the mean velocity in the *x*-direction (parallel to the wall) and y is the distance from the wall. The turbulent velocity is defined as:

di mana μ dan μ_t adalah kelikatan molekul dan kelikatan gelora, $\overline{v_x}$ ialah halaju min pada arah-x (selari dengan dinding) dan y adalah jarak daripada dinding. Halaju gelora ditakrifkan sebagai:

$$\mu_t = -\frac{\rho \overline{v_x' v_y'}}{\partial \overline{v}_x / \partial y} \qquad \text{Q.5.3.}$$

S.5.3

where v_x' and v_y' are the instantaneous fluctuations in the velocity components in the *x* and *y*-direction respectively. Show that for the region immediately adjacent to the wall (where $\mu >> \mu_t$), equation (Q.5.2.) leads to expression:

di mana v_x' dan v_y' adalah ketidakstabilan ketika pada komponen-komponen halaju pada arah x dan y. Tunjukkan bahawa kawasan yang paling berdekatan dengan dinding ($\mu >> \mu_t$), persamaan (S.5.2.) membawa kepada persamaan:

$$u^+ = y^+$$

where u^+ and y^+ are given by;

di mana u^+ dan y^+ diberi sebagai;

$$u^+ = \frac{\overline{v_x}}{u^*} \quad \text{and} \quad y^+ = \frac{u^* \rho y}{\mu}$$

where u^* is the friction velocity defined as;
di mana u^* adalah halaju geseran yang dimaksudkan sebagai;

$$u^* = \sqrt{\tau_0 / \rho}$$

where τ_0 is the wall shear stress.
di mana τ_0 adalah tekanan ricih dinding.

[8 marks/markah]

[d] For the case of fluid further away from the wall, where $\mu_t >> \mu$, the Prandtl mixing length hypothesis may be invoked;

Untuk kes bendalir yang berjauhan dari dinding, di mana $\mu_t >> \mu$, hipotesis panjang campuran Prandtl boleh digunakan;

$$\overline{v_x' v_y'} = l^2 \left| \frac{\partial \overline{v_x}}{\partial y} \right| \frac{\partial \overline{v}_x}{\partial y}$$

where the mixing length l is given by;
di mana panjang campuran, l *diberi sebagai;*

$$l = \kappa y$$

where κ, is the von Karman constant. Show that it follows from equation Q.5.2. above and from the above mixing length hypothesis that;
di mana κ, adalah pemalar von Karman. Tunjukkan bahawa ia menuruti persamaan S.5.2 di atas. Tunjukkan juga hipotesis panjang campuran di mana;

$$u^+ = \frac{1}{\kappa} \ln y^+ + C \qquad \text{Q.5.4.}$$

S.5.4.

where C is a constant.
di mana C adalah suatu pemalar.

[2 marks/markah]

[e] Assuming that $\kappa = 0.41$ and that equations Q.5.3. and Q.5.4. above give ideal values for u^+ at $y^+ = 11$, estimate the value of the constant C in equation Q.5.4.

Mengandaikan bahawa κ = 0.41 dan persamaan S.5.3. dan S.5.4. di atas diberi sebagai nilai-nilai unggul untuk u^+ pada $y^+ = 11$, anggarkan nilai pemalar C pada persamaan S.5.4.

[2 marks/markah]

6. A porous sphere of radius '*a*' encloses a concentric spherical cavity packed with silica gel of radius '*b*' as shown in the Figure Q.6. The system is exposed to a pool of liquid at 25 °C having water of concentration C_w moles/m^3. The water commences diffusing with unsteady state diffusion.

Sebuah sfera poros dengan jejari 'a' meliputi suatu rongga sfera sepusat yang terpadat dengan gel silika dan mempunyai jejari 'b' seperti yang ditunjukkan dalam Rajah S.6. Sistem tersebut didedahkan ke suatu takungan cecair pada 25 °C yang mempunyai kepekatan C_w mol/m^3. Air tersebut mula meresap dengan resapan tak berkeadaan mantap.

