UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2007/2008

October/November 2007

MAA 101 – Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>FIVE</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>LIMA</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

*[**Arahan:**   Jawab **semua sembilan** [9] soalan.]*

1. Evaluate the limit.

   (a) $\lim\limits_{x \to 4} \dfrac{\sqrt{x}-2}{x-4}$ (b) $\lim\limits_{x \to 0} \dfrac{\sin 9x}{\sin 4x}$ (c) $\lim\limits_{x \to \frac{\pi}{2}}(x-\frac{\pi}{2})\tan x$ (d) $\lim\limits_{x \to 0} \dfrac{x-\sin x}{x^3}$

   [20 marks]

2. (a) State the *Intermediate Value Theorem*.

   (b) If $f(x) = x^3 - x^2 + x - 2$, then show that there is a number c such that
   $c^3 - c^2 + c - 2 = 0$.

   [14 marks]

3. Differentiate the function.

   (a) $y = e^{\ln(2^x + \sec x)} - \ln(e^{\log_2(\tan^{-1}x)})$ (b) $y = \left(\dfrac{1+3x}{3x}\right)(3-x)$.

   [14 marks]

4. Suppose $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, where $f(2) = 3$, $g(2) = 5$, $g'(2) = 4$, $f'(2) = -2$ and $f'(5) = 11$. Find $h'(2)$ and $F'(2)$.

   [12 marks]

5. Consider the function $f(x) = x^3 - 3x + 1$ on the interval $[-2, 3]$.

   (a) Find the critical numbers of f.

   (b) Find the local maximum and local minimum values of f.

   (c) Are these local extreme values also absolute extreme values of f? If not, find the absolute maximum and absolute minimum values of f.

   [15 marks]

6. Consider the equation $6 + \displaystyle\int_a^x \dfrac{f(t)}{t^2}\,dt = 2\sqrt{x}$, $x > 0$.

   (a) By differentiating both sides, find a function f that satisfies the equation.

   (b) With the function f found in part (a), find the number a that satisfies the equation.

   [10 marks]

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1. *Nilaikan had berikut.*

   (a) $\displaystyle \lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$ *(b)* $\displaystyle \lim_{x \to 0} \frac{\sin 9x}{\sin 4x}$ *(c)* $\displaystyle \lim_{x \to \frac{\pi}{2}} (x - \tfrac{\pi}{2}) \tan x$ *(d)* $\displaystyle \lim_{x \to 0} \frac{x - \sin x}{x^3}$

   [20 markah]

2. *(a) Nyatakan Teorem Nilai Pertengahan.*

   (b) Jika $f(x) = x^3 - x^2 + x - 2$, *maka tunjukkan bahawa wujud suatu nombor c sedemikian* $c^3 - c^2 + c - 2 = 0$.

   [14 markah]

3. *Bezakan fungsi berikut.*

   (a) $y = e^{\ln(2^x + \sec x)} - \ln(e^{\log_2(\tan^{-1} x)})$ *(b)* $y = \left(\dfrac{1+3x}{3x} \right)(3-x)$.

   [14 markah]

4. *Andaikan* $h(x) = f(x)g(x)$ *dan* $F(x) = f(g(x))$, *dengan* $f(2) = 3$, $g(2) = 5$, $g'(2) = 4$, $f'(2) = -2$ *dan* $f'(5) = 11$. *Cari* $h'(2)$ *dan* $F'(2)$.

   [12 markah]

5. *Pertimbangkan fungsi* $f(x) = x^3 - 3x + 1$ *pada selang* $[-2,3]$.

   (a) Cari nombor genting untuk f.

   (b) Cari nilai-nilai maksimum and minimum setempat untuk f.

   (c) Adakah nilai-nilai ekstrem setempat ini juga nilai-nilai ekstrem mutlak untuk f *? Jika tidak, cari nilai maksimum dan minimum mutlak untuk* f.

   [15 markah]

6. *Pertimbangkan persamaan* $6 + \displaystyle\int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x}$, $x > 0$.

   (a) Dengan membezakan kedua-dua belah persamaan, cari suatu fungsi f *yang memenuhi persamaan tersebut.*

   (b) Dengan fungsi yang didapati di bahagian (a), cari nombor a *yang memenuhi persamaan tersebut.*

   [10 markah]

7. (a) Write out the form of the partial fraction decomposition of the function

$$\frac{2}{x^2 - 6x + 8}.$$

Then determine the numerical values of the coefficients.

(b) Evaluate the integral $\int \frac{1}{x^2 - 6x + 8} dx$.

(c) Find the area of the region under the curve $y = \frac{1}{x^2 - 6x + 8}$ from $x = 5$ to $x = 10$.

[16 marks]

8. Set up, but **do not** evaluate, an integral for the volume of the solid obtained by rotating about the y-axis the region bounded by

$$y = \sqrt{x-1},\ y = 0,\ x = 5,$$

using the method of

(a) disk/washer   (b) cylindrical shell.

[15 marks]

9. *True* or *False*.

(a) If f is continuous on $[a,b]$, then its derivative is also continuous on $[a,b]$.

(b) If f' is continuous on $[0,2]$, then $\int_0^2 f(x)\,dx = f'(2) - f'(0)$.

(c) $\int_{-1}^{1} \frac{1}{x} dx = \ln x \Big]_{-1}^{1} = \ln|1| - \ln|-1| = 0$.

(d) If $\frac{d}{dx}\sin(x°) = \frac{d}{dx}\sin\left(\frac{\pi x}{180}\right) = \frac{\pi}{180}\cos\left(\frac{\pi x}{180}\right) = \frac{\pi}{180}\sin(x°)$.

(e) If f is differentiable and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.

[10 marks]

7.　(a) Tuliskan bentuk komposisi pecahan separa untuk fungsi

$$\frac{2}{x^2 - 6x + 8}.$$

　　　Kemudian tentukan nilai berangka untuk koefisien tersebut.

(b) Nilaikan kamiran $\int \frac{1}{x^2 - 6x + 8} dx$.

(c) Cari luas rantau di bawah lengkung $y = \frac{1}{x^2 - 6x + 8}$ dari $x = 5$ ke $x = 10$.

[16 markah]

8.　Bentukkan, **tanpa** menilai, suatu kamiran untuk isipadu bungkah yang diperoleh dengan memutarkan sekitar paksi-y rantau dibatasi oleh

$$y = \sqrt{x-1}, \ y = 0, \ x = 5,$$

dengan menggunakan kaedah

(a) cakera/pembasuh　　　　　　　　　(b) kerang silinder.

[15 markah]

9.　**Benar** atau **Salah**.

(a) Jika f selanjar pada $[a,b]$, maka terbitannya juga selanjar pada $[a,b]$.

(b) Jika f' selanjar pada $[0,2]$, maka $\int_0^2 f(x)\,dx = f'(2) - f'(0)$.

(c) $\int_{-1}^{1} \frac{1}{x} dx = \ln x \big]_{-1}^{1} = \ln|1| - \ln|-1| = 0$.

(d) $\frac{d}{dx}\sin(x°) = \frac{d}{dx}\sin\left(\frac{\pi x}{180}\right) = \frac{\pi}{180}\cos\left(\frac{\pi x}{180}\right) = \frac{\pi}{180}\sin(x°)$.

(e) Jika f terbezakan dan $f(-1) = f(1)$, maka terdapat suatu nombor c sedemikian $|c| < 1$ dan $f'(c) = 0$.

[10 markah]

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