UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2007/2008

October/November 2007

MAA 111 – Algebra for Science Students
[Aljabar untuk Pelajar Sains]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eleven** [11] questions.

*[**Arahan:** Jawab **semua sebelas** [11] soalan.]*

1. Write $B=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ as a product of elementary matrices.

[5 marks]

2. If A is a nonsingular matrix and $\left(5A^T\right)^{-1}=\begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$, find A.

[5 marks]

3. Find the value(s) of k so that the linear system

$$\begin{aligned} x + ky - z &= 1 \\ -x + (k+2)y + z &= -1 \\ 2x - 2y + kz &= 1 \end{aligned}$$

(a) has a unique solution
(b) has infinitely many solutions
(c) inconsistent

[10 marks]

4. Let $A=\begin{bmatrix} 6 & 2 & 8 \\ 9 & 5 & 11 \\ 3 & 1 & 6 \end{bmatrix}$, $L=\begin{bmatrix} 2 & 0 & 0 \\ t & s & 0 \\ 1 & 0 & -1 \end{bmatrix}$ and $U=\begin{bmatrix} r & 1 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & p \end{bmatrix}$.

Find the scalars r, s, t and p so that $A = LU$.

Hence, solve the linear system

$$\begin{bmatrix} 6 & 2 & 8 \\ 9 & 5 & 11 \\ 3 & 1 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}.$$

[10 marks]

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1. *Tuliskan* $B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ *sebagai hasildarab matriks permulaan.*

[5 markah]

2. *Jika A matriks tak singular dan* $\left(5A^T\right)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$, *cari A.*

[5 markah]

3. *Dapatkan nilai k supaya sistem Linear*

$$\begin{aligned} x &+ ky &- z &= 1 \\ -x &+ (k+2)y &+ z &= -1 \\ 2x &- 2y &+ kz &= 1 \end{aligned}$$

(a) *mempunyai penyelesaian unik*
(b) *mempunyai penyelesaian yang tak terhingga*
(c) *tak konsisten*

[10 markah]

4. *Katakan* $A = \begin{bmatrix} 6 & 2 & 8 \\ 9 & 5 & 11 \\ 3 & 1 & 6 \end{bmatrix}$, $L = \begin{bmatrix} 2 & 0 & 0 \\ t & s & 0 \\ 1 & 0 & -1 \end{bmatrix}$ and $U = \begin{bmatrix} r & 1 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & p \end{bmatrix}$.

Cari pemalar r, s, t dan p supaya $A = LU$.

Seterusnya, selesaikan sistem linear

$$\begin{bmatrix} 6 & 2 & 8 \\ 9 & 5 & 11 \\ 3 & 1 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}.$$

[10 markah]

5. Given that $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 2$. Find $\det\begin{bmatrix} -g & -h & -i \\ 3a+d & b+3e & c+3f \\ 2d & 2e & 2f \end{bmatrix}$.

[8 marks]

6. Determine whether the given set together with the operations of addition and scalar multiplication is a vector space.

(a) $V=\{(x,y)\,;\, x,y\in\mathbb{R}\}$, $(x_1,y_1)+(x_2,y_2)=(x_1+y_2,x_2+y_1)$ and $c(x,y)=(cx,cy)$.

(b) $V=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix};\ ad=0 \text{ and } a,b,c,d\in\mathbb{R}\right\}$ with the usual matrix addition and scalar multiplication.

[8 marks]

7. Determine whether W is a subspace of V.

(a) $V=\mathbb{R}^3$, $W=\left\{\begin{pmatrix} a \\ b \\ c \end{pmatrix};\ b=a^2 \text{ and } a,b,c\in\mathbb{R}\right\}$.

(b) $V=P_2$, $W=\{a_0+a_1t+a_2t^2\,;\, a_0=a_1+a_2\}$.

[9 marks]

8. Let $A=\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$.

Find,

(a) eigenvalues and corresponding eigenvectors
(b) a nonsingular matrix P such that $P^{-1}AP$ is diagonal
(c) A^4.