[a] Starting from the three dimensional diffusion and continuity equations, show that the concentration of water C at any point within the outer packing could be expressed as

Bermula dengan persamaan-persamaan resapan dan keselanjaran tiga dimensi, tunjukkan bahawa kepekatan air C di mana-mana titik dalam padatan luaran boleh diungkapkan sebagai

$$\frac{\partial C}{\partial t} = D \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial C}{\partial r} \right)$$

[6 marks/markah]

[b] The silica gel acts as a sink for the moisture where the concentration of water at a radius b is maintained at C_b, show that at steady state conditions the concentration C_w at a radius r could be expressed as

Gel silika bertindak sebagai sebuah sinki untuk kelembapan di mana kepekatan air pada jejari b dikekalkan pada C_b. Tunjukkan bahawa pada keadaan mantap, kepekatan C_w pada jejari r boleh diungkapkan sebagai

$$C = \frac{aC_w(b-r) + bC_b(r-a)}{r(b-a)}$$

[6 marks/markah]

[c] If the moisture diffusivity in the porous packing is $6\text{x}10^{-9}$ m^2/s, C_w = 55.56 $kmol/m^3$ and C_b =0, estimate the rate of moisture transport across the outer surface of the packing at steady state.

Jika kemeresapan kelembapan dalam padatan poros ialah $6\text{x}10^{-9}$ m^2/s, C_w = 55.56 $kmol/m^3$ dan C_b =0, anggarkan kadar pengangkutan kelembapan merentasi permukaan luaran padatan pada keadaan mantap.

[8 marks/markah]

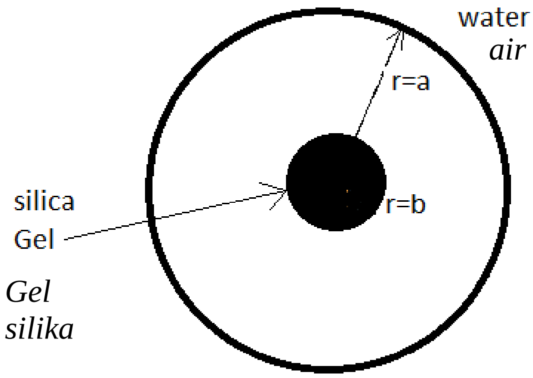

Figure Q.6.
Rajah S.6.

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Appendix

Useful formulae for Mass transport analysis

NOTATION AND DEFINITIONS

A. Notation

- c_A= concentration of A (kmol of A/m3)
- D_{AB}=Diffusion coefficient of A in B (m^2/s)
- *i,j,k* are unit vectors in three perpendicular directions associated with the system considered.
- J_A=Diffusional flux in kmol/(m^2.s) = $J_x.i+J_y.j+J_z.k$ (vector)
- *Jx,Jy,Jz*= Diffusional flux in kmol/(m^2.s)in the directions *i,j,k* respectively
- n_A= Overall mass transfer flux in kg/(m^2.s)= *Nx.i+Ny.j+Nz.k* (vector)
- n_x,n_y,n_z =Overall mass transfer fluxes in kg/(m^2.s)in directions of the unit vectors *i,j,k*
- N_A= Overall mass transfer flux in kmol/(m^2.s)= *Nx.i+Ny.j+Nz.k* (vector)
- *Nx,Ny,Nz* =Overall mass transfer fluxes in kmol/(m^2.s) in directions of the unit vectors *i,j,k*
- r_A= rate of reaction in kg/(m^3.s)
- R_A=rate of reaction in kmoles/(m^3.s)
- *s*= scalar
- w_A=weight fraction of component A (kg of A /total kg) w_A=weight fraction of component A (kg of A /total kg)
- *x,y,z* = distances (m) in directions of the unit vectors *i,j,k*
- *u,v,w*= velocities (m/s) in i, j,k directions represented by *v*=u.i+v.j+w.k* (vector)
- $\tilde{v}$ = a vector
- ρ =*density of medium kg/m^3*

B. Definition of the operator ∇ Cartesian Co-ordinates

VECTOR

$$\nabla.v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

SCALAR

$$\nabla s = \frac{\partial s}{\partial x} i + \frac{\partial s}{\partial y} j + \frac{\partial s}{\partial z} k$$

C Definition of the operator ∇ Cylindrical co-ordinates

$$\nabla.\tilde{v} = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\nabla s = \frac{\partial s}{\partial r} i + \frac{1}{r}\frac{\partial s}{\partial \theta} j + \frac{\partial s}{\partial z} k$$

D. **Definition of the operator ∇** Spherical co-ordinates

$$\nabla.v = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r.Sin\theta}\frac{\partial(v_\theta.Sin\theta)}{\partial\theta} + \frac{1}{rSin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\nabla s = \frac{\partial s}{\partial r}i + \frac{1}{r}\frac{\partial s}{\partial\theta}j + \frac{1}{rSin\theta}\frac{\partial s}{\partial\phi}k$$

E. **Definition of the operator ∇^2** Cartesian Co-ordinates

$$\nabla^2 s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

F. **Definition of the operator ∇^2** Cylindrical co-ordinates

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial\theta^2} + \frac{\partial^2 s}{\partial z^2}$$