[15 marks]

5. *Di beri* $\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 2$. *Cari* $\det\begin{bmatrix} -g & -h & -i \\ 3a+d & b+3e & c+3f \\ 2d & 2e & 2f \end{bmatrix}$.

[8 markah]

6. *Tentukan sama ada set yang diberikan bersama dengan operasi penambahan dan perdaraban skalar merupakan suatu ruang vektor.*

(a) $V = \{ (x,y) \,;\, x,y \in \mathbb{R} \}$, $(x_1,y_1)+(x_2,y_2)=(x_1+y_2, x_2+y_1)$ *dan* $c(x,y)=(cx,cy)$.

(b) $V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} ;\ ad = 0 \text{ dan } a,b,c,d \in \mathbb{R} \right\}$ *dengan operasi penambahan dan pendaraban skalar matriks yang biasa.*

[8 markah]

7. *Tentukan sama ada W merupakan suatu subruang bagi V.*

(a) $V = \mathbb{R}^3$, $W = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} ;\ b = a^2 \text{ and } a,b,c \in \mathbb{R} \right\}$.

(b) $V = P_2$, $W = \{ a_0 + a_1 t + a_2 t^2 \,;\, a_0 = a_1 + a_2 \}$.

[9 markah]

8. *Katakan* $A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$.

Dapatkan,

(a) *Nilai eigen dan vektor eigen yang sepadan dengannya.*
(b) *Matriks P yang tak singular sedemikian* $P^{-1}AP$ *adalah matriks pepenjuru*
(c) A^4.

[15 markah]

9. Find a basis and dimension of the solution space of the system

$$\begin{aligned} x - 3y + z &= 0 \\ 2x - 6y + 2z &= 0 \\ 3x - 9y + 3z &= 0 \end{aligned}.$$

[10 marks]

10. (a) Show that if $\{u, v, w\}$ is a linearly independent set of vectors, then $\{u+v, u-v, u-2v+w\}$ is also linearly independent.

(b) Find the conditions on a, b, c so that $v = (a, b, c)$ in $\mathbb{R}^3$ is spanned by the vectors $u_1 = (1,2,0)$, $u_2 = (-1,1,2)$, $u_3 = (3,0,-4)$.

[10 marks]

11. State whether the following statement TRUE or FALSE :

(a) If A is nonsingular and skew-symmetric matrix, then A^{-1} is skew-symmetric.

(b) A homogeneous system of n linear equations with $m > n$ variables always has a unique nontrivial solution.

(c) The zero vector is an element of every vector space.

(d) If A is an $n \times n$ matrix and rank $(A) = n$, then det $(A) = 0$.

(e) Every diagonalizable matrix has at least one eigenvalue.

[10 marks]

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9. *Dapatkan asas dan dimensi bagi ruang penyelesaian bagi sistem*

$$\begin{aligned} x - 3y + z &= 0 \\ 2x - 6y + 2z &= 0. \\ 3x - 9y + 3z &= 0 \end{aligned}$$

[10 markah]

10. (a) *Tunjukkan bahawa jika* $\{u, v, w\}$ *adalah set vektor yang tak bersandar linear, maka* $\{u+v, u-v, u-2v+w\}$ *juga tak bersandar linear.*

(b) *Dapatkan syarat atas a, b, c supaya* $v = (a, b, c)$ dalam $\mathbb{R}^3$ *direntangi oleh vektor* $u_1 = (1,2,0)$, $u_2 = (-1,1,2)$, $u_3 = (3,0,-4)$.

[10 markah]

11. *Nyatakan samada penyataan berikut BETUL atau PALSU:*

(a) *Jika A matrik tak singular dan simetri pencong, maka* A^{-1} *simetri pencong*

(b) *Sistem homogen bagi n persamaan linear dengan* $m > n$ *pembolehubah selalunya mempunyai penyelesaian unik yang tak remeh.*

(c) *Vektor zero merupakan unsur bagi setiap ruang vektor.*

(d) *Jika A merupakan matriks* $n \times n$ *dan pangkat (A) = n, maka penentu A = 0.*

(e) *Setiap matrik terpepenjurukan mempunyai sekurang-kurangnya satu nilai eigen.*

[10 markah]

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