G. **Definition of the operator ∇^2** Spherical co-ordinates

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2.Sin\theta}\frac{\partial}{\partial\theta}\left(Sin\theta\frac{\partial s}{\partial\theta}\right) + \frac{1}{r^2 Sin^2\theta}\frac{\partial^2 s}{\partial\phi^2}$$

DIFFERENTIAL EQUATIONS FOR MASS TRANSPORT ANALYSIS

A.Equivalent forms of Fick's law of binary diffusion and Mass/Molar flux equations

(mass flux)

$$n_A = \rho w_A.v - D_{AB}\nabla.(\rho w_A)$$

(molar flux)

$$N_A = c_A.v^* - D_{AB}\nabla.c_A$$

B. The equations of continuity for a multi-component mixture

(mass flux)

$$\frac{\partial(\rho w_A)}{\partial t} = -(\nabla.n_A) + r_A$$

(molar flux)

$$\frac{\partial c_A}{\partial t} = -(\nabla.N_A) + R_A$$

SOLUTIONS OF DIFFERENTIAL EQUATIONS:

1. $$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial x^2}$$

Solution

$$C_A = A + B\int_0^{x/\sqrt{4Dt}} exp(-\eta^2)d\eta$$

Given : $$\int_0^{\infty} exp(-\eta^2)d\eta = \frac{\sqrt{\pi}}{2}$$

2. $$\frac{\partial^2 c_A}{\partial x^2} = 0$$

Solution: $$\frac{\partial c_A}{\partial x} = \text{constant} = K$$

3. $$\frac{\partial^2 y}{\partial x^2} = \alpha^2 y$$

Solution

: $$y = K_1 . sinh\,\alpha x + K_2\, cosh\,\alpha x$$

where K_1 and K_2 are constants.

or $$y = K_1 e^{+ax} + K_2 e^{-ax}$$

$$\frac{\partial^2 y}{\partial z^2} = \alpha^2 y$$

Solution

$$y = K_1 . sinh\,\alpha z + K_2\, cosh\,\alpha z$$

where K_1 and K_2 are constants.

4.

$$\frac{\partial c_A}{\partial r} = K/r^2$$ **Solution is**

$$c_A = -K/r + K1$$

5.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial c_A}{\partial r}\right) = \alpha^2 c_A$$

Solution is $$\frac{c_A}{c_{AR}} = \frac{C_1}{r} cosh\,\alpha r + \frac{C_2}{r} sinh\,\alpha$$

6. **Hyperbolic functions**

$$\frac{d}{dx}(\cosh u) = (\sinh u)\frac{du}{dx}$$ $$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

$$\frac{d}{dx}(\sinh u) = (\cosh u)\frac{du}{dx}$$

Equations related to heat transport analysis

1. Combined energy flux vector:

$$\mathbf{e} = \left(\frac{1}{2}\rho v^2 + \rho\hat{U} \right)\mathbf{v} + [\boldsymbol{\pi} \cdot \mathbf{v}] + \mathbf{q}$$

$$\mathbf{e} = \left(\frac{1}{2}\rho v^2 + \rho\hat{H} \right)\mathbf{v} + [\boldsymbol{\pi} \cdot \mathbf{v}] + \mathbf{q}$$

2. Stress tensor:

$$[\boldsymbol{\pi} \cdot \mathbf{v}] = p\mathbf{v} + [\boldsymbol{\tau} \cdot \mathbf{v}]$$

3. Velocity distribution in the z direction for laminar, incompressible flow of a Newtonian fluid in a long

$$v_z = v_{z,\max}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

4. Work flux:

$$(\boldsymbol{\pi}_x \cdot \mathbf{v}) = \pi_{xx}v_x + \pi_{xy}v_y + \pi_{xz}v_z \equiv [\boldsymbol{\pi} \cdot \mathbf{v}]_x$$

$$(\boldsymbol{\pi}_y \cdot \mathbf{v}) = \pi_{yx}v_x + \pi_{yy}v_y + \pi_{yz}v_z \equiv [\boldsymbol{\pi} \cdot \mathbf{v}]_y$$

$$(\boldsymbol{\pi}_z \cdot \mathbf{v}) = \pi_{zx}v_x + \pi_{zy}v_y + \pi_{zz}v_z \equiv [\boldsymbol{\pi} \cdot \mathbf{v}]_z$$

5. Hyperbolic function:

$$\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$$

$$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right)$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

6. Heat diffusion equation for extended surface

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0$